Parametrizing hybrid RANS-LES approaches with the renormalization group

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Abstract

A two-equation turbulence model that can adjust itself from RANS to LES regimes is constructed using the renormalization group. When the grid-spacing gets too coarse for LES, the model tends to a RANS model. In well-resolved regions the model results in a LES subgrid-model. The renormalization group procedure leads to transport-equations that are dependent on the filterwidth and contain no adjustable constants.

1 Abstract

Hybrid RANS-LES simulations (also called 'Very Large Eddy Simulations' (VLES)) are a relatively new approach to simulating turbulent flows. Very Large Eddy Simulations can be seen as Large Eddy Simulations in which the cut-off wavelength of the filter does not necessarily lie in the inertial range. An obvious advantage over LES is that the grid spacing, at a wall for instance, can be much coarser. In VLES, one can put the cut-off at a wall in the energy containing range. This means that the subgrid model in a VLES should transform into a RANS model when the filterwidth gets coarse enough. The renormalization group (RNG) technique seems to be well suited to derive such a model from first principles. Namely, in RNG one builds models by means of an iterative procedure that removes the smallest scales from a system and mimics the effect of these removed scales on the larger by adjusting some transport coefficients. To get a LES subgrid model one lets the iteration procedure continue until all the scales below some wavenumber in the inertial range are removed. A RANS model is obtained when one iterates all the way up to the energy containing scales. The consequence of this procedure for VLES is that with RNG, model coefficients can be obtained that depend on the cut-off wavenumber of the iterative procedure, so that the model naturally adjusts itself from a LES regime (when the cut-off corresponds to a wavenumber in the inertial range), to a RANS regime (when the cut-off corresponds to a wavenumber in the
energy containing range). In other approaches to VLES ([1, 2]) generally the model coefficients do not have this cut-off wavenumber dependence. In [3] there is a dependence of the filterwidth in the $\overline{K}$- and $\overline{\varepsilon}$-transport equations, but the constants in these equations are calibrated with RANS computations.

2 The RNG procedure

In this section a short description of the renormalization group will be given. Only the general idea will be explained. For the actual application of the method, the reader is referred to the literature ([4, 5, 6]), and the construction of the VLES-model presented below is explained in [7].

The renormalization group is a general method for eliminating modes from a system with a large number of degrees of freedom. In turbulence this means for instance that the small scales are removed from the turbulent field, and the effect of this loss in information has to be accounted for by adjusting the transport parameters of the retained modes (the large scales). The first step RNG technique that was used by Yakhot and Orszag consists of removing the finest (dissipation-range) scales from the turbulent velocity field, with a subsequent (analytically prescribed) altering of a turbulent transport-coefficient, in this case the viscosity. This step is based on the fact that the dissipation-range is strongly affected by the molecular viscosity, so that the nonlinear term is small in comparison with the diffusive term, and the small-scale velocity field can be linearized. As the result of performing the first step is again the Navier-Stokes equation, but with a higher viscosity, the procedure can be iterated, the result being a model for the large scales.

Although the YO-RNG procedure is not a completely rigorous one (many assumptions were made in their derivations), it was remarkably successful in deriving turbulence models. It was shown in [5] that two-equation turbulence models and large-eddy subgrid models can be constructed directly from the Navier-Stokes equations, without having to rely on ad-hoc modelling. As was already mentioned in the introduction, to obtain a LES-subgrid model the iteration was stopped in the inertial range, and to obtain a RANS model the iteration continued until the energy containing range was reached. In the present work, the cut-off can be in both the inertial or the energy containing range. This makes the model capable of adjusting itself from LES to RANS regimes and vice versa, as will be shown below.

3 The model

In this section the RNG VLES-model as constructed in [7] is summarized. The three important wavenumbers in the model are the 'integral wavenumber' $\Lambda_e$ corresponding to the integral length-scale, the 'cut-off wavenumber' $\Lambda_c$ corresponding to the cut-off of the iterative procedure (which we identify with the wavenumber corresponding to the width of the VLES filter) and the 'initial wavenumber' (the UV cutoff) $\Lambda_0$ corresponding to the molecular length scales where the RNG iteration procedure is started from (see Fig.1). These quantities will be estimated further in this section.
The effective viscosity $\nu$ is given by

$$\nu(\Lambda_c) = \nu_0 \left( 1 + \frac{0.119 \bar{\varepsilon}}{\nu_0^3} (\Lambda_c^{-4} - \Lambda_0^{-4}) \right)^{1/3}. \tag{1}$$

The transport equations for the mean turbulent kinetic energy $\overline{K}$ and the mean rate of dissipation $\bar{\varepsilon}$ of the subgrid scales are

$$\frac{D\overline{K}}{Dt} = P_R - \bar{\varepsilon} + \frac{\partial}{\partial x_i} \left( \alpha(\Lambda_c) \nu(\Lambda_c) \frac{\partial \overline{K}}{\partial x_i} \right)$$

$$\frac{D\bar{\varepsilon}}{Dt} = \frac{6}{5} \nu(\Lambda_c) \Lambda_c^2 P_R - \nu^2(\Lambda_c) (D_1 + D_2)$$

$$+ \frac{\nu(\Lambda_c)S^3(1 - \eta/\eta_0)}{1 + \beta \eta^3} + \frac{\partial}{\partial x_i} \left( \alpha(\Lambda_c) \nu(\Lambda_c) \frac{\partial \bar{\varepsilon}}{\partial x_i} \right) \tag{3}$$

where $\alpha(\Lambda_c)$ is related to $\nu(\Lambda_c)$ by

$$\begin{vmatrix} 1.3929 - \alpha \ \ \ \ \ \ \ \ \ \ 0.63 \ \ \ \ \ \ \ \ \ \ 0.37 \\
0.3929 \ \ \ \ \ \ \ \ \ \ 2.3929 - \alpha \ \ \ \ \ \ \ \ \ \ \ \ \ 3.3929 \\
\end{vmatrix} = \frac{\nu_0}{\nu(\Lambda_c)}. \tag{4}$$

and $D_1$ and $D_2$ are given by

$$D_1 = \frac{10 \Lambda_c^2}{\nu(\Lambda_c)} \left( \bar{\varepsilon} - \frac{3}{2} C_K \nu(\Lambda_c) \bar{\varepsilon}^{2/3} \left( \Lambda_c^{4/3} - \Lambda_c^{4/3} \right) \right)$$

$$D_2 = \frac{6}{5} C_K \bar{\varepsilon}^{2/3} \left( \Lambda_c^{10/3} - \Lambda_c^{10/3} \right). \tag{5}$$

$D_1$ and $D_2$ contain extra terms that don’t occur in the RNG RANS model. These extra terms vanish identically when $\Lambda_c = \Lambda_e$, i.e. in the RANS limit. The constants $\eta_0$ and $\beta$ are ([8]) $\eta_0 = 4.38$ and $\beta = 0.012$, and the Kolmogorov constant.

Figure 1: A typical energy spectrum of a turbulent flow. Indicated are the integral and initial wavenumbers.
\( C_K \) as determined by the Yakhot and Orszag RNG theory is 1.615. The cut-off wavenumber is simply related to the width of the VLES-filter \( \Delta \) as

\[
\Lambda_c = \frac{\pi}{\Delta},
\]

and the initial wavenumber was estimated by Yakhot and Orszag to be

\[
\Lambda_0 = 0.2 \left( \frac{\varepsilon}{\nu_0^3} \right)^{1/4}.
\]

To estimate the integral wavenumber \( \Lambda_e \) the formula

\[
\Lambda_e = \left( \frac{3}{2} C_K \right)^{3/2} \frac{1}{\tau^{3/2} \varepsilon_t^{1/2}}
\]

can be used. Here the turbulent timescale \( \tau \) is defined as

\[
\tau = \frac{K_t}{\varepsilon_t} + \sqrt{\frac{\nu_0}{\varepsilon_t}}
\]

to avoid singularities near the wall. To evaluate (8) and (9) the total RANS \( \varepsilon_t \) and \( K_t \) are needed. These can be reconstructed from the VLES computation in the following way [3]. One decomposes the RANS \( K_t \) and \( \varepsilon_t \) in a resolved contribution \( (K_r \text{ and } \varepsilon_r) \) and a modelled one \( (K \text{ and } \varepsilon) \):

\[
K_t = K_r + \bar{K}, \quad \varepsilon_t = \varepsilon_r + \bar{\varepsilon}.
\]

The \( K_r \)- and \( \varepsilon_r \)-parts can be directly constructed from the computed velocity field and \( \bar{K} \) and \( \bar{\varepsilon} \) follow from their transport equations.

4 Discussion

The only term that is different from the standard RANS RNG model is the destruction term in the transport equation of the mean dissipation rate. All the other terms are in fact the same ones as in the RANS RNG model, but written in a form where the cut-off wavenumber dependence is kept. One sees from the equations above that the model indeed goes from DNS in the limit \( \Lambda_c \to \Lambda_0 \) (according to (1) then \( \nu \to \nu_0 \)) and to RANS in the limit \( \Lambda_c \to \Lambda_e \). More specifically, when in (1) \( \Lambda_c = \Lambda_e \) is taken, and the assumptions \( \Lambda_0 \ll \Lambda_e \) and \( \nu_0 \ll \nu(\Lambda_e) \) are made (i.e. a high Reynolds number is assumed), one gets for the effective viscosity (using (8))

\[
\nu(\Lambda_e) = 0.085 \frac{\bar{K}^2}{\bar{\varepsilon}}.
\]

Under the same conditions it can easily be checked that (2) and (3) reduce to their RANS versions derived in [5] and [9]: \( D_2 \) vanishes, \( D_1 \) becomes equal to
$-1.68\bar{e}^2/\bar{K}$ and the production term in the $\bar{e}$-equation becomes $1.42P_{\bar{e}}\bar{e}/\bar{K}$. Furthermore, when $v_0 \ll \nu(\Lambda_e)$, (4) leads to $\alpha \approx 1.39$. The reader can verify that this is exactly the RANS model as described in [9]. In the region $\Lambda_e < \Lambda_c < \Lambda_0$ one has an effective viscosity that can be used as a subgrid model for LES, which is driven by transport equations with transport-coefficients that adapt themselves to the filterwidth cut-off wavenumber.

5 Results

The first testcase is a simulation of turbulent flow over a backward-facing step at $Re = 5100$, based on the step height. When $h$ denotes the step-height, an entry section of length $10h$ was put before the step, and the section behind the step measures $20h$. The spanwise direction is $4h$ wide. The dimensionless distance from the wall for the nearest grid cell is approximately 1, based on the friction velocity at the end of the region behind the step. The dimensionless streamwise spacing ranges between 1 and 125. The uniform spanwise spacing $(\Delta z^+)$ is around 20. The total mesh consists of approximately 480000 cells. To show the LES-behaviour of the model, the coherent structures behind the step are shown in Fig.2. More specifically, an isosurface of a positive value of the second invariant of the rate of strain-tensor, $Q = 1/2(\Omega_{ij}\Omega_{ij} - S_{ij}S_{ij})$ is shown here, which is known to depict well regions of low pressure. In the plot we can see how the vortex rolls that are shed behind the step are broken up by streamwise streaks, a typical feature of this flow. In Fig.3 the skin-friction coefficient behind the step is shown in comparison with the DNS results of Le et al. [10]. One sees that the reattachment length is well predicted. The obtained length of 6.4 is within 2% of the DNS value. The Strouhal-frequency $S = f h / U_0$ of the vortex shedding behind the step is approximately 0.073, which is also well within the experimentaly obtained range (Eaton and Johnston found that the spectral peak occurs in the Strouhal number range $0.066 < S < 0.08$, [11]). The amplitudes of the skin-friction coefficient are not completely satisfying yet, which is probably due to the near-wall behaviour of the model: for this calculation no use was made of any damping function, which can lead to problems in the coarse-mesh regions.

The second testcase is channel flow at $Re = 7890$ based on half the channel width (corresponding to $Re_f = 395$). The turbulence production in this flow is completely determined by the walls, which makes the correct description of the near-wall region crucial. In a well resolved LES simulation this means that the span- and streamwise gridspacings should be such that they can accurately resolve the relevant physical phenomena in that region, i.e. the streamwise streaks. For a completely resolved LES this means that $z^+$ should be smaller than 30, and $x^+$ no more than about 80. In our simulation we use a much coarser grid near the wall $(\Delta x^+ \approx 100, y^+ \approx 1.5$ and $\Delta z^+ \approx 100$), and the VLES-model is in RANS-mode in that region. To obtain good near-wall behaviour of the RANS model, low-Reynolds modifications are necessary. The modifications as proposed by Yang and Shih [12] are used in this work. The grid counted approximately 9000 cells in total. The geometry was $l \times w \times h = 3H \times 2H \times 9H$. Shown in Fig.4 is the mean
Figure 2: Isosurfaces of $Q = 11$ show the coherent flow structures behind the step. The flow comes from the left and is shed at $x = 0$.

Figure 3: Skin friction coefficient behind the step. Symbols denote DNS data, the solid line the result of the VLES computation.
velocity profile, compared with the DNS-data of Kim et al. [13]. It is clear that the comparison is good.

6 Conclusions

A RNG $K-\varepsilon$ model was made dependent of a cut-off wavelength $\Lambda_c$. This form makes the model appropriate for Very Large Eddy Simulations, in which a turbulence model is desired that can go continuously from a LES-regime to a RANS-regime.

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References


