Aerodynamics of a moving curveball in Newtonian flow

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Abstract

We propose a dynamical system of the interaction between a ball and a viscid flow in two-dimensional space. It's the combination of Navier-Stokes equations for the fluid and Newton's 2nd law for the ball. The ball's trajectory is unknown and determined by the dynamical system. The author has successfully rewritten the dynamics in coordinates moving with the ball and it makes numerical simulations possible. The curved flight path of the ball is obtained and so is the behavior of the flow around it.

1 Introduction

It is well-known that the trajectory of a moving ball is not a straight line if we spin the ball initially. This is so-called Magnus effect. The first scientist to comment on the curved flight path of sports balls is Sir Isaac Newton. Newton [7] (1672) had noted how the flight of a tennis ball was affected by spin and his explanation is: "For, a circular as well as a progressive motion..., its parts on that side, where the motions conspire, must press and beat the contiguous air more violently than on the other, and there excite a reluctancy and reaction of the air proportionably greater.". In 1686, Newton published Philosophiae Naturalis Principia Mathematica. Newton changed the science forever: he set up the first mathematical system to describe the dynamics of universe. However, he still can't explain the curved flight path from his three basic laws of motion. The explanation of a curveball was still philosophy.

The association of this effect with the name of Magnus was due to Rayleigh. His paper [8] in 1877 is credited as the first "true explanation" of the effect. Since the introduction of the boundary-layer concept by Prandtl in 1904, the Magnus effect has been attributed to asymmetric boundary-layer separation. On the other hand, Kutta and Joukowsky used complex
variables to describe the irrotational steady inviscid flow and their theorem states that the drag force is zero and the side force is proportional to the circulation around the object and its velocity.

However, the explanation by asymmetric boundary-layer separation is still philosophy. Kutta-Joukowski theorem is correct for a very special case only. Without viscosity, the spin of a ball or cylinder will never produce circulation. Furthermore, the flow around a moving ball is not irrotational and steady.

In the modern era, we can use computers to simulate the behavior of fluids. Many people had computed the motion of the 2D Navier-Stokes flow around a moving 2D ball (cylinder) when the ball’s constant velocity and rotational speed are specified. Mathematically, the velocity and rotational speed of the ball “serve” as the constant boundary conditions of the fluid velocity in Navier-Stokes equations. This approach can give us pressure and stress tensor around the ball and we can get both drag and side forces. Magnus effect was thus explained or verified by computation experiments.

But, how about the trajectory of a moving curveball? The velocity and rotational speed of a ball are not constants all the time. Furthermore, the changes of the velocity and rotational speed are not given. They’re determined by the behavior of the fluid around the ball for every moment. Let’s go back to where Magnus effect is from originally: the sports balls. Of course a baseball or tennis player wants to know the motion of the ball more than the fluid around it.

In fact, the curved flight path is caused by the complicated interaction between the ball and the fluid. Another classic problem about interaction between an object and a fluid is the oscillation of a falling paper. Until now, there is still no perfect explanation for this phenomenon. However, some simplified models are proposed. Tanabe & Kaneko [9] (1994) assumed that the drag force is proportional to the velocity and used Kutta-Joukowski theorem to compute the motion of a falling paper. Thus the velocity and rotational speed of a falling paper can determine the drag and side forces and it becomes a closed dynamical system “without” the fluid – the fluid just produces the drag and side forces to the falling paper. We can also use this approach to compute the trajectory of a curveball. But it’s not proper to combine the results from two different kinds of flows. Even though they claimed their model might be a crude approximation, this paper was criticized by Mahadevan, Aref & Jones [5] (1995) and Tanabe & Kaneko [10] tried to defend their model by claiming “we constructed our model as simply as possible by composing forces that we thought essential and should not be omitted.”.

The general problem of interaction between a solid body and an inviscid flow has been studied for over a century following the work by Kelvin and Kirchhoff around 1870 (see Lamb [4]). However, a general theory for the interaction between a rigid body and viscid flow is still not available. Like what the author mentioned, we can’t get the curved trajectory without
viscosity because the spin of a ball won’t change the behavior of the flow around it.

Motivated by the curved path of sports balls, the author would like to consider the motion of a ball and an viscous fluid around it simultaneously. We don’t want to “construct” a new model here. We use Navier-Stokes equations for the fluid and Newton’s 2nd law for the ball to set up the dynamics system. Up to the no-slip condition, the fluid and ball’s velocities on the boundary should be the same. However, It's extremely complicated to consider the dynamics in three-dimensional space. So, We try to figure out how to handle the interaction in two-dimensional space here. The “ball” in this paper should be a cylinder floating in a fluid in our three-dimensional world. We find the 2D dynamical system is still very complicated but fortunately we have figured out how to handle it. We really get a curveball in the numerical simulations.

2 The dynamics system

The dynamics system (PDEs for the fluid and ODEs for the ball) we study here is:

\[ u_t + (u \cdot \nabla) u = -\nabla \left( \frac{p}{\rho} \right) + \nu \Delta u \]  
\[ \nabla \cdot u = 0 \]  
\[ q_t = \nu \]  
\[ M v_t = - \int_{\partial B} (p n - \sigma \cdot n) \, ds + Mg + F \]  
\[ I \omega_t = R \int_{\partial B} s \cdot (\sigma \cdot n) \, ds + L \]  
\[ \sigma = \rho \nu \left( \nabla u + (\nabla u)^T \right) \]

where \( u, p, \rho \) are the velocity, pressure and constant density of the fluid. \( q, v, \omega, M, I, R \) are the position, velocity, rotational speed, mass, inertia tensor and radius of the ball. \( \nu \) is the viscosity constant. \( s \) and \( n \) are tangent and normal of the ball’s boundary. \( F \) and \( L \) are the force and torque from other sources (for example, from a pitcher’s hand). Up to the no-slip condition, the fluid and ball’s velocities on the boundary should be the same:

\[ u_{|\partial B} = v + R \omega s \]  
\[ u_{|\infty} = 0 \]

The vorticity \( \xi = \nabla \times u \) satisfies

\[ \xi_t + (u \cdot \nabla) \xi = \nu \Delta \xi \]
Note that the trajectory $q(t)$ of the ball is not given - it’s determined by this complex system, and that is the difficulty to handle the system because the boundary between the ball and fluid is moving and the motion is unknown. So, we’ll try to rewrite the system in coordinates moving with the ball to “fix” the boundary.

3 Coordinates transformation

Even though $q(t)$ is unknown, let’s try this coordinates transformation (and remember it’s not a Galileo transformation):

$$\begin{cases}
x' = x - q(t) \\
t' = t
\end{cases}$$

$$\partial_t = \partial_{t'} - v \cdot \nabla', \nabla = \nabla'$$

Let $u' = u - v$ be the fluid velocity in the moving coordinates, we can rewrite the PDEs as

$$(u' + v)_{t'} + (u' \cdot \nabla') u' = -\nabla \left( \frac{p}{\rho} \right) + \nu \Delta u'$$

$$\nabla \cdot u' = 0$$

$$u'|_{\partial B} = R \omega s$$

$$u'|_{\infty} = -v$$

Note that $\xi = \nabla \times u = \nabla' \times u'$. Eqn (4) is not changed in the new coordinates system:

$$\xi_{t'} + (u' \cdot \nabla') \xi = \nu \Delta \xi$$

Now the boundary is fixed and it’s straightforward to use complex variables and polar coordinates:

$$Re^s e^{i\theta} = x_1' + ix_2'$$

Let $\psi'$ be the stream function in the moving coordinates. Eqn (6) becomes

$$\xi_{t'} + \frac{\psi'_s \xi_{\theta} - \psi'_\theta \xi_s}{R^2 e^{2s}} = \nu \Delta \xi$$

$$\Delta \psi' = \xi$$

$$\Delta = \frac{1}{R^2 e^{2s}} (\partial_s^2 + \partial_{\theta}^2)$$

We would like to use eqn (7) and the stream-function vorticity methods to compute the fluid around the ball. However, we need the pressure on
the boundary (see eqn (2)) for $v_t$. It doesn't worth to solve a Poisson's equation to evaluate pressure everywhere when we just need pressure on the boundary. Let's look at the "only" term including $p$ we need:

$$\int_{\partial B} p\,ds = R \int_0^{2\pi} p_{\partial B} e^{i\theta} d\theta = iR \int_0^{2\pi} p_\theta |_{\partial B} e^{i\theta} d\theta$$

(integral by part)

Because $p_\theta |_{\partial B} = R\nabla p |_{\partial B} \cdot s$, by eqn (5), we can get

$$-R^2\omega_t + iR \cdot \text{Re} \left( e^{-i\theta} v_{t'} \right) = \frac{p_\theta |_{\partial B}}{\rho} - \nu \xi_s |_{\partial B}$$

(9)

On the other hand, we can get

$$\sigma \cdot n |_{\partial B} = i\rho \nu e^{i\theta} (\xi |_{\partial B} - 2\omega)$$

(10)
in polar coordinates. We can rewrite eqns (2) and (3) as

$$(M - \pi R^2 \rho) v_{t'} = iR \rho \nu \int e^{i\theta} (\xi - \xi_s) |_{\partial B} d\theta + F$$

(11)

$$I \omega_t = R^2 \rho \nu \int \xi |_{\partial B} d\theta - 4\pi R^2 \rho \nu \omega + L$$

(12)

The extra term $-\pi R^2 \rho v_{t'}$ in eqn (11) is "produced" from the integral of pressure around the ball.

Integral eqn (9) over $\theta$, we can get

$$2\pi R^2 \omega_t = \nu \int \xi_s |_{\partial B} d\theta$$

(13)

It's weird that $\omega_{t'}$ is determined by two different equations when $v_{t'}$ is determined by one only. We'll see the reason why later.

So, we have successfully rewritten the dynamical system in a moving polar coordinates by eqns (7), (8), (11), (12) and (13). Furthermore, we can see that the integrals in eqns (11), (12) and (13) are Fourier transformation of $\xi$ and $\xi_s$ over $\theta$. So, we can rewrite the system as (the symbol $'$ is ignored)

$$R^2 e^{2s} \xi_t = \nu \left( \xi_{ss} - n^2 \xi^n \right)$$

(14)

$$R^2 e^{2s} \xi_t = \nu \xi^n - n^2 \xi^n$$

(15)

$$(M - \pi R^2 \rho) v_t = 2\pi i \rho \nu \left( \xi^{-1} - \xi_s^{-1} \right) |_0 + F$$

(16)

$$I \omega_t = 2\pi R^2 \rho \nu \left( \xi_0 |_0 - 2\omega \right) + L$$

(17)

$$R^2 \omega_t = \nu \xi_0 |_0$$

(18)
The boundary conditions for $\tilde{\psi}^n$ are listed in the table:

| $n$ | $\psi^n|_0$ | $\psi^n_s|_0$ | $e^{-s}\tilde{\psi}^n|_s \to \infty$ | $e^{-s}\tilde{\psi}^n_s|_s \to \infty$ |
|-----|-------------|-------------|-------------------------------|-------------------------------|
| 0   | 0           | $R^2\omega$ | Unknown                       | 0                            |
| -1  | 0           | 0           | $iR\nu/2$                     | $iR\nu/2$                     |
| 1   | 0           | 0           | $-iR\nu/2$                    | $-iR\nu/2$                    |
| Others | 0           | 0           | 0                             | 0                            |

We can see that $e^{-s}\tilde{\psi}^0|_s \to \infty$ is unknown when $e^{-s}\tilde{\psi}^n|_s \to \infty$ are known for all other $n$. That’s probably why we need two equations for $\omega_t$ to close the system.

Now the boundary is fixed and we don’t need to integral $\xi$ around the ball for every time step to get $v_t$ and $\omega_t$. We can get the trajectory of the ball by integral the velocity $v$ over $t$ numerically.

4 Numerical simulations

Let’s pick up parameters and give the dynamics a try. After we nondimensionalize the system, $R$, $\nu$, $\rho = 1$ and only two parameters are left: $M$ and $I$. If the ball has uniform density, $M$ and $I$ are determined by one parameter – density $d$ of the ball only. What we expect is to see a curveball and see how Magnus effect works. By Newton’s 2nd law, if we want to increase the change of direction, we need to decrease $d$. So, we choose $d = 2$ (when the density $\rho$ of the fluid is 1). We pick up $v_0 = 100$ and $\omega_0 = 50$ and the simulations are amazing: we get a curveball which changes its direction for over 360º. We also get the vorticity of the fluid for every moment. See figures 1 and 2 for the trajectory and vorticity contour.

Figure 3 is a close-up shot of the vorticity contour when $t = 0.15$. An animated movie, colored pictures and more close-up shots are available in the web site http://www.csm.ornl.gov/~huang/curveball.htm.

5 Conclusions

We have figured out how to handle the interaction between a rigid body and a fluid in a special case and got some interesting simulations. The skills presented in the paper show the possibility to compute the interaction between a two-dimensional viscous flow and a rigid body without “constructing” a simplified model like what Tanabe & Kaneko [9] did. The author believes it’s possible to compute the motion of a falling paper by skills similar to what have been developed here.

Even though this research is inspired by baseball, what we have done is still far away from simulating a real moving baseball in computer. It’ll be extremely complicated if we want to handle the system in three-dimensional space: the vorticity will become a vector; singularity in spherical polar coordinates will probably cause trouble..... Even though we can figure it out, remember that a baseball is not a perfect sphere – there are strings around it. There is still no explanation why pitchers can pitch knuckleball, slider,
Figure 1: The trajectory and vorticity contour (1)
Figure 2: The trajectory and vorticity contour (2)
Figure 3: Close-up shot of vorticity contour. $t = 0.15$
split-finger fastball, forkball, sinker...... Some physicists tried to understand the trajectory of a baseball by experiments – for example, Briggs [3] for a curveball; Watts & Sawyer [11] for a knuckleball. It’s still a long road to simulate a real moving baseball and explain how the erratic changes of direction happen.

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References


