Experimental and numerical investigation of cellular Stokes flow in a special wedge-shaped channel bounded by a cylindrical surface

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Abstract

Cellular Stokes flow induced by rotation of a circular cylinder in a wedge-shaped channel bounded by a cylindrical surface is analysed experimentally and numerically. Flow visualization experiments have been carried out in the wedge-shaped channel. In the numerical calculations, series in terms of polar coordinates are used to represent the stream function. Numerical solution of the biharmonic equation has been obtained for this geometry. Streamline patterns are obtained for different wedge lengths. Excellent agreement is found between the experimental and the corresponding numerical results.

1 Introduction

Many problems of physical interest involve the flow of fluids at low Reynolds numbers. In recent literature much work has been devoted to the study of cavity flow, to separation phenomena and to viscous cells. The most studied case is naturally a cavity of a rectangular shape. This type of flow in which viscous cells occur is important not only from a fundamental point of view, but is encountered in many practical applications, cited in Shen & Floryan, Higdon and Rybicki & Floryan. Since the geometry is simple and finite, the problem has also been an important test case for numerical schemes.

Although the rectangular cavity flow has been investigated extensively, publications are very scarce on the subject of the triangular cavity. In fact, the triangular shape is more common in practice. For example, it is easier to mill a triangular groove, which is wider at the opening, than a square one. Also, the study of triangular grooves is important in the design of fluid flow across corrugated boundaries. Such corrugations are required due to flexibility (see Savvides & Gerrard) or the enhancement of heat transfer (see Sparrow & Charmchi). The former also includes the medical application of the work. Ribbens, Watson & Wang investigated numerically steady recirculating
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viscous flow inside an equilateral triangular cavity generated by translating one side.

It is worthwhile to note that the result for the square cavity cannot be applied to the triangular cavity (also to the cavities of complex shape). On the other hand, from the experimental point of view, the gap in the literature is more serious. There is one visualization photograph of Taneda\(^7\), who visualized the first two cells in the flow confined by two \(28^\circ5'\) intersecting planes and driven by a rotating cylinder at the fluid surface. Thus, there is a need for knowledge of the detailed flow structure both numerically and experimentally in a wedge-shaped cavity and this motivated the present work.

The purpose of the present investigation is to show, both experimentally and numerically the global structure of the flow field induced by a rotating cylinder placed in a wedge-shaped channel bounded by a cylindrical surface. Flow visualization experiments have been carried out in the wedge-shaped channel. In the calculations, series in terms of polar coordinates are employed to represent the stream function and the quadratic minimization method is employed to satisfy the outer boundary conditions. Streamline patterns are obtained for different wedge lengths. Excellent agreement is found between the experimental and the corresponding numerical results.

2 Formulation of the problem

Let us consider an infinitely long vertical circular cylinder of radius \(R_1\), placed in a wedge-shaped channel bounded by a cylindrical surface of radius \(R_2\), as shown in Figure 1.

![Figure 1: Horizontal cross-section of the channel.](image)

The cylindrical surface and the wedge are matched at \(A\) and \(A'\). So there are no sharp corners at \(A\) and \(A'\). The distance between the centre of the cylinder and the corner is denoted by \(y_o\). The wedge angle is \(2\alpha\). The confining aspect ratio \(R\) is \(R_1/R_2\).

The reason for the selection of this special geometry channel is two-fold. First, it contains only one singular point at \(P\). Second, each value of \(y_o\)
corresponds to a different wedge angle \(2\alpha\). This permits to investigate the influence of the wedge-angle on the cellular structure in the channel.

This channel is filled with a highly viscous fluid of constant physical properties; its kinematic viscosity is denoted by \(\nu\). The motion is obtained by a very slow rotation of the inner cylinder with a uniform angular velocity \(\omega_o\). Thus the Reynolds number of the flow, defined by

\[
\text{Re} = \frac{(\omega_o R_1^2)}{\nu}
\]

is supposed to be sufficiently small. Under these conditions, the Stokes regime hypothesis is valid and the equation of motion in the horizontal cross-section is

\[
\nabla^4 \psi = 0
\]

where \(\psi\) is the stream function.

The radial coordinate \(r\) and the radial and the tangential velocity components \(V_r\) and \(V_\theta\) are normalized by \(R_1\) and \(\omega_o R_1\) respectively.

The boundary conditions are no slip on the inner cylinder:

\[
V_r = 0, \quad V_\theta = 1
\]

no slip on the outer channel walls:

\[
V_r = 0, \quad V_\theta = 0
\]

and the symmetry condition related to the antisymmetric behaviour of the flow:

\[
\psi(r,\theta) = \psi(r,-\theta)
\]

### 3 The numerical results

The numerical solution of the biharmonic equation (2) subject to the boundary conditions (3)-(5) has been obtained. The details of the solution will not be given here and can be found in Kent \(^8\).

In Figure 2, the streamline patterns are given for \(y_o = 7.5\), \(2\alpha = 30^\circ\) and \(R = 0.5\). Figure 3 presents the streamline patterns for \(y_o = 2.5\), \(2\alpha = 86^\circ\) and \(R = 0.5\).

Hancock \(^9\) and Hancock \(^10\) indicated an interesting result concerning the shape of the dividing streamline. He showed that for a corner angle of \(2\alpha\) less than about \(69^\circ 4'\) the separating streamline would be concave outwards. As \(2\alpha\) increases the dividing streamline will be convex outwards. In Figure 3 the corner angle \(2\alpha\) is greater than \(69^\circ 4'\) and the shape of the dividing streamline is convex outwards. On figure 2 it is concave outwards. This indicates the high precision of our numerical results.
Figure 2: The streamline patterns for $y_o=7.5$, $R=0.5$ and $2\alpha=30^\circ$.

Figure 3: The streamline patterns for $y_o=2.5$, $R=0.5$ and $2\alpha=86^\circ$. 
4 Experimental results

The apparatus constructed to visualize the flow is shown in Figure 4. The channel is filled with a highly viscous silicon oil of viscosity 300 cm²/s. The motion is obtained by the rotation of the circular cylinder at a constant speed of \( \omega_0 = 3 \) r.p.m. Thus the Reynolds number is about 0.001. The visualization is carried out using solid tracers of magnesium of about 40 \( \mu \text{m} \) in length and 4 \( \mu \text{m} \) in thickness. The solid traces are illuminated by a thin sheet of light coming from a laser device. By means of very long time exposure, we visualize the flow in the channel.

![Experimental apparatus diagram](image)

Figure 4: The experimental apparatus.

In Figure 5 the visualization photograph is presented. The exposure time for this picture is 15 minutes.

Many other visualization photographs can be found in Kenf⁸.

5 Comparison of the numerical and experimental results

By comparing Figure 2 with the corresponding visualization photograph of Figure 5, excellent agreement is found between the experimental and numerical results. Both the location of the separating line between the main current and the first cell, and the first cell centre are almost identical on these figures.

The agreement between the numerical results and the corresponding experimental results indicates that the experimental flow is effectively two dimensional in the region of interest.
Experimental and numerical analysis of cellular Stokes flow induced by rotation of a cylinder in a wedge-shaped channel bounded by a cylindrical surface has been made. The main purpose is to visualize the cellular motion in the wedge-shaped region and to determine the structure of streamline pattern numerically. The selection of this special geometry permits to investigate the influence of the wedge angle on the flow structure in the channel.

By comparing the numerical results with the corresponding visualization photographs, excellent agreement has been found. Concave-convex outwards of the shape of the dividing streamline indicated by Hancock and Hancock are consistent with our numerical results. This indicates the high precision of our numerical calculations.

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References


