Hydrodynamic interaction and coalescent motion of air-bubbles floating in liquid flow

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Abstract

A new theoretical method is proposed to estimate the hydrodynamic interaction among air-bubbles in a viscous liquid at low Reynolds numbers in relation to multi-phase flow in industrial applications. This method is a superposition method of Oseen's flow fields around each air-bubble, of which the shape is deformable during its motion. The experiment is carried out using a glycerol tank and high-speed video-cameras with 10000 frames/s.

Equal-sized air-bubbles located vertically with the same spacings at the initial time showed an interesting motion to make pairs of air-bubbles from the preceding two air-bubbles successively in turn. In these pairs of air-bubbles the following air-bubble approached the preceding air-bubble. During the approaching motion the shape of the preceding air-bubble became oblate, while the following air-bubble became prolate. Then the two air-bubbles contacted and coalesced finally. During the coalescent motion of two air-bubbles there were recognized an occurrence of several micro-bubbles.

The numerical results predicted by using the superposition method of Oseen's flow field agree well with the experimental results.

1 Introduction

The motion of small gas-bubbles in a liquid at low Reynolds numbers is not only a classical subject, but is directly related to such problems as multi-phase flows in industrial applications. Thus this subject has been treated theoretically by many authors. The analytical solutions to Stokes equation for a rigid sphere and
a single bubble (Stokes [1], Hadamard [2], Rybczynski [3], and so on) are well-known. Taylor and Acrivos [4] derived the matched asymptotic solution to Stokes’s solution near the bubble and to Oseen’s solution at infinity. Brabston and Keller [5] obtained the numerical solution to Navier-Stokes equation directly. Morrison and Stewart [6] derived the basic equation for the unsteady motion of a single bubble. The motion of particles, bubbles and drops are summarized systematically by Clift, Grace and Weber [7]. However there are left rooms to study the motion of a cluster of gas-bubbles in liquid under the consideration of hydrodynamic interaction among the gas-bubbles. It should be noted that the motion of solid particles is not predicted by using Stokes’s approximation even at the low Reynolds numbers below 0.5, but that the motion is predicted well by using Oseen’s approximation (Kumagai [8]).

In this paper the analytical solution to Oseen’s equation (Kumagai [9]) is newly applied to the superposition method of Oseen’s flow field. This method is considered to be the first approximation to the reflection method in the microhydrodynamics for the motion of solid particles (Happel and Brenner [10], Kim and Karrila [11], Kumagai [8],[12],[13]). The numerical results obtained by using the above-described superposition method of Oseen’s flow field are discussed with the experimental results for the motion of equal-sized air-bubbles located vertically with the same spacings at the initial time in glycerol. Equal-sized air-bubbles located vertically with the same spacings at the initial time in glycerol showed an interesting motion to make pairs of air-bubbles from the preceding two air-bubbles successively in turn (Kumagai et al [14]). In the motion of a pair of air-bubbles the preceding air-bubble became oblate, while the following air-bubble became prolate to contact with the preceding air-bubble. The two air-bubbles coalesced to form a single air-bubble. The approaching and coalescent motion of the pair of air-bubbles is discussed in detail by using the high-speed video-cameras with 10000 frames/s.

2 Basic equations and numerical procedures

The motion of several bubbles \((a,b,\cdots,n)\) is considered in the cartesian coordinates. The equation of motion for a spherical bubble \(j\) may be represented (Morrison and Stewart [6]) as

\[
\frac{\pi d_j^3}{6} \rho_j \frac{dU_{bj}}{dt} = \frac{\pi d_j^3}{6} \rho_j \frac{dU_{bj}}{dt} - \frac{1}{6} \frac{\pi d_j^3}{6} \rho_j \frac{dv_j}{dt} + k \frac{\pi d_j^3}{6} (\rho_i - \rho_j)g
\]

\[
-\pi \mu_j d_j v_j \frac{3 + 2\sigma_j}{Y_j} - \frac{d_j^2 \rho_j \sqrt{\pi \nu_j} \int_{t_s}^{t} \left( \frac{dv_j}{dt} \right)_{t-s} ds}{\sqrt{(t-s)}} ,
\]

\[
-\frac{d_j^2}{36} \rho_j \int_{t_s}^{t} \frac{d^2 v_j}{dt^2} (t-s) ds / \sqrt{(t-s)}
\]

where

\[
Y_j = 1 + \sigma_j - \frac{Re_j (3 + 2\sigma_j)}{16}, \quad \sigma_j = \frac{\mu_j}{\rho_j}.
\]
Here \( f \) denotes fluid, \( b \); bubble, \( d_j \); diameter of the bubble, \( \mu \); viscosity, \( \nu \); kinematic viscosity, \( \rho \); density, \( U_j \); velocity at the position \( j \), \( v_j (= U_{bj} - U_b) \); relative velocity of the bubble \( j \), \( \text{Re}_j (= |v_j| d_j / \nu) \); Reynolds number for the bubble \( j \), and \( k \); unit vector along \( z \)-axis. For non-spherical bubble we assume that the drag of bubble may be represented as \( \lambda \) times Oseen’s drag for the spherical bubble, where \( \lambda \) means the modified factor for the deformation of spherical bubble.

Here we will consider the freely-rising motion of several equal bubbles at low Reynolds numbers. The "equal bubbles" means bubbles with equal volume, density and viscosity. In this case we may assume the fluid velocity \( U_{bj} \approx 0 \). Since the inertia term in eqn (1) is equivalent to \( 10^{-3} \) of the gravitational force, the freely-rising motion of bubbles at low Reynolds numbers may be considered as quasi-steady. Under these assumptions the equation of motion may be simply described as

\[
k \frac{\pi d_j^3}{6} (\rho_j - \rho_b) g = \lambda \pi \mu_i d_j v_j \frac{3 + 2 \sigma_j}{Y_j}.
\]

(3)

Considering the hydrodynamic interaction among bubbles, the relative velocity of the bubbles \( j \) may be represented as

\[
v_j = U_j - v'_j = U_j - \sum_{k \neq j} v'_{kj} \quad (k \neq j).
\]

(4)

Here \( v'_{kj} \) means the fluid velocity at the position \( j \) produced by another bubble \( k \) translating with the relative velocity \( v_k \). For a single spherical bubble \( t \), which has equal volume, density and viscosity with the bubble \( j \), the quasi-steady motion of the bubble \( t \) may be described as

\[
k \frac{\pi d_t^3}{6} (\rho_t - \rho_b) g = \lambda \pi \mu_i d_t v_t \frac{3 + 2 \sigma_t}{Y_t}.
\]

(5)

Thus we may put \( v_j = v_t \) and we may assume that

\[
U_j = v_j + \sum_{k \neq j} v'_{kj} \quad (k \neq j).
\]

(6)

The local fluid motion around the bubble \( k \) is assumed to satisfy the quasi-steady Oseen’s equations:

\[
div \mathbf{v} = 0 \quad \text{and} \quad (\mathbf{v}_k \cdot \nabla) \mathbf{v} = -\frac{\mathbf{V} \rho}{\rho_i} + \nu \nabla^2 \mathbf{v}.
\]

(7)

The appropriate boundary conditions for these bubbles may be described as

\[
\mathbf{v} \rightarrow 0 \quad \text{at} \quad \mathbf{r} \rightarrow \infty, \quad \mathbf{v} = \mathbf{v}_a \quad \text{at} \quad \mathbf{r} = \mathbf{r}_a, \quad \cdots \cdots, \quad \mathbf{v} = \mathbf{v}_n \quad \text{at} \quad \mathbf{r} = \mathbf{r}_n.
\]

(8)

Since there are many boundary conditions to be satisfied simultaneously as shown in eqn (8), it is difficult to determine the exact solution to eqns (7) and (8). Thus the conventional method of reflections, a quite asymptotic method, under the assumption of Stokes’s approximation (Happel and Brenner [10] and Kim and Karrila [11]) is developed here to be suitable for the problem under the assumption of Oseen’s approximation. The relative velocity \( v_j \) of the bubble \( j \) is
determined by using the reflection method (Kumagai [8]) as follows:

\[ \mathbf{v}_j = \sum_{n=0}^{\infty} \mathbf{v}_j^{(2n+1)}, \]

(1) 

\[ \mathbf{v}_j^{(1)} = \mathbf{U}_j \quad \text{(on the bubble } j \text{)} \]

\[ \Rightarrow \mathbf{v}^{(1)} = \left| \mathbf{v}_j^{(1)} \right| \left( \mathbf{i}_{\mathbf{y}} \mathbf{v}_j^{(1)} + \mathbf{i}_{\mathbf{z}} \mathbf{v}_j^{(1)} \right) \]

\[ \Rightarrow \mathbf{v}_{jk}^{(1)} = \left| \mathbf{U}_j \right| \left\{ \mathbf{i} \left( \mathbf{v}_j^{(1)} \times \mathbf{r}_{jk} \right) / \left| \mathbf{r}_{jk} \right| + \mathbf{v}_{jk}^{(1)} \mathbf{R}_{jk}^{(1)} / \left| \mathbf{R}_{jk}^{(1)} \right| \right\} \\
+ \mathbf{j} \left( \mathbf{v}_j^{(1)} \mathbf{y}_{jk} / \left| \mathbf{r}_{jk} \right| + \mathbf{v}_{jk}^{(1)} \mathbf{R}_{jk}^{(1)} / \left| \mathbf{R}_{jk}^{(1)} \right| \right) \\
+ \mathbf{k} \left( \mathbf{v}_j^{(1)} \mathbf{z}_{jk} / \left| \mathbf{r}_{jk} \right| + \mathbf{v}_{jk}^{(1)} \mathbf{R}_{jk}^{(1)} / \left| \mathbf{R}_{jk}^{(1)} \right| \right), \]

(2) 

\[ \mathbf{v}_{k}^{(2)} = \mathbf{U}_k - \mathbf{v}_j^{(1)} \quad \text{(on the bubble } j \text{)} \]

\[ \Rightarrow \mathbf{v}^{(2)} = \sum_{k=1}^{n} \mathbf{v}_{k}^{(2)} = \sum_{k=1}^{n} \left( \mathbf{U}_k - \mathbf{v}_j^{(1)} \right) \quad (k \neq j) \]

\[ \Rightarrow \mathbf{v}_j^{(2)} = \sum_{k=1}^{n} \mathbf{v}_{k}^{(2)} \quad (k \neq j) \]

\[ \cdots \cdots \cdots, \quad (9) \]

where

\[ \mathbf{U}_j = \mathbf{i} \mathbf{U}_{xj} + \mathbf{j} \mathbf{U}_{yj} + \mathbf{k} \mathbf{U}_{zj}, \]

\[ \mathbf{r}_{jk} = \mathbf{r}_k - \mathbf{r}_j = \mathbf{i} \mathbf{x}_{jk} + \mathbf{j} \mathbf{y}_{jk} + \mathbf{k} \mathbf{z}_{jk}, \]

\[ \mathbf{i}_{\mathbf{y}} \mathbf{z}_{jk} = \mathbf{r}_{jk} / \left| \mathbf{r}_{jk} \right| , \]

\[ \mathbf{i}_{\mathbf{z}} \mathbf{y}_{jk} = \mathbf{r}_{jk} \times \left( \mathbf{U}_j \times \mathbf{r}_{jk} \right) / \left| \mathbf{r}_{jk} \times \left( \mathbf{U}_j \times \mathbf{r}_{jk} \right) \right| = \mathbf{R}_{jk} / \left| \mathbf{R}_{jk} \right| \]

(10)

and \( \mathbf{i}_x \) and \( \mathbf{i}_y \) denote unit vectors along the direction of \( \mathbf{r} \) and \( \theta \) respectively.

In general cases of the freely-rising motion of a cluster of bubbles, the higher-order terms of \( \mathbf{v}_j^{(2m+1)} \) may be small to be neglected and thus we may simply assume that

\[ \mathbf{v}_j = \mathbf{v}_j^{(1)} + \mathbf{v}_j^{(3)} = \mathbf{U}_j - \sum_{k=1}^{n} \mathbf{v}_{kj}^{(1)} \quad (k \neq j). \]

The velocity \( \mathbf{v}_j^{(1)} \) is defined as the velocity produced by bubble \( k \) translating with velocity \( \mathbf{v}_k \) and is simply described as

\[ \mathbf{v}_j^{(1)} = \lambda \left| \mathbf{r}_{kj} \right| \left( \mathbf{i}_{\mathbf{y}} \mathbf{v}_{kj} + \mathbf{i}_{\mathbf{z}} \mathbf{v}_{kj} \right). \]

(11)

Here the first order solution to Osele's equation is given (Kumagai [9]) as

\[ \mathbf{v}_{ij}^{(m)} = \cos \theta_{ij}^{(m)} \left[ 1/(2Y_{ij}^{(m)})^3 - (3 + 2\sigma)/(2Y_{ij}^{(m)}) \right] + \mathbf{R}_{ij}^{(m)} \left( 3 + 2\sigma \right)/(16Y_{ij}^{(m)}) \]

\[ -3 \mathbf{R}_{ij}^{(m)} (3 + 2\sigma) \cos \theta_{ij}^{(m)} / (1/r_{ij}^4 / 2 - r_{ij}^2 + 1)/(32Y_{ij}^{(m)}) \]

\[ + \mathbf{R}_{ij}^{(m)} (3 + 2\sigma) / (1/r_{ij}^4 / 2 - r_{ij}^2 + 1)/(32Y_{ij}^{(m)}) \quad (r_{ij} \geq 1), \]

(13)
\[ -\text{Re}^{(m)}(3 + 2\sigma') \cos \theta_{ij}^{(m)} \frac{1}{r_{ij}^4} \left[ \frac{1}{(16Y^{(m)})} \right] (r_{ij} \geq 1). \] (14)

This simple method is the first approximation to the conventional reflection method, and we may call here the superposition method of Oseen's flow field. Substitution of eqn (12) into eqn (6) completes the equation of motion for a cluster of bubbles in fluid at low Reynolds numbers. Using eqns (12) and (6) the position \( \mathbf{r}_j \) and the velocity \( \mathbf{U}_j \) of the bubble \( j \) for the time lapse are calculated numerically.

### 3 Experiments

The experimental apparatus is schematically shown in Figure 1. The test tank filled with pure glycerol has a cross-section measuring \( 400 \times 400 \text{mm} \) and a depth of \( 600 \text{mm} \). The repetitive pipet is fixed to the bubble-releasing device at the bottom via a vinyl tube to pour air with the desired volume. The locations and shapes of air-bubbles are measured using high-speed video-cameras with 10000 frames/s and computers.

![Experimental apparatus](image)

**Figure 1**: Experimental apparatus

### 4 Results and discussions

#### 4.1 Freely-rising motion of two equal bubbles located vertically in series

Figure 2 (a) shows a typical example of the rising motion of two equal bubbles
located vertically in series at the initial time in quiescent glycerol. The freely-rising motion of two equal bubbles at the low Reynolds numbers below 0.5 is classified into three stages. When the center-to-center distance $L$ is larger than three times bubble diameter $d$, the shapes of both bubbles are regarded as spherical. In the 1st stage the following bubble approaches the preceding bubble gradually. When $L$ reaches $3d$, the preceding bubble becomes oblate while the following bubble becomes prolate. In this 2nd stage the approaching speed of two bubbles becomes a little higher than the speed in the 1st stage because of the deformation of two bubbles. In the 3rd stage two bubbles contact and after a short time two bubbles coalesce to make a single larger bubble. The coalescent motion will be discussed in 4.3 in this paper.

The numerical results obtained by using the superposition method of Oseen’s flow field agree well with the experimental results as shown in Figure 2 (b). It should be noted that the numerical results obtained using the conventional reflection method of Stokes’s flow field does not predict this type of approaching motion of two equal bubbles even at the low Reynolds numbers below 0.5.

![Video pictures and Experimental and numerical result](image)

**Figure 2** Motion of 2 bubbles rising vertically in a quiescent glycerol

### 4.2 Freely-rising motion of four equal bubbles located vertically in series

Figure 3 (a) shows a typical example of the rising motion of four equal bubbles located vertically in series with the same spacings at the initial time in quiescent glycerol at the Reynolds numbers below 0.5. First the preceding two bubbles approach to make a pair of bubbles. Then the following two bubbles approach to make a pair of bubbles. If the number of equal bubbles located vertically in series at the initial time is even, they show the interesting motion to make pairs of bubbles from the preceding two bubbles successively in turn. If the number is
odd, the preceding bubbles rises in the same manner as the even case while the lowest bubble rises alone. Pairs of bubbles rise in the same manner as shown in Figure 2. The superposition method of Oseen’s flow field predicts well the freely-rising motion of four equal bubbles as shown in Figure 3 (b).

Figure 3: Motion of 4 bubbles rising vertically in a quiescent glycerol

4.3 Deformation of a pair of equal bubbles

Figure 4 shows a typical example of deformation of two equal bubbles vertically in series obtained by using the high-speed video-cameras which move vertically with the mean speed of the two bubbles. This shows an approaching motion of two equal bubbles. Figure 5 shows the modulus of deformation $\varepsilon$ depending on the center-to-center distance between two bubbles. Here the modulus of deformation from the spherical shape is defined as $\varepsilon = (d_h - d_v) / (d_h + d_v)$ in Figure 6. We can determine the modified factor $\lambda = \lambda (\varepsilon) = \lambda (l/d)$ in eqns (1) and (3) by using the drag ratio of non-spherical particle to spherical particle given by Happel and Benner [10].
Figure 4: Approaching motions of 2 air bubbles rising vertically in a quiescent glycerol in a case of $M=73.4$, $Eo=7.39$ ($Re=0.191$, $\sigma=6.70\times10^4$, $de=6.1\,\text{mm}$)

Figure 5: Modulus of deformation for 2 bubbles

Figure 6: Definition of $\varepsilon$

### 4.4 Coalescent motion of two equal bubbles

Figure 7 shows a typical example of coalescent motion of two equal bubbles after contact of two bubbles. The lower bubble pushes the upper bubble. Then a crack occurs on the central part of interfacial plane between two bubbles. This crack extends most rapidly on the interfacial plane with wavy motion. During this bursting-like motion there are occurred a small number of micro-bubbles as shown in Figure 7. This interesting motion will be reported in near future.
Figure 7: Coalescent motions of 2 air bubbles rising vertically in a quiescent glycerol in case of $Re_t=0.19$, $d_{m1}=8.58\text{[mm]}$, $d_{m2}=9.78\text{[mm]}$

5 Conclusions

The numerical results obtained by using the newly-developed superposition method of Oseen's flow field around deformable bubbles agree well with the experimental results for the freely-rising motion of equal-sized air-bubbles located vertically in series with the same spacings in quiescent glycerol at the initial time. Equal-sized air-bubbles located vertically in series with the same spacings at the initial time show the interesting motion to make pairs of air-bubbles from the preceding two air-bubbles successively in turn. In the motion of pairs of air-bubbles the following air-bubble approaches to catch up the preceding air-bubble. During the coalescent motion of the two air-bubbles there is recognized an occurrence of a small number of micro-bubbles.

References


