# Pressure drop in pipe lines for compressed air: comparison between experimental and theoretical analysis 

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## Abstract

In this paper a test methodology to determine the flow parameters, such as pressure drop and flow-rate, in straight pipes for compressed air is presented.
The experimental test were carried out using a properly instrumented test bench and the curves pressure drop vs. flow-rate, varying the upstream pressure, were obtained. The tests were made on pipes with different internal diameters, corresponding to common industrial size.
A theoretical analysis of the pressure drop losses was carried out taking into account Blasius and Prandtl formulations. This approach provides a formula that, considering the roughness and the internal diameter of the pipe, the test conditions (pressure, temperature, density), allows us to calculate the pressure losses at different flow-rate.
The formula permits us to calculate the theoretical curves (pressure drop vs. flow-rate) which are in a good agreement with the experimental ones.

## 1 Introduction

A pneumatic network must be provided to distribute the compressed air from a compressor installation to the various points of consumption with low pressure losses.

In particular, it is important to evaluate the pressure losses along the line that can be provoked by the length of the tubes or by the connecting elements. In this way the user can size up the line itself.

The evaluation of the flow-rate parameters is possible by an experimental investigation, using an instrumented test rig, as well as by means of theoretical formulations.

To establish which approach is more reliable it is necessary to compare the experimental and theoretical data. Several approaches, both theoretical and experimental, allow us to determine the pressure drop as a function of the flow-rate, for a certain upstream pressure, taken into account the pipes wall friction as well as the pipes roughness (Reynolds ${ }^{1}$, Barenblatt ${ }^{2}$, Zagarola ${ }^{3}$, AA.VV. ${ }^{4}$ ).

In this paper a formula which calculates the pressure losses along straight pipe lines, based on a proper theoretical formulation, is presented and the theoretical and experimental curves are superimposed.

## 2 Measuring test bench and test procedure

The experimental tests were carried out using the test bench shown in figure 1.

The test rig (Belforte ${ }^{5}$ ) has a diaphragm flow measurement $D$ and it allows the automatic calculation of the flow-rate with the acquisition of: temperature $T$, pressure upstream of the diaphragm $p_{u}$ and the pressure drop $\Delta p_{d}$ between upstream and downstream the diaphragm (International Standard ${ }^{6}$ ). The pressure regulator $R_{l}$ allows us the regulation of the pressure $p_{u}$, whereas the pressure regulator $R_{2}$ allows us to establish the pressure $p_{l}$.

The element under test $E$ is inserted between two pressure measurement tubes, to regularise the flow before the pressure measuring point $p_{1}$ e $p_{2}$; this procedure is established by the International Standard ${ }^{7}$.

The tests were carried out fixing the pressure $p_{l}$, and by means of the control valve $L$ the flow is regulated. The flow-rate $Q_{N}$ through the valve and the pressure drop $\Delta p=\left(p_{1}-p_{2}\right)$ across the element are recorded.

The flow-rate $Q_{N}$ measured by the test rig is referred to the standard conditions ANR (International Standard ${ }^{8}$ ).

The configuration of the test bench was chosen to reach a maximum flow-rate of $12000 \mathrm{dm}^{3} / \mathrm{s}$ (ANR) and a maximum pressure of 12 bar. This configuration was obtained according to the International Standard ${ }^{7}$ that establish the procedure to test pneumatic components.


Figure 1: Measuring test bench

The pipes considered in this paper are aluminium and with an average roughness of $1 \mu \mathrm{~m}$ and can be considered to being close to smooth pipes.

Tubes with internal diameter $D$ equal to $25 \mathrm{~mm}, 32 \mathrm{~mm}, 50 \mathrm{~mm}$, 63 mm and a total length $L_{0}=5 \mathrm{~m}$ have been considered.

During the experimental tests particular attention has been put on the pressure measuring point, according to the International Standard ${ }^{6}$. $L_{l}=10 \cdot D$ is the distance between the inlet of the flow and the pressure tapping to measure the upstream pressure; $L_{2}=3 \cdot D$ is the distance between the pressure tapping to measure the downstream pressure and the outlet of the flow.

Figure 2 shows a scheme of the pipe; it is possible to note the pressure tappings where the pressures $p_{1}$ and $p_{2}$ are measured. The pressure drop (or pressure loss) $\Delta p=\left(p_{1}-p_{2}\right)$ experimentally measured is relative to the length $L$, that depends on the internal diameter of the tube D.

In particular, $L$ equal to $4,675 \mathrm{~m} ; 4,584 \mathrm{~m} ; 4,350 \mathrm{~m}$ and $4,181 \mathrm{~m}$ relative to diameters $25 \mathrm{~mm}, 32 \mathrm{~mm}, 50 \mathrm{~mm}$ and 63 mm were considered.


Figure 2: Pressure tappings on the pipe

## 3 Theoretical formulation

Different formulations suitable for calculating the pressure drop in pipe lines for compressed air which are referred to (Reynolds ${ }^{1}$, Barenblatt ${ }^{2}$, Zagarola ${ }^{3}$, AA.VV. ${ }^{4}$ ). These can be theoretical or empirical and both are referred to the case of turbulent flow in rough pipes.

The different approaches are based on expression (1), which is useful either in laminar and turbulent flow:

$$
\begin{equation*}
\Delta p=p_{1}-p_{2}=P_{1}-P_{2}=\lambda \cdot \rho \cdot \frac{w^{2}}{2} \cdot \frac{L}{D} \tag{1}
\end{equation*}
$$

where: $\Delta p$ is the pressure drop in a pipe of length $L$ and diameter $D ; \lambda$ is the dimensionless friction coefficient; $\rho$ is the fluid density and $w$ is the mean value of the flow velocity. $P$ and $p$ are, respectively, the absolute and relative pressure.

In eqn. (1) the air medium velocity $w$ is related to the flow-rate by the eqns. (2) and (3):

$$
\begin{align*}
& G=\rho \cdot Q_{V}=\rho \cdot \frac{\pi \cdot D^{2}}{4} \cdot w  \tag{2}\\
& G=\rho_{N} \cdot Q_{N} \tag{3}
\end{align*}
$$

where: $G$ is the mass flow-rate, $Q_{\mathrm{V}}$ is the volume flow-rate in operative conditions and $Q_{N}$ is the volume flow-rate in standard conditions (ANR). The air density in operative conditions $\rho$ can be related to the air density in standard conditions by means of:

$$
\begin{equation*}
\rho=\rho_{N} \cdot \frac{P_{1}}{P_{N}} \cdot \frac{T_{N}}{T} \tag{4}
\end{equation*}
$$

where: $\rho_{N}, P_{N}, T_{N}$ are, respectively, the density, the pressure and the temperature of the air in standard conditions (International Standard ${ }^{8}$ ). Substituting eqn. (4) into eqn. (2) and by means of eqn. (3) we obtain the following expression for the air medium velocity:

$$
\begin{equation*}
w=\frac{4 \cdot Q_{N}}{\pi \cdot D^{2}} \cdot \frac{P_{N}}{P_{1}} \cdot \frac{T}{T_{N}} \tag{5}
\end{equation*}
$$

where: $Q_{N}, P_{l}, T$ are directly measured by the test bench.
By means of eqn. (5) the eqn. (1) becomes:

$$
\begin{equation*}
\Delta p=\lambda \cdot \frac{8 \cdot L \cdot \rho_{N} \cdot Q_{N}^{2}}{\pi^{2} \cdot D^{5}} \cdot \frac{T}{T_{N}} \cdot \frac{P_{N}}{P_{1}} \tag{6}
\end{equation*}
$$

In eqn. (6) it is important to define correctly the friction coefficient $\lambda$ defining its dependence on the other parameters defining the pipe flow. The dimensionless form of this relationship is:

$$
\begin{equation*}
\lambda=\lambda\left(\operatorname{Re}, \frac{\varepsilon}{D}\right) \tag{7}
\end{equation*}
$$

where: $R e$ is the Reynolds number, $\varepsilon$ is the pipe wall roughness and $\varepsilon / D$ is the relative roughness. The dependence of $\lambda$ on the parameter $\varepsilon / D$ is negligible in the case of smooth pipes. According to Blasius, for values of Reynolds number up to 80.000, the dimensionless friction coefficient $\lambda$ can be obtained from:

$$
\begin{equation*}
\lambda=0.3164(\mathrm{Re})^{-\frac{1}{4}} \tag{8}
\end{equation*}
$$

The Reynolds number for a cylindrical pipe (Reynolds ${ }^{1}$ ) of constant cross section can be calculated as follows:
$\operatorname{Re}=\frac{4 \cdot \rho_{N} \cdot Q_{N}}{\pi \cdot D \cdot \mu}$
The air viscosity $\mu$ was computed by means of the following relationship:

$$
\begin{equation*}
\mu=\left(1.84-\left(\frac{300-T}{300}\right)\right) \cdot 10^{-5} \tag{10}
\end{equation*}
$$

Which has been obtained considering the data from (CRC Handbook ${ }^{9}$ ).

In the case of high Reynolds numbers, up to $10^{8}$, Prandtl derived theoretically the value of $\lambda$ for smooth pipes as follows:

$$
\begin{equation*}
\lambda=\frac{1}{(2 \log (\operatorname{Re} \sqrt{\lambda})-0.8)^{2}} \tag{11}
\end{equation*}
$$

The disadvantage of this formula is that $\lambda$ is implicit. Using both Blasius and Prandtl formulations it is possible to obtain a general expression of $\lambda$ which has a good approximation for Reynolds numbers up to $10^{8}$ namely:

$$
\begin{equation*}
\lambda=\frac{1}{\left(2 \log \left(0.5625 \operatorname{Re}^{\frac{7}{8}}\right)-0.8\right)^{2}} \tag{12}
\end{equation*}
$$

Substituting eqn. (12) in eqn. (6) it is possible to calculate the pressure drop versus the standard flow-rate. The theoretical calculated
values of $\Delta p$ can be compared to the values obtained by the experimental tests.

To have a better approximation for pipes which are not smooth, the eqn. (12) of $\lambda$ was modified by a corrective coefficient $c$, obtaining:

$$
\begin{equation*}
\lambda=\frac{1}{\left(2 \log \left(0.5625 \operatorname{Re}^{\frac{7}{8}}\right)-0.8-c \operatorname{Re}\right)^{2}} \tag{13}
\end{equation*}
$$

where $c$ is a dimensionless coefficient which depends on the pipe diameter. In particular $c$ is equal to: $1.110^{-6}, 0.910^{-6}, 0.610^{-6}, 0.510^{-}$ 6 for pipes diameter $25 \mathrm{~mm}, 32 \mathrm{~mm}, 50 \mathrm{~mm}, 63 \mathrm{~mm}$ respectively.

This coefficient takes into account that the pipes under test have a very little roughness; this explains the dependence of $c$ on the pipe diameter and its decreasing as the diameter increase.

By means of eqn. (6), using for $\lambda$ the eqn. (13), the theoretical data fit with good approximation the experimental data over the entire range of flow-rates considered.

Moreover the pressure losses of the pipes can be calculated using the following empirical expression proposed in (AA.VV. ${ }^{4}$ ):

$$
\begin{equation*}
\Delta p=1.6 \cdot 10^{3} \cdot Q_{N}^{1.85} \cdot \frac{L}{D^{5} \cdot P_{1}} \tag{14}
\end{equation*}
$$

Eqn. (14) is referred to compressed air pipework in general made up of commercial steel pipe.

## 4 Experimental and theoretical results

The experimental and theoretical results were obtained with an upstream pressure $p_{I}$ equal to 4,6 and 8 bar (relative pressure). The curves show the pressure drop $\Delta p$ vs. the flow-rate $Q_{N}$.

In figures $3,4,5$ and 6 are shown the curves for the four diameters considered. In every figure the dots are the experimental data while the continuous lines have been obtained from the theoretical formulation (eqns. (6) and (13)).

For every size of tubes it is possible to note that the curves have a regular behaviour. Obviously, the pressure loss decreases for the same flow-rate and upstream pressure if the pipe diameter increases. At equal diameter and flow-rate the pressure loss decreases if the upstream pressure increases.

The good fitting of experimental and theoretical results encourages the authors in the use of the proposed formulation for working pressure different from the tested one.


Figure 3: Pressure drop vs. flow-rate for $\mathrm{D}=25 \mathrm{~mm}$


Figure 4: Pressure drop vs. flow-rate for $\mathrm{D}=32 \mathrm{~mm}$

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Figure 5: Pressure drop vs. flow-rate for $\mathrm{D}=50 \mathrm{~mm}$


Figure 6: Pressure drop vs. flow-rate for $\mathrm{D}=63 \mathrm{~mm}$


Figure 7: Pressure drop vs. flow-rate for $\mathrm{D}=32 \mathrm{~mm}$

In figure 7 the results obtained with eqn. (14) compared with experimental data for a pipe of $D=32 \mathrm{~mm}$ and length $L=4,584 \mathrm{~m}$ are referred. The dots are referred to the experimental data, the continuous lines were obtained with the theoretical formulation (eqns. (6) and (13)) and the dashed lines were obtained by eqn. (14).

## 5 Conclusions

In an air distribution pipe line it is important to evaluate the pressure losses along the line itself, which can be provoked by the length of the pipes or by the connecting elements. For this reason is necessary to estimate the flow characteristics of the line.

A test methodology to measure the pressure drop losses and the flow-rate, as a function of the upstream pressure, in straight pipes for compressed air was fitted out. In this paper results of experimental tests are presented.

Different pipes, with sizes corresponding to common industrial applications, were tested and the experimental results were used to verify an empirical formula, based on Blasius and Prandtl theory. This formula, which considers the roughness and the internal diameter of the
tube, the test conditions (pressure, temperature, density), allows to calculate the pressure losses at different flow-rate.

The theoretical curves (pressure drop vs. flow-rate) reproduce, a good approximation the corresponding experimental ones; this was verified for every size of tubes and in different test conditions. Moreover the formula can be used to evaluate the pressure losses for different size and in other operating conditions.

This formula seems to be a useful tool for study aimed at designing industrial pipework where establishing the pressure drop losses is of fundamental importance.

## References

[1] Reynolds A.J., "Thermofluid dynamics", J.Wiley \& Sons, Budapest, 1971.
[2] Barenblatt G.I., Chorin A.J., "Scaling laws for fully developed turbulent flow in pipes", Appl.Mech.Rev., ASME, vol.50, n.7, pp.413-429, july 1997.
[3] Zagarola M.V., Smits A.J., Orszag S.A., Yakhot V., "Experiments in high Reynolds number turbulent pipe flow", 34th Aerospace Sciences Meeting \& Exhibit, Reno, pp.1-8, January 15-18, 1996.
[4] AA.VV., "Atlas Copco Air Compendium", Stoccolma, 1975.
[5] Belforte G., D'Alfio N., Ferraresi C., "Banco prova computerizzato per valvole pneumatiche", Convegno Oleodinamica - Pneumatica Amma, n.6, 1986.
[6] International Standard ISO 5167, "Mesure de débit des fluides au moyen de diaphragmes, tuyères et tubes de Venturi insérés dans des conduites en charge de section circulaire", 1980.
[7] International Standard ISO 6358, "Pneumatic fluid power. Components using compressible fluids. Determination of flow-rate characteristics", 1987.
[8] International Standard ISO 8778, "Transmission pneumatiquesAtmosphere normale de reference", 1990.
[9] CRC Handbook of Chemistry and Physics, CRC Press, Florida USA, 1981.

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