Bluff-body wake evolution and interaction in two dimensions

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Abstract

Bluff-body wakes have been extensively studied for more than a century, however, many issues related to wake instabilities still need elucidation. For a cylinder wake, instabilities develop in directions normal and parallel to the cylinder axis. What is the respective influence of the former and the latter upon such important parameters as the vortex-shedding frequency? Our experiments with cylinder wakes in a nearly two-dimensional soap film flow show differences between the wake parameters in two- and three-dimensional (2D and 3D) wakes. In 2D, there is no evidence of discontinuities in the dependence of the dimensionless vortex-shedding frequency (the Strouhal number St) upon the Reynolds number Re due to spanwise instabilities. For Re<180, the 3D flow is neutrally stable to spanwise perturbations and thus not substantially different from 2D. A recently developed fit for St in terms of Re⁻¹/² valid for Re<180 does not describe the behaviour of 3D flow for higher Re, however, we show that it remains valid for 2D data. In the absence of 3D effects, the trend of the dependence of St upon Re to weaken at higher Re is attributable entirely to shear-driven instabilities. We also examine the features of 2D wakes far downstream of the cylinders (far wake region) and interaction of multiple wakes. The most striking feature of the far wake is the onset of the secondary vortex street (second wake), which exists in 3D only for low Reynolds numbers (Re<300). In soap film flows, it persists to Re>1200. We find the dimensionless downstream distance to the onset of the second wake to scale with Re⁻¹/². Finally, we present previously unknown features of the interaction of 2D wakes, such as coupling between disordered structures in the far wake.
1 Introduction

Many practical fluid dynamics problems involve flows around a combination of bluff bodies. The latter can be parts of a cooling system, fins on a chip, or downtown skyscrapers—regardless, presence of several bodies seriously complicates the problem as compared with that of a single-body wake, because one has to consider the interaction between the wakes. Given that even the single bluff-body wake is anything but trivial, the general understanding of the problem of a flow around a system of bluff bodies must be built gradually, beginning from studies of relatively simple bluff-body arrangements. The simplest possible bluff-body combination is a pair of parallel circular cylinders of diameter \(d\) separated by a distance \(s\). Relative to the freestream direction, the cylinders can be positioned in tandem, with separation normal to the freestream, or at an arbitrary angle. The direction of the separation and the spacing \(s/d\) are the dimensionless parameters defining the problem along with the Reynolds number \(Re\). The latter can be defined based on either the diameter or the spacing. In this paper, we will use the former definition, namely \(Re = u_o d / \nu\), where \(u_o\) is the freestream velocity and \(\nu\) the kinematic viscosity. Please note that our work deals with cylinders separated in the direction normal to the freestream.

Flow around a pair of cylinders has attracted considerable attention in the last century, although nowhere near the popularity of the problem of a single-cylinder wake (see Zdravkovich [1] for a detailed review). The 1930s studies of forces on a cylinder pair separated in the direction normal to the freestream [2] quantified interference drag, i.e., the difference between the drag of the cylinder pair and the drag of a single cylinder times two. The measurements showed the interference drag to be a function of both \(Re\) and \(s/d\) and generally become negligible for \(s/d > 5\). More than a decade later, studies of the predominant vortex-shedding frequencies in the near-wake of a cylinder pair with normal separation [3] revealed existence of at least two shedding regimes. The dominant frequency is the same as the Bénard - von Kármán frequency of a single cylinder for \(s/d > 2\), while for \(s/d < 1.5\), the dominant frequency is consistent with that of a single body twice the individual cylinder size.

From point measurements, it was rather hard to infer the physics underlying these phenomena, even without taking into account the three-dimensional (spanwise) effects of the interaction. A fair notion of the appearance of the flow field in the plane normal to the spanwise direction was acquired by Bearman and Wadcock [4] with smoke visualisation. This work is also noteworthy for the use of an analog correlator to correlate hot-wire measurements at two points in the wake, thus assessing the degree of interaction of the wakes. A later dye- and smoke-visualization study [5] investigated synchronisation between vortices in the wakes for \(2 < s/d < 6\) and found regimes of vortex-shedding with in-phase and antiphase \((180^\circ\) out of phase\) synchronisation. This study also noted that vortices forming simultaneously at the cylinders rotate around each other as they are advected downstream, forming a combined wake comprised of such vortex pairs. A recent attempt to explain the different synchronisation regimes used two coupled Landau equations with coefficients estimated from experimental
observations [6]. For Re close enough to the critical value characterising the onset of vortex-shedding, the model can reproduce most of the experimentally observed features.

The possible role of spanwise effects in wake interaction was first mentioned as early as 1963 [4]. The wake behind a single cylinder is neutrally stable to spanwise perturbations only for Re<170 [7]. It is noteworthy that the flow-visualisation studies mentioned above typically deal with Reynolds numbers 200 or lower. How does the presence of two cylinders modify the flow stability to spanwise perturbations? We are not aware of a theoretical study that would provide the definitive answer, but studies of a “tandem” cylinder-pair configuration [1] show some enhancement of spatial two-dimensionality, which would suggest increased spanwise stability for this geometry. One could also argue that, as the limiting case of the normally-separated cylinders \((d/d+1)\) is a bluff-body with the Reynolds number twice that of a single cylinder, the spanwise stability of the flow of the normally-separated cylinders should be somewhat lower than that of a single cylinder.

Elimination of the spanwise effects motivated the experimental part of our study, which was carried out in a gravity-driven soap-film tunnel described in the next section of this paper. Under certain conditions, the governing equations of soap-film flow are very close to two-dimensional Navier-Stokes [8]. The results and discussion section begins with the description of our findings, experimental and numerical, for the case of a single-cylinder wake in 2D. After the features peculiar to a 2D cylinder wake are described, we proceed to the case of two interacting wakes, where we observe some types of behaviour not reported for 3D flows.

2 Experimental arrangement and diagnostics

The experimental apparatus is a tilted, gravity-driven soap film channel, where a constantly replenished soap film (characteristic thickness 10 microns) runs between two support wires (characteristic spacing 7 cm) connecting the top and bottom reservoirs. (Fig. 1). A detailed description of the system is provided elsewhere [9]. The channel consists of an expanding, parallel-flow, and contracting sections. The expanding section is tilted in the longitudinal direction at 5°, which makes it possible to sustain a gravity-driven flow with the mean freestream velocity of about 1 m/s. We use a 2% household liquid soap solution in tap water seeded with a small volume fraction \((\sim 10^{-7})\) of highly reflective tracer particles (titanium dioxide, characteristic size 200 nm).

The flow of the soap film is in many ways similar to a channel flow confined to 2D and described by Navier-Stokes equations. The difference is in the additional dissipative force due to air drag. The influence of this force can be taken into account in a simple analytical model for the fully-developed flow profile [9]. For the energy balance of soap-film flow structures, air drag typically becomes important on the scale of 1 cm or greater [10]. The relative influence of air drag is also decreased by producing slower and thicker films, which motivates the design features of our tunnel, specifically the independently adjustable
discharge rate and tilt angle. In the better-known vertical soap-film tunnels (for a review of flowing soap film research, see [11]), the soap-film thickness and the freestream velocity cannot be easily adjusted independently.

Figure 1: Photograph of the experimental setup (a) and close-up view of the cylinder arrangement (b). View from the downstream direction. Features of the experimental arrangement (a) are: 1 – top reservoir, 2 – camera, 3 – support wires, 4 – cylinder pair inserted into the flow shown in (b), 5 – bottom reservoir, 6 – recirculating pump. The lasers are outside the field of view.

A pair of Nd:YAG lasers (pulse duration ~5 ns, pulse energy 0.1 J, wavelength 532 nm) illuminates the film at 2° from above. The laser beam is defocused by a combination of a cylindrical and spherical lens. A digital delay generator controls the time interval between the pulses. The pulse pair double-exposes the flow images. Displacements between the first and second exposure of each particle can be interpreted in terms of velocity. For quantitative flow analysis (velocity field measurement), we use a single-frame cross-correlation particle-image velocimetry (PIV) algorithm [12]. Our acquisition device is a 2048 by 2048 pixel, 15 frames per second CCD camera. Digital images interrogated by the PIV software produce gridded velocity fields of 127 by 127 vectors spaced by 640 μm. For PIV analysis of the data presented in this paper, the physical size of the field of view was about 8 cm on the side. The same visualisation arrangement with single exposure produces semi-quantitative maps of the film-thickness gradient by highlighting interference fringes in the film. Film thickness in soap-film hydrodynamics usually behaves as a scalar field strongly coupled with vorticity [13], and is an attractive quantity to visualise.
It is also noteworthy that the viscosity of a soap film depends on the type of surfactant, its concentration, and film thickness [8-10]. Thus even the measurement of a Reynolds number \( Re \) of the flow becomes far from trivial [14]. For a 2% solution of a surfactant with well-known properties, we have to measure the film freestream velocity and the discharge rate to find the mean film thickness and thus estimate the viscosity. This procedure involves PIV for velocity measurements. It is employed throughout the following section whenever the Reynolds number is referred to, although space constraints limit us to presenting only visualisation results in the figures. Laser visualisation, while standard in 3D hydrodynamics, has not been previously applied to soap-film flows, to the best of our knowledge.

3 Results and discussion

Prior to describing the two-wake interaction, let us outline the features of a single cylinder wake in 2D, whose behavior is different from that of its well-known 3D analog.

3.1 Single-cylinder wake

At low Reynolds numbers \((Re<100)\), both 2D and 3D cylinder wakes evolve through several flow patterns as the downstream distance increases. In the near-wake, the dominant structure is the Bénard – von Kármán vortex street of staggered, counter-rotating vortices shed off the alternating sides of the cylinder. The large-scale structure of this street is two-dimensional, with instabilities in the spanwise direction beginning to grow only as \( Re \) exceeds 170 [7]. At downstream distances \( x \) on the order of 30 cylinder diameters \( d (x/d=30) \), the vortex street deforms into a pair of near-parallel shear layers. Even farther downstream, these layers roll up into a vortex street once again at the onset of the second wake [15].

The characteristic scale of the second wake is much larger than that of the near-wake vortex street. In 3D, however, the second wake can be reliably observed only for \( Re<300 \), because at higher Reynolds numbers it is apparently destroyed by spanwise instabilities [16].

The situation is radically changed in 2D, where the spanwise instabilities do not exist. We observe the second wake in 2D in the range \( 100<Re<1200 \). Figure 2 shows laser visualisations of our second-wake observations for \( Re=300, 600 \) and 1200. These images were obtained in the flow with a freestream velocity 1.9 m/s and kinematic viscosity \( 3.2\times10^{-6} \) m²/s by inserting cylinders of diameters 0.5, 1 and 2 mm to produce the corresponding Reynolds number. The film mean thickness is about 12 microns. The images were scaled to make the cylinder diameters look the same size and inverted (positive to negative) for easier interpretation. In Fig. 2, we visualize a pattern of thickness fluctuations following the vortices shed by the cylinder forming the near-wake staggered pattern (indicated by “1” in the figure), and later the second wake (labeled “3”). The most interesting feature of the sequence of images is the apparent dependence of
the onset of the second wake on the Reynolds number. A detailed study [17] of the dependence of the dimensionless downstream location \( x/d \) of the second wake onset upon \( Re \) reveals that the results are consistent with a power-law formula \( x/d \sim Re^{1/2} \).

![Figure 2: Laser visualisation of cylinder wakes at \( Re \sim 300 \) (a), \( Re \sim 600 \) (b), and \( Re \sim 1200 \) (c). Numbers indicate location of the wake features: 1 – first (Bénard – von Kármán) wake, 2 – near-parallel shear layers, 3 – second wake onset.](image)

Another difference between the 2D and 3D case which highlights the role of 3D instabilities is in the behaviour of the Strouhal number \( St=fd/U \), where \( f \) is the frequency of vortex shedding off one side of the cylinder and \( U \) is the freestream velocity. Thus the Strouhal number is the dimensionless vortex-shedding frequency characterizing the near wake.

The relationship between \( Re \) and \( St \) has been studied extensively in 3D (again, refer to [1] for a detailed review), with many empirical relationships proposed for experimental data. Because of the spanwise instabilities changing the character of the flow near \( Re \sim 200 \), many authors propose distinctly different fits for \( Re<200 \) and \( Re>200 \). The now-classic paper of Roshko [18] introduced two fits describing a large number of 3D measurements: \( St=0.212(1-21.2/Re) \) for \( Re<200 \) and \( St=0.212(1-12.7/Re) \) for \( 200<Re<30000 \).

A recent work of Williamson and Brown [19] revisited the issue of describing the \( Re - St \) relationship and provided physical arguments to support the notion that this relationship should be described as a power series of \( Re^{-1/2} \). Figure 3 shows the comparison of our experimental results with the fits of Roshko [18] described above and with the two-term fit proposed by Williamson and Brown [19] for \( Re<170 \): \( St=0.2665-1.018 Re^{-1/2} \).
Figure 3: $Re - St$ relationship from soap-film experiment and curve fits of Roshko [18] and of Williamson and Brown [19] obtained for 3D cylinder-wake data.

It is apparent that the 2D case at $Re > 200$ is characterized by higher $St$ values than those reported in 3D (Roshko fit for high $Re$, dotted line). The $Re^{1/2}$ fit [19] for 3D data at $Re < 170$, where spanwise perturbations in a 3D flow do not grow, provides a good agreement with our data not just for the low Reynolds numbers, but throughout the entire range of our measurements, up to $Re = 1200$. This result supports the notion of physical relevance of $Re^{1/2}$ for the description of the wake behaviour. We have already encountered this scaling in the description of the onset of the far wake. It also appears important for the vortex shedding in the near wake. Additional study is needed to elucidate the issue. Specifically, we should measure the vorticity in the shear layers on the sides of the cylinder where the vortices roll up and check for $Re^{1/2}$ scaling there as well.

If one considers the rate of change $d(St)/d(Re)$, it clearly decreases with $Re$ both in 2D and in 3D, but in 3D the decrease becomes more abrupt at $Re = 200$. One could argue that the $Re - St$ relationship becomes weaker as the secondary instabilities in the near-wake begin to consume more energy. This would explain why $St$ reaches higher values in 2D: while the influence of the shear-driven instabilities in the plane of the flow increases, there are no spanwise instabilities.

### 3.2 Interaction of wakes from two cylinders

Ideally, the flow regimes of two interacting cylinder wakes should be mapped in the $Re - s/d$ plane [6]. This remains a task of the immediate future for us. Presently we have sufficient evidence that 2D experiments show features not previously observed in 3D. The likely explanation for the difference is the
absence of spanwise instabilities, which radically alter the 3D cylinder wake at higher Reynolds numbers.

Figure 4 shows laser visualisation of the two limit cases, when the wake behaviour is essentially the same as in the case of a single cylinder. This occurs when the cylinders are far apart, Fig. 4 (a), or when they approach very close, Fig. 4 (b). The cylinder diameter is $d=1$ mm, the same as in Fig. 2 (b), the Reynolds number is about 500. The limit cases are consistent with the 3D observations, except for the differences in the single-body wake behaviour noted in the previous section.

![Figure 4](image)

Figure 4: Non-interacting 2D cylinder wakes at $s/d=12$ (a) and completely merged wake at $s/d=1$ (b).

We begin to observe wake interaction as $s/d$ decreases past 6. The feature apparently peculiar to 2D wakes, at least at higher Reynolds numbers, is synchronisation between the second wakes while the first wakes are unsynchronised (Fig. 5). The evident coupling of disordered vortical structures in the far wake is particularly interesting and, to the best of our knowledge, has not been previously reported.

![Figure 5](image)

Figure 5: Synchronisation between the second wakes at $s/d=5$. The image is a mosaic of two fields of view.

As $s/d$ decreases further, anti-phase and then in-phase synchronisation of the near-wakes take place, qualitatively similar to the results reported in 3D for more moderate Reynolds numbers (Fig. 6). The values of $s/d$ characterising the locking appear to be dependent on $Re$. Additional study is necessary to determine the nature of this dependence.
The flow structure morphologically similar to structures observed in 3D [5] also characterises the flow regime preceding the formation of a single wake behind the paired cylinders as $s/d$ decreases below 2. There exists a difference in appearance between our 2D results and the 3D observations [5] that can be attributed either to a higher Reynolds number of our flow or to its two-dimensionality. Vortices shed off the cylinders move in pairs to the periphery of the wake in both cases, however, in our case these vortex pairs drift apart with downstream distance, producing a distinctive “Christmas tree” pattern (Fig. 7).

In summary, we have presented a new look on an old and important problem of bluff-body wakes and wake interaction. Studies of 2D flows and comparison of results from 2D and 3D studies facilitate better understanding of the complex interplay of several instabilities developing in the wake flows. The laser-visualisation technique described here has the advantage of allowing quantitative interpretation in terms of film velocity and thickness fields, and we will continue to employ it. In the immediate future, our studies will focus on the interaction of two cylinder wakes and on the influence of the Reynolds number on the wake synchronisation regimes.

Acknowledgements. The authors thank R.E. Ecke, M.K. Rivera, M.S. Ingber, and J. Vigil for useful discussions. This research is partially supported by NASA through PURSUE (Preparation for University Research of Students in Undergraduate Education) program, grant no. PP-108-01S, and by Los Alamos National Laboratory through LDRD (Lab-Directed Research and Development) program.
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