Solution of inverse problem for reservoir permeability

S. P. Panawalage, M. Rahman, J. Biazar & M. R. Islam
Department of Engineering Mathematics & Oil and Gas Research Group, Faculty of Engineering, Dalhousie University, Canada

Abstract

A precise knowledge of permeability is crucial for accurate simulation of reservoir performance and enhanced reservoir management. At present permeability is measured by taking a core sample to a lab and measuring flow rates, but this is both expensive and less accurate because there can be significant problems with measurement error, and the value is measured only at the point of the sample. The best way to resolve this problem is to obtain in-situ permeability with the aid of reservoir characteristics by means of solving differential equations which govern the fluid flow in porous media.

Keywords: Adomian decomposition method, inverse problem, machine technique, reservoir permeability, well data.

1 Introduction

We discussed mathematical models of permeability, which are of the “inverse” type; that is, the behavior of the reservoir flow is assumed known, as well as the initial condition. A model which calculates the permeability of reservoir is called inverse (backward) type. It is highlighted that due to complexity, there is a lack of interest to researchers in the inverse problem of inferring the permeability of the porous media from flow data. Among different techniques, a machine technique has been put forward to overcome the traditional complexity that is usually arisen in solving typical inverse problem. This work is intended to solve the inverse problem easily that would be a great advancement of knowledge in reservoir characterization.
2 Governing mathematical equations of porous flow

A porous medium is a solid with holes in it. In this section we summarize the governing flow equations for movement of fluid through porous media which are used to solve inverse problem for reservoir permeability. In 1856, a French hydraulic engineer named Henry Darcy defined this rock characterization mathematically and experimentally. Darcy’s experiment resulted in the formulation of a mathematical law that describes fluid motion in porous media and, Darcy’s mathematical equation can be expressed in the following diffusion form by assuming permeability and viscosity are constant over pressure, time, and distance ranges [3]

\[
\frac{\partial p}{\partial t} = \frac{k}{\phi \mu c_t} \left[ \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \right] \tag{1}
\]

It is well established in oil and ground-water flow that velocities are generally sufficiently low such that Darcy’s law is usually valid as the law of flow. Therefore, Darcy’s law cannot describe fluid flow accurately when the flow rate is high it is called non-Darcy flow. In 1901, Forchheimer added a non-Darcy term to Darcy’s law to adequately describe gas flow behavior at higher flow rates. Forchheimer’s modified equation can be written in diffusion-Cartesian form [3]

\[
\frac{\partial p}{\partial t} = D \left[ \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} \right] - \frac{v}{\phi} \left[ \frac{\partial p}{\partial x} + \frac{\partial p}{\partial y} + \frac{\partial p}{\partial z} \right] \tag{2}
\]

Above diffusion equation (2) can be written in cylindrical coordinates system

\[
\frac{\partial p}{\partial t} = D \left[ \frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} + \frac{1}{r^2} \frac{\partial^2 p}{\partial \theta^2} + \frac{\partial^2 p}{\partial z^2} \right] - \frac{v}{\phi} \left[ \frac{\partial p}{\partial r} + \frac{p h(\theta)}{r} + \frac{\partial p}{\partial z} \right] \tag{3}
\]

where

\[f(\theta) = \cos \theta + \sin \theta, h(\theta) = \cos \theta - \sin \theta, \quad D = 1/c_1\{(\mu/k) + 2\beta \rho v\},\]

\(p\) is the reservoir pressure, \(\beta\) is the non-Darcy coefficient, \(\phi\) is the porosity, \(\rho\) is fluid density, \(v\) is fluid flowing velocity, \(c\) is the compressibility of flowing fluid, \(c_t\) is the total compressibility and, \((x, y, z)\) & \((r, \theta, z)\) are the Cartesian and Cylindrical coordinates respectively.
3 Mathematical formulations

The reservoir is assumed to be cylindrical shape (Figure 1) and it is symmetrical about its axis. The well is located at centre of the cylindrical reservoir. Let us consider differential equation (3), which governs the typical reservoir flow in three-dimensional cylindrical coordinates and, the initial condition of (3)

\[ p(r, \theta, z, 0) = g(r, \theta, z) \quad \text{\{initial condition\}} \]

where \( p(r, \theta, z, t) \) denotes reservoir pressure.

![Fluid flowing in right circular cylinder shape reservoir.](image)

Figure 1: Fluid flowing in right circular cylinder shape reservoir.

Recently a great deal of interest has been focused on the application of Adomian’s decomposition method, well addressed in [1,2], to solve a wide variety of stochastic and deterministic problems. This method is discussed the solution of both linear and non-linear differential equations as convergent series. To solve (3) by Adomian decomposition method, first we must derive canonical form of this equation. To derive canonical form, let us apply the inverse operator of \( \int_0^1 \partial/\partial t \) to (3), hence obtain

\[ p(r, \theta, z, t) = g(r, \theta, z) + N(p) \tag{4} \]

where

\[
N(p) = \int_0^1 \left[ \frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} + \frac{1}{r^2} \frac{\partial^2 p}{\partial \theta^2} + \frac{\partial^2 p}{\partial z^2} - \frac{v}{\phi} \left( \frac{\partial p}{\partial r} (f(\theta)) + \frac{\partial p}{\partial \theta} \left( \frac{h(\theta)}{r} + \frac{\partial p}{\partial z} \right) \right) \right] dt
\]
As usual in the Adomian decomposition method, the solution \( p \), is considered as the summation of a series like

\[
p = \sum_{n=0}^{\infty} p_n
\]

and \( N(p) \), is considered as

\[
N(p) = \sum_{n=0}^{\infty} A_n(p_0, p_1, \ldots, p_n)
\]

where \( A_n(p_0, \ldots, p_n) \) are called Adomian polynomials.

Substituting (5) and (6) into (4), and using an alternate algorithm for computing Adomian polynomials, the following scheme would be defined \[1, 2\]

\[
p_0 = g(r, \theta, z)
\]

\[
p_{n+1} = \int_0^t \left\{ D \left[ \frac{\partial^2 p_n}{\partial r^2} + \frac{1}{r} \frac{\partial p_n}{\partial r} + \frac{1}{r^2} \frac{\partial^2 p_n}{\partial \theta^2} + \frac{\partial^2 p_n}{\partial z^2} \right]
\]

\[
- \frac{\partial}{\partial \theta} \left[ \frac{\partial p_n}{\partial r} (f(\theta)) + \frac{\partial p_n}{\partial \theta} (h(\theta)) + \frac{\partial p_n}{\partial z} \right] \right\} dt, n = 0, 1, \ldots
\]

Hence, an analytical approximated solution can be derived from the following series

\[
p(r, \theta, z, t) = g(r, \theta, z) + t[D \times g_1(r, \theta, z) + g_2(r, \theta, z)] + t^2[D^2 \times g_3(r, \theta, z) + D \times g_4(r, \theta, z) + g_5(r, \theta, z)] + \ldots
\]

Depending on the accuracy desired we could choose as many terms as necessary.

Similarly an analytical approximation solution of (2) is easily obtained by Adomian decomposition method as follows

\[
p(x, y, z, t) = g(x, y, z) + D^0[g_a(x, y, z, t)] + D^1[g_b(x, y, z, t)] + D^2[g_c(x, y, z, t)] + D^3[g_d(x, y, z, t)] + \ldots + D^{n+1}[g_*(x, y, z, t)]
\]

where \( p(x, y, z, 0) = g(x, y, z) \) \{initial condition\}

4 Solution techniques of inverse problem by an analytical approximation method

In general, there is a lack of interest to researchers in the inverse problem of inferring the permeability of the porous media from flow data because of complexity. A machine technique has been put forward with the aid of Adomian decomposition method to overcome the traditional complexity that is usually arisen in solving typical inverse problem. Estimation of the permeability from
observed well data is attempted via matching technique and is summarized as follows.

- Utilizing various solution techniques of differential equations, solve (analytically or numerically) partial differential equations that govern the fluid flowing through porous media.
- Obtain reservoir permeability when matching analytic or numerical solution with observed well test data.

It is noted that this method is quite simple and estimated permeability fairly represents whole area of reservoir measured. The matching technique seems to be the most powerful method for further developments to achieve precise permeability for wide classes flow behaviors in the porous media. At this point taking into account both analytic approximated solutions (7 or 8) and observed well test data for the same position \((r_1, \theta_1, z_1)\), or \((x_1, y_1, z_1)\) and put forward inverse solution by means of matching technique which is summarized in following five steps.

1. Complete standard plot such as \(p(r_1, \theta_1, z_1, t)\) vs \(t'\) by means of observed well data at the location \((r_1, \theta_1, z_1)\).
2. Varying “D” for the promising limit, construct analytic plots such as \(p(r_1, \theta_1, z_1, t)\) vs \(t\) for the same location \((r_1, \theta_1, z_1)\) by means of equation (7).
3. Obtain \(D_{\text{Match}}\) when both analytic and standard plots match each other.
4. Calculate reservoir permeability \((k)\) from the equation,
   \[D_{\text{Match}} = \frac{1}{\phi c} \left\{ \frac{\mu}{k} + 2\beta \rho \nu \right\} \text{ throughout non-Darcy flow}\]
5. Letting \(v = 0, c = c_i\) and repeating above procedures obtain reservoir permeability from the equation \(k = D_{\text{Match}} \times c_i \phi \mu\) when the flow obeys Darcy’s law.

### 4.1 Numerical examples

In this part, two examples are presented to illustrate the overall idea. Fluid flowing in the reservoir is considered to obey Darcy law and observed well test data is available at location \((1,1,100)\). Firstly, obtain standard plot from given observed well test data as, i.e, \(p(1,1,100,t)\) vs \(t\) (time). Then, consider equation (8) for analytic plot at the same location \((1,1,100)\); (Note: \(v = 0, c = c_i, D = k / \mu \phi c_i\) for the Darcy flow [3])

\[\dot{p}(x, y, z, t) = g(x, y, z) + D[g_i(x, y, z, t)] + \ldots \cdot D^{n+1}[g_{n+1}(x, y, z, t)],\]

where \(p_0 = g(x, y, z)\), {known initial condition}
\[
\frac{p_{n+1}}{D} = \left[ \frac{\partial^2 p_n}{\partial x^2} + \frac{\partial^2 p_n}{\partial y^2} + \frac{\partial^2 p_n}{\partial z^2} \right] dt = D^{n+1} \times g_{n+1}(x, y, z, t)
\]

Now, studying observed well test data (or standard plot) obtain initial condition-
p(x, y, z, 0) = p_0 = g(x, y, z) as follows

Example 1: Assume standard plot, \( p(1,1,100, t) \) vs \( t \) is linear, let
\[
p(x, y, z, 0) = ax^2 + bx + cy^2 + dy + ez^2 + fz
\]
Hence obtain
\[
p_0 = ax^2 + bx + cy^2 + dy + ez^2 + fz
\]
\[
p_1 = 2D[a + c + e]t
\]
\[
p_2 = 0
\]
Therefore, analytic solution can be obtained from Eqn. (8) as
\[
p(x, y, z, t) = ax^2 + bx + cy^2 + dy + ez^2 + fz + 2D[a + c + e]t
\]
Then, obtain analytic solution for the observed well location (1,1,100) as
\[
p(1,1,100, t) = a + b + c + d + 10000e + 100f + 2D[a + c + e]t
\]
Now, utilizing knowledge on observed well test data (or standard plot) estimate constants \( a, b, c, d, \) and \( e \) for analytic plot.

e.g. take \( a=b=c=d=-0.2010 \) and \( e=f=0.4011 \), then simplifies (11) as;
\[
p(1,1,100, t) = 4050 - D[1.8 \times 10^{-3}]t
\]
Now varying “\( D \)” for possible range, complete analytic plot (Figure 2) using (12). Then varying “\( D \)” and obtain \( D_{\text{Match}} \) by matching analytic plot (equation 12) with standard plot (observed well test data). Hence obtain reservoir permeability, \( k = D_{\text{Match}}[\mu\phi\psi_t] \).

Example 2: Assume standard plot, "\( p(1,1,100, t) \) vs \( t \)" not linear, and let
\[
p(x, y, z, 0) = [ax^4 + bx^3 + cx^2 + dx] +
\]
\[
[ey^4 + fy^3 + gy^2 + hy] + [iz^4 + jz^3 + kz^2 + mz]
\]
Similarly obtain an analytic solution for the observed well location (1,1,100) from (8)
\[
p(1,1,100, t) = A + [D \times B]t + [D^2 \times C]t^2
\]
Figure 2: Plot of $p(1,1,100, t)$ vs $t$, for linear form by (12).

where

$$A = [a + b + c + d + e + f + g + h + 100^4 i + 100^3 j + 100^2 k + 100m]$$

$$B = [12a + 6b + 2c] + [12e + 6f + 2g] + [12i \times 100^2 + 600j + 2k]$$

$$C = [12a + 12e + 12i]$$

As before, utilizing knowledge on standard plot (or well test data), estimate constants $A$, $B$ and $C$ for analytic plot. Hence assume

$$A = 5000, B = -4.16666 \times 10^{-4} & C = -6.944444 \times 10^{-10}$$

Similarly to example 1, changing “$D$” for promising range, complete analytic plot (Figure 3) using condition (14).

Finally, obtaining $D_{\text{Match}}$ when analytic plot matches standard plot and, then calculate reservoir permeability

$$k = D_{\text{Match}} [\phi \mu c, i]$$

It is highlighted that this approach is highly sensitive to accuracy of initial condition. Therefore, thorough knowledge on observed well data is vital to estimate initial condition accurately.

Note: For examples 1 & 2, following unit were considered; $k$ (md), $p$ (psia), $c_t$ (psi$^{-1}$), $t$ (hrs), $\phi$ (fraction), $\mu$ (cp), and $(x, y, z)$ (ft).
4.2 Estimation of variation of reservoir permeability field

It is clearly understood that permeability is not quite constant and it may show significant variance over the space. But, it makes such a dilemma when it is considered directly as variable. Therefore, to overcome this drawback; initially assume constant permeability for the specific zone, i.e., observed well area. Thus, solve inverse problem for the specific zone and obtain permeability for said zone which is called spot permeability. Similarly, obtain spot permeability for different known well locations by solving inverse problem with aid of observed well data. Hence, accuracy of estimated permeability can be improved by taking average value of spot permeability.

Otherwise, examine variation of spot permeability throughout the area observed and then, reservoir permeability field would be expressed with respect to space. This can be done by easily plotting of spot permeability against space. Thus, relationship of specific permeability field can be put forward accurately by analyzing spot permeability using knowledge of mathematics/computer or both.

5 Conclusion

To be able to predict the reservoir's performance or to optimize reservoir's production, the determination of reservoir properties such as permeability is first required. The permeability is deterministic, typically in laboratory but the experimental determination of the permeability for reservoir engineering scale is difficult and less than perfect; generally relying on a highly simplified Darcy flow analysis. The best way to resolve this problem is to obtain in-situ permeability with aid of reservoir characteristics by means of solving differential equations which governs the fluid flow in porous media. Analytic analysis could
greatly improve the resolution, reduce the testing costs, and allow reliable parametric investigation over the reservoir.

It is noted that during the last few years there has been an increasing tendency to solve inverse problem but much complicated way. This study highlighted that matching techniques have been put forward to overcome the limitations of traditional inverse method. Consequently, this work indicates some encouragement to tackle inverse problem effectively.

This work summarized mathematical models of permeability, which are of the “inverse” type: that is, the behavior of the reservoir flow is assumed known, as well as the initial condition. A model which calculates the permeability of reservoir is called inverse (backward) type. This research effort to discover comfort way to solve inverse problem that would be a great extent in reservoir characterization. Accuracy of this method can be enhanced by utilizing experience and thorough knowledge on reservoir pressure behavior.

References