Compressible flow of saturated moist air with condensation phenomena

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Abstract

Compressible flow of saturated moist air with various condensation processes around thin airfoils is investigated. The study uses an extended transonic small-disturbance (TSD) model of Rusak and Lee [11] which includes effects of heat addition to the flow due to condensation. Two possible limit types of condensation processes are considered. In the nonequilibrium and homogeneous process, the condensate mass fraction is calculated according to classical nucleation and droplet growth rate models. In the equilibrium process, the condensate mass fraction is calculated by assuming an isentropic process. The flow and condensation equations are solved numerically by iterative computations. Results under same upstream conditions describe the flow structure, field of condensate, and pressure distribution on airfoils' surfaces. It is found that flow characteristics, such as position and strength of shock waves and airfoils' pressure distribution, are different for the two condensation processes. Yet, in each case, heat addition as a result of condensation causes significant changes in flow behavior and affects the aerodynamic performance of airfoils.

1 Introduction

Air in the lower levels of the atmosphere may be considered as a mixture of perfect gases which are dry air and water vapor. The gaseous components of moist air are thermally and calorically perfect gases, obey Dalton's law of partial pressures [1,2], and are characterized by the relative humidity, which is the ratio of water vapor
pressure to saturation pressure of water at the temperature of the mixture, \( 0 \leq \Phi = p_0/p_g(T) \leq 1 \). In most relevant cases of transonic flows, moist air reaches the saturation conditions (\( \Phi = 100\% \)) and condensation of water vapor may occur. Such a process causes changes in the flow field around airfoils which may result in large variations in the aerodynamic performance of airfoils [3,4].

Wegener [2] suggested that the condensation of water vapor may develop in one of two limit types of processes. The first type is an equilibrium process. It occurs when many foreign nuclei (i.e. dust, aerosols, and ions) exist as seeds of condensation and then the phase transition is relatively "slow". In this process, condensation starts immediately after water vapor reaches saturation conditions and it may be modeled as an isentropic process which evolves according to the local flow properties. This type of condensation is typical of transonic atmospheric flight. The second type of condensation is a nonequilibrium process [1-10]. This process usually happens in rapid expansions of highly purified vapors in supersonic nozzles and around airfoils in cloud chambers. For example, in this process the supersaturation ratio \( S = p_u/p_g(T) > 1 \) (an extension of the relative humidity concept into the saturation region) may increase much above one (\( S >> 1 \)) without any condensation. This may happen because the liquid droplets do not reach a critical size for growth and stay as water vapor. Only at a critical state, known as a supersaturation state, do the liquid droplets reach the critical size. A significant nucleation of water droplets is suddenly initiated by spontaneous fluctuations in the water vapor itself, known as homogeneous condensation.

In an equilibrium condensation process, the specific entropy of moist air along stream lines is assumed constant:

\[
s = (1 - \omega)s_a + (\omega - g)s_v + gs_L = \text{constant.} \tag{1}
\]

Here, \( \omega \) is local initial specific humidity and \( g \) is condensate mass fraction. \( s_a, s_v, \) and \( s_L \) are specific entropies of dry air, water vapor, and condensate, respectively. Using free-stream conditions and assuming \( s_v \sim s_g \) and \( s_L \sim s_f \), (1) reduces to:

\[
g = \frac{(1 - \omega_\infty)(C_{uv}\ln(p/p_\infty) - C_{v0}\ln(p/p_\infty))}{s_g(T) - s_f(T)} + \frac{\omega_\infty(s_g(T) - s_v_\infty)}{s_g(T) - s_f(T)}. \tag{2}
\]

In a nonequilibrium and homogeneous condensation process, the classical nucleation theory is used. Following Hill [5], the condensate rate equation can be generalized for a steady and two-dimensional flow, resulting in:

\[
(puQ)_x + (pvQ)_y = 4\pi\rho_L(\rho Q_1 \frac{d\rho}{dt} + \frac{1}{3} Jr_0^3), \tag{3}
\]

\[
(puQ)_x + (pvQ)_y = 2\rho Q_2 \frac{d\rho}{dt} + Jr_0^2, \tag{4}
\]

\[
(puQ)_x + (pvQ)_y = \rho Q_3 \frac{d\rho}{dt} + Jr_0, \tag{5}
\]

\[
(puQ)_x + (pvQ)_y = J. \tag{6}
\]

Here \( \rho \) is density of a mixture, and \( \rho_L \) is density of condensate, \( d\rho/dt \) is droplet growth rate, \( J \) is nucleation rate, and \( r_0 \) is initial radius of a nucleus. Also, \( Q_1, Q_2, \)
and \( Q_3 \) are the sum of droplet surfaces per unit mass, droplet radii per unit mass, and droplets per unit mass, respectively. These equations reflect the balance between convection of properties \( g, Q_1, Q_2, \) and \( Q_3 \) and production of the properties as it is related to \( J \) and \( d\tau /dt \). The nucleation rate equation \( J \) is given by:

\[
J = \mathcal{K} \exp \left( \frac{-W^*}{kT} \right), \quad \mathcal{K} = \sqrt{\frac{2\sigma_{\infty}(T)}{\pi}} \frac{\tilde{m}^{-3/2}}{\rho_0^2},
\]

\[
W^* = \frac{16}{3} \pi \left( \frac{\tilde{m}}{\rho_0 \ln(S) kT} \right)^2 \sigma_{\infty}^3(T).
\]

Here, \( k \) is Boltzmann constant, \( T \) is temperature, \( \sigma_{\infty} \) is surface tension of a plane surface, \( \tilde{m} \) is mass per molecule of water, \( S \) is supersaturation ratio, and \( \rho_0 \) is partial density of water vapor. Also, the droplet growth rate is given by [2-5] as:

\[
\frac{dr}{dt} = \frac{\alpha(T) p_v - p_g(T)}{\rho_L \sqrt{2\pi R_v T}}.
\]

Here \( \alpha \) is a condensation parameter given by empirical relations in [3], \( p_v \) is water vapor pressure, \( p_g \) is saturation pressure, and \( R_v \) is water vapor gas constant.

This paper studies the behavior of a compressible flow of moist air with various condensation processes around thin airfoils. It is based on the modified TSD model of Rusak and Lee [11] which includes effects of heat addition to the flow due to condensation. Asymptotic models for computing transonic flow behavior with the two condensation processes are presented. The numerical method to solve the problem is described and results of computations under the same upstream flow conditions are compared. It is found that flow characteristics, such as position and strength of shock waves and pressure distribution, are different in the two cases.

2 Asymptotic model

The theory [11] studied a transonic flow of a uniform stream of moist air with Mach number \( M_\infty \sim 1 \), temperature \( T_\infty \), pressure \( p_\infty \), and initial specific humidity \( \omega_\infty \ll 1 \) around a thin airfoil of chord \( c \), thickness ratio \( 0 < \epsilon \ll 1 \), and shape functions \( F_{u,i}(x/c) \) of the upper and lower surfaces. The flow may be described by a modified Kármán-Guderley equation for the velocity perturbation potential \( \phi_1 \):

\[
[K - (\gamma_0 + 1) M_\infty^2 \phi_1 x] \phi_1 \bar{x} + \phi_1 \bar{y} = \bar{g}_1 x K_\omega \left( \frac{h_{f3}(T_\infty)}{C_{p0} T_\infty} - \frac{\mu_0}{\mu_v} \right).
\]

Here \( \bar{x} = x/c, \bar{y} = \epsilon^{3/2} y/c \) is the stretched vertical coordinate, and \( K = (1 - M_\infty^2) / \epsilon^{3/2} \) is the classical transonic similarity parameter. The moist air similarity parameter \( K_\omega = \omega_\infty / \epsilon^{3/2} \) represents the small amount of water vapor in moist air in terms of airfoil’s small thickness ratio and \( g_1 \) is the condensate mass fraction.

The boundary conditions for solving Eq. (10) are the linearized tangency condition along the airfoil chord, the Kutta condition at the trailing edge, and the decay
of flow perturbations in the far field:

\[ \phi_{1\tau}(x, 0^\pm) = F_{\tau,\nu}(x) \quad \text{for} \quad 0 \leq x \leq 1, \]
\[ \phi_{1\tau}(1, 0^+) = \phi_{1\tau}(1, 0^-), \quad \phi_{1\tau} \rightarrow 0 \quad \text{as} \quad x \rightarrow -\infty. \]  

Eqs. (10) and (11) describe a transonic flow of moist air around a thin airfoil with a heat addition due to condensation. The condensation effects depend on the field of \( \tilde{g}_1 \) which is different in each one of the condensation processes.

When an equilibrium condensation process is considered, Eq. (2) gives:

\[ \tilde{g}_1 = s_g(T_\infty) - s_{v\infty} \left( \frac{d s_g}{dT} \right)_{\infty} \left[ 1 - \frac{s_g(T_\infty) - s_{v\infty}}{s_{fg}(T_\infty)} \right]. \]  

Note that the condensate mass fraction \( \tilde{g}_1 \) is related to the change of temperature in the flow which is given by \( \tilde{T}_1 = \tilde{\tau}_1 T_\infty \) and \( \Phi_\infty = 100\% \), \( s_{v\infty} = s_g(T_\infty) \) and then Eq. (12) is simplified to:

\[ \tilde{g}_1 = \frac{\tilde{T}_1}{s_{fg}(T_\infty)} \left( \frac{d s_g}{dT} \right)_{\infty} = \frac{1}{\gamma} \frac{T_\infty}{s_{fg}(T_\infty)} \left( \frac{d s_g}{dT} \right)_{\infty} \phi_{1\tau}. \]  

When a nonequilibrium and homogeneous condensation process is considered, Eqs. (3)-(6) show that \( \tilde{g}_1 \) is given by a set of ordinary differential equations:

\[ \tilde{\tau}_1 = 4\pi K_\tau \left( \frac{Q_{11}}{\omega} + \frac{1}{3} J \tau_0^3 \right), \]  
\[ \tilde{Q}_{11} = K_t \left( 2 \tilde{Q}_{21} \frac{d \tilde{\tau}}{dt} + J \tau_0 \right), \]  
\[ \tilde{Q}_{21} = K_t \left( \tilde{Q}_{31} \frac{d \tilde{\tau}}{dt} + J \tau_0 \right), \]  
\[ \tilde{Q}_{31} = K_t J. \]  

The similarity parameter of this condensation process is

\[ K_t = \sqrt{\frac{(\mu_0/\mu_v)^3}{8\pi \gamma_\alpha} \frac{p_{\infty} c_\rho \omega_{\infty}}{\sigma_{\infty}(T_\infty) M_{\infty}}}, \]  

which is the ratio of the characteristic time of the condensation process to the characteristic convection time of the flow. It relates between the airfoil’s chord, pressure and Mach number of the upstream flow, and initial specific humidity. Other non-dimensional parameters in Eqs. (14)-(17) are:

\[ \tilde{Q}_{11} = \frac{Q_1}{\omega_{\infty} / \rho L_\infty l_c}, \quad \tilde{Q}_{21} = \frac{Q_2}{\omega_{\infty} / \rho L_\infty l_c}, \quad \tilde{Q}_{31} = \frac{Q_3}{\omega_{\infty} / \rho L_\infty l_c}, \]  

where

\[ l_c = 2\sigma_{\infty}(T_\infty)/(\rho L_\infty R T_\infty), \]  

\[ \sigma_{\infty} = 2\pi \omega_{\infty} / (\rho L_\infty R T_\infty). \]
\[
\frac{d\bar{T}}{dt} = \frac{\alpha(T)}{\sqrt{T}} \left( \bar{p}(1 - \bar{g}_1) - K_{pg} \bar{p}_g(T) \right), \quad K_{pg} = 1/S_\infty, \quad (21)
\]

\[
\bar{r}_0 = \frac{1.3r^*}{l_c}, \quad r^* = \frac{2\sigma_\infty(T)}{\rho_l R_0 T \ln(S)}, \quad (22)
\]

\[
\mathcal{J} = \sqrt{\frac{2T}{32\pi^3}} n_c^{3/2} \bar{\rho}^2 \sqrt{\sigma_\infty(T)} (1 - \bar{g}_1)^2 \exp \left( -\frac{n_c \sigma_\infty^3}{2T^3 (\ln S)^2} \right), \quad (23)
\]

\[
n_c = (4\pi/3) \rho_{f,\infty} l_c^3/m, \quad S = S_\infty \bar{p}(1 - \bar{g}_1)/\bar{p}_g(T). \quad (24)
\]

The parameters \( \sigma_\infty(T) = \sigma_\infty(T)/\sigma_\infty(T_\infty) \) and \( \bar{p}_g(T) = p_g(T)/p_g(T_\infty) \) are the nondimensional plane surface tension and saturation pressure. The surface tension function \( \sigma_\infty(T) \) and the condensation coefficient \( \alpha(T) \) are given by the empirical relations according to Schnerr and Dohrmann [3]. The saturation pressure function \( p_g(T) \) is given by Pruppacher and Klett [12].

The condensation equations are subjected to no condensation at the far field:

\[
\bar{g}_1 = \bar{Q}_{11} = \bar{Q}_{21} = \bar{Q}_{31} = 0 \text{ as } \bar{x} \to -\infty. \quad (25)
\]

The solution of Eqs. (10) and (11) with either (13) or (14)-(24) and with condition (25) gives the fields of \( \phi_1 \) and \( \bar{g}_1 \) from which the fields of temperature, pressure and density can be computed according to:

\[
\bar{T} = 1 - e^{2/3}(\gamma_a - 1) M_\infty^2 \phi_1 \bar{x} + e^{4/3} \left[ \frac{K_{\omega}}{\frac{h_{fg}(T_\infty)}{C_{pa} T_\infty}} \bar{g}_1 - \frac{\gamma_a - 1}{2} M_\infty^2 \phi_{1z}^2 \right], \quad (26)
\]

\[
\bar{p} = 1 - e^{2/3} \gamma_a M_\infty^2 \phi_1 \bar{x} + 2e^{4/3} \gamma_a M_\infty^2 \phi_{1z}^2, \quad (27)
\]

\[
\bar{\rho} = 1 - e^{2/3} M_\infty^2 \phi_1 \bar{x} - e^{4/3} \left[ \frac{K_{\omega}}{\frac{h_{fg}(T_\infty)}{C_{pa} T_\infty}} - \frac{\mu_a}{\mu_v} \right] \bar{g}_1 - \frac{3\gamma_a + 1}{2} M_\infty^2 \phi_{1z}^2. \quad (28)
\]

The solution of the problem also gives the pressure coefficient: \( c_p = (p - p_\infty)/(1/2 \rho_\infty U_\infty^2) = -2e^{2/3} \phi_1 \bar{x} \), the field of the condensate mass fraction, \( \bar{g}_1 \), and the vapor pressure field, \( p_v = S_\infty p_g(T_\infty) \bar{p}(1 - \bar{g}_1) \).

3 Numerical method

The type of Eq. (10) may change according to the local flow and condensation properties. In order to accurately compute these changes, the type-sensitive difference scheme of Murman and Cole [13] is used in a numerical solution of the problem. According to this method, a test has to be devised to decide if a computational grid point is of elliptic type, hyperbolic type, or mixed type. At each step of the iterations, the computations use the suitable difference scheme at that point. We repeat the iterative procedure until the solution of Eq. (10)-(11) with either (13) or (14)-(24) and (25) converges to a steady state solution. Then, we compute the pressure coefficient along the airfoil’s surfaces and the field of the condensate mass fraction. A detailed description of the numerical scheme, extensive mesh convergence studies, and the close correlation of the results with benchmark solutions of [3,4] are shown in Rusak and Lee [11].
4 Numerical results

A uniform flow of saturated air with free-stream Mach number $M_\infty = 0.8$, temperature $T_\infty = 313$ K, pressure $p_\infty = 1$ atm, $\Phi_\infty = 100\%$, and $\omega_\infty = 0.0466$ around various airfoils at zero angle of attack is considered. The different characteristics between nonequilibrium and equilibrium condensation processes are studied.

Figures 1-3 show the computed results around an airfoil with chord $c = 4$ m for both condensation processes. At the given conditions, $K = 1.48$, $K_\omega = 0.789$, and $K_t = 1.16$ is used for nonequilibrium condensation. With respect to a dry air solution, the equilibrium condensation causes the shock wave near the mid chord to move downstream and become stronger whereas the nonequilibrium condensation forms a compression wave in front of the shock wave and causes it to slightly move upstream and become weaker (see Fig. 1). The condensate mass fraction for the equilibrium condensation is similar to $c_p$ when $c_p < 0$ and builds up gradually. For the nonequilibrium process, it suddenly appears at a certain point on the airfoil's chord and changes nonlinearly with $T$ (see Fig. 2). The change of the water vapor pressure as function of temperature along a streamline that runs close to the airfoil surface is shown in a $p - T$ phase diagram in Fig. 3. It can be seen that equilibrium condensation is reversible whereas nonequilibrium condensation is irreversible and the phase diagram is more complicated as condensation occurs.

![Figure 1](image)

Figure 1: The pressure distribution along a NACA0012 airfoil for an upstream flow with $\Phi = 100\%$, $T_\infty = 313$ K, and $p_\infty = 1$ atm.
Figure 2: The distribution of condensate mass fraction along a NACA0012 airfoil for an upstream flow with $\Phi = 100\%$, $T_\infty = 313K$, and $p_\infty = 1$ atm.

Figure 3: The vapor pressure as a function of temperature along a streamline that runs close to a NACA0012 surface for an upstream flow with $\Phi = 100\%$, $T_\infty = 313K$, and $p_\infty = 1$ atm.
Figures 4-6 provide another example on the comparison between the two condensation processes. A flow around a circular arc airfoil with thickness ratio $e = 0.1$ and chord $c = 4 \text{ m}$ is studied in which $K = 1.13$, $K_{\omega} = 1.00$, and $K_t = 0.53$ is used for the nonequilibrium condensation. It can be seen from Figure 4 that the shock wave for each process is shifted upstream against the dry air solution. A strong compression waves appears in front of shock wave in both cases. Figure 5 demonstrates that the maximum value of condensate mass fraction for the nonequilibrium condensation process is about 2.5 times higher than that for the equilibrium condensation and that most of the condensation for the nonequilibrium case occurs near the airfoil's rear part. Figure 6 demonstrates again that the equilibrium condensation process is reversible whereas the nonequilibrium process is irreversible.

**Figure 4:** The distribution of condensate mass fraction along a circular arc airfoil for an upstream flow with $\Phi = 100\%$, $T_\infty = 313 K$, and $p_\infty = 1 \text{ atm}$.

**Figure 5:** The distribution of condensate mass fraction along a circular arc airfoil for an upstream flow with $\Phi = 100\%$, $T_\infty = 313 K$, and $p_\infty = 1 \text{ atm}$.
Figure 6: The vapor pressure as a function of temperature along a streamline that runs close to a circular arc surface for an upstream flow with $\Phi = 100\%$, $T_\infty = 313 K$, and $p_\infty = 1 \text{ atm}$.

### 6. Conclusions

Compressible flow of saturated moist air with various condensation processes around thin airfoils can be investigated using an extended transonic small-disturbance model by Rusak and Lee [11] which includes the effects of heat addition to the flow due to condensation. Two possible limit types of condensation processes are considered. In the nonequilibrium and homogeneous process, the condensate mass fraction is calculated according to the classical nucleation and droplet growth rate models of Wegener and Mack [1] and Hill [5]. In the equilibrium process, the condensate mass fraction is calculated by assuming an isentropic process. The flow and condensation equations are solved numerically by iterative computations. Under the same upstream flow conditions, results of the flow structure, field of condensate, and pressure distribution on airfoils’ surfaces are described for the two condensation processes. It is found that the flow characteristics, such as the position and strength of shock waves and airfoils’ pressure distributions, are different in the two cases. Yet, in each case, heat addition as a result of condensation causes significant changes in the flow behavior and affects the aerodynamic performance of airfoils.
References


