Stenosis-induced flow disturbances in an artery segment assessed by numerical simulation

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Abstract

The origin of sounds detectable in arteries with serious stenosis is still a questionable matter. To explore the hypothesis that stenosis-induced flow disturbances may be the cause of vascular sounds we numerically simulated the onset of flow oscillations in a vessel with a severe constriction. The model, based on the axi-symmetric Navier-Stokes equations, includes fluid non-linear inertial forces, viscoelastic wall motion, anatomical taper and reflexion in the vessel extremities due to proximal and distal circulations. By this model unstable circulations and self-excited flow oscillations were reproduced. Depending on the steady entry blood flow level in the stenotic artery segment different flow oscillatory patterns were simulated. A common characteristic of such flow oscillations was an high frequency component in the audible band (550 Hz). Thus, results support the hypothesis that vascular sounds may be generated by high frequency flow oscillations.

1 Introduction

When the natural vessel geometry of an arterial vessel is locally altered by pathological conditions, such as a stenosis, hemodynamics may be strongly disturbed, and phenomena such as flow separation, vortex circulation and turbulence can take place.

Flow disturbances are believed to have a diagnostic relevance in detecting the stenotic site and in determining the extent of the atherosclerotic
narrowing. Non-invasive techniques, such as Doppler or MRI, are currently used in a clinical context to obtain a detailed view of local blood flow and to extract sufficient information to determine the actual degree of occlusion. However, despite the considerable progress in such diagnostic techniques, precise, quantitative knowledge of hemodynamics in a stenotic vessel segment is still lacking. This justifies the considerable investigative effort, both theoretical and experimental, towards elucidating the basic features of flow circulations occurring in a vessel with a stenotic-like geometry [1].

A peculiar phenomenon that takes place in regions with severe stenosis is the presence of unstable circulations. These circulations are unstable in time and in space, and they can cause temporal oscillations in the flow with frequencies much greater than those related to the cardiac cycle.

The interest in this phenomenon is due to its possible implication in the generation of vascular sounds detectable at the body surface near a severe stenotic site [2]. These sounds are associated with exacerbated pathological conditions, and phonoangiography has long been used in an attempt to diagnose cardiovascular diseases by estimating the residual lumen diameter from the characteristic frequency of sound. However, quantitative diagnoses based on these sounds have been impeded by the lack of understanding of the sound genesis.

In order to gain a deeper understanding of hemodynamics in a stenotic artery, we developed a novel computer model of fluid-dynamics in a viscoelastic constricted tube. By this model, unstable circulations causing flow oscillations with frequency in the audible range were simulated.

2 Model equations

The numerical simulator was based on the Navier-Stokes and continuity equations, assuming the blood to be an incompressible newtonian fluid, the flow axially symmetric and the vessel a rectilinear, deformable, thick, tapered tube of isotropic, incompressible, viscoelastic material with circular cross section. In cylindrical coordinates $\eta, z$ ($\eta$ radial and $z$ axial coordinate) these equations take respectively the following form [3]:

\[
\frac{\partial w}{\partial t} = \frac{1}{R} \left( \eta w \frac{\partial R}{\partial z} - u + \eta \frac{\partial R}{\partial t} \right) \frac{\partial w}{\partial \eta} - \frac{w}{\partial z} \\
- \frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{\nu}{R^2} \left( 1 + 2\eta^2 \left( \frac{\partial R}{\partial z} \right)^2 \right) \\
- \eta^2 R \frac{\partial^2 R}{\partial z^2} \frac{1}{\eta} \frac{\partial w}{\partial \eta} + \left( 1 + \eta^2 \left( \frac{\partial R}{\partial z} \right)^2 \right) \frac{\partial^2 w}{\partial \eta^2} \\
+ \left( R^2 \frac{\partial^2 w}{\partial z^2} - 2\eta R \frac{\partial R}{\partial z} \frac{\partial^2 w}{\partial \eta \partial z} \right),
\]  

(1)
\[ \frac{1}{R} \frac{\partial u}{\partial \eta} + \frac{u}{\eta R} + \frac{\partial w}{\partial z} - \eta \frac{\partial R}{\partial z} \frac{\partial w}{\partial \eta} = 0, \quad (2) \]

where \( t \) is the time, \( \rho \) the density, \( \nu \) the kinematic viscosity of blood, \( w(\eta, z, t) \) and \( u(\eta, z, t) \) the velocity components in the axial and radial directions respectively, \( p(z, t) \) the transmural pressure, and \( R(z, t) \) the radius of the vessel lumen, which is a function of both \( z \) and \( t \) because of the vessel deformability. The dimensionless radial coordinate, \( \eta = r/R(z, t) \) \[4\], was used to avoid the difficulty arising from the boundary conditions with respect to the radial coordinate prescribed on a moving surface:

\[
\begin{align*}
\left. u(\eta, z, t) \right|_{\eta=1} &= \frac{\partial R}{\partial t} \\
\left. w(\eta, z, t) \right|_{\eta=1} &= 0 \\
\left. u(\eta, z, t) \right|_{\eta=0} &= 0 \\
\left. \frac{\partial w}{\partial \eta} \right|_{\eta=0} &= 0.
\end{align*}
\quad (3)
\]

The first two conditions reflect the assumption that the longitudinal movements of the vessel are negligible, the last two impose the axial symmetry of the blood flow.

The boundary conditions with respect to \( z \) were calculated by means of a classic three-element Windkessel load at the proximal and distal ends of the vessel segment similar to \[1\]. The Windkessel loads simulate the effects of the circulatory system upstream and downstream of the vessel, lumping them in convenient input impedances. In the proximal Windkessel load the presence of a flow generator simulates the artery input blood flow.

By multiplying eqn (2) by \( \eta R \) and integrating with respect to the radial coordinate between zero and \( \eta \), one obtains an equation to be inserted in eqn (1); the same equation, obtained as described from eqn (2), is then specialized for \( \eta = 1 \), and the boundary conditions at the wall (eqn (3)) are imposed; thus, another equation is obtained, which is inserted in eqn (1) as well. Thus, the following blood motion equation was obtained:

\[
\frac{\partial w}{\partial t} = \frac{1}{\eta R^2} \left( \int_0^\eta \lambda \frac{\partial (R^2 w)}{\partial z} d\lambda - \eta^2 \int_0^1 \lambda \frac{\partial (R^2 w)}{\partial z} d\lambda \right) - \frac{1}{\rho} \frac{\partial p}{\partial z} + \frac{\nu}{R^2} \left( 1 + 2\eta^2 \left( \frac{\partial R}{\partial z} \right)^2 \right) - \eta^2 R \frac{\partial^2 w}{\partial \eta \partial z} + \frac{\nu R}{R^2} \left( R^2 \frac{\partial^2 w}{\partial z^2} - 2 \eta R \frac{\partial R}{\partial z} \frac{\partial^2 w}{\partial \eta \partial z} \right). \quad (4)
\]

In order to solve eqn (4) a weighted residual method was used \[5\].
The motion of the vessel wall was prescribed according to the continuity equation in its integral form:

\[
\frac{\partial R}{\partial t} = -\frac{1}{2\pi R} \frac{\partial Q}{\partial z},
\]

(5)

where the flow \( Q(z,t) \) is defined as

\[
Q(z,t) = 2\pi R^2 \int_0^1 \eta \, w \, d\eta.
\]

(6)

The constitutive equation of the vessel wall was deduced by assuming both the transverse stress in the wall and the inertial effect due to wall mass to be negligible:

\[
p(z,t) = \frac{\sigma_o \left( e^{\beta(\delta - 1)} - 1 \right) + \mu \left( \frac{\partial R}{\partial t} \right)}{(r_m/2h)\delta^2 - 1/2},
\]

(7)

where \( \mu \) is the wall viscosity, \( \sigma_o \) and \( \beta \) are constant parameters, and the extension ratio, \( \delta \), was expressed as a function of inner radius, \( R(z,t) \), as follows:

\[
\delta(z,R(z,t)) = \frac{R_m}{r_m} = \frac{R + \sqrt{R^2 + 2 r_m h}}{2 r_m},
\]

(8)

where \( R_m(z,t) \) indicates the midwall radius of the deformed vessel, and \( r_m \) and \( h \) respectively denote the midwall radius and the wall thickness in the unstressed steady state (when transmural pressure is null).

In order to solve these equations numerically, the \( z \)-axis was discretized by uniformly spaced mesh points and the partial derivatives with respect to \( z \) were approximated by a finite difference centered scheme. The value of the parameters used for the simulations are: mid-wall radius in \( z = 0 \), \( r_{m0} \): 1.8 mm; wall thickness (in the non-stenotic region), \( h(z) \): 0.16 mm; natural tapering, \( \tau \): 0.03°; length of the vessel segment, \( L \): 20 mm; length of the sinusoidal-shaped stenotic region, \( L_{st} \): 12 mm; distance of the center of stenosis from \( z = 0 \), \( D_{st} \): 10 mm; maximum wall thickness (in center of stenosis), \( h_{max} \): 0.80 mm; discretization length of the \( z \)-axis, \( \Delta z \): 0.2 mm; wall viscosity, \( \mu \): 100 dyn s cm\(^{-3}\); blood density, \( \rho \): 1.06 g cm\(^{-3}\); kinematic blood viscosity, \( \nu \): 3.3x10\(^{-2}\) cm\(^2\) s\(^{-1}\); parameter, \( \sigma_o \): 1.0x10\(^5\) dyn cm\(^{-2}\); parameter, \( \beta \): 8.36.
3 Periodic flow oscillations

Since the blood was initially at rest, the input flow applied to the proximal Windkessel load started from zero and slowly increased to a constant value according to a logistic law. The steady inflow was reached after 1.8 s and it was maintained constant up to 6 s, when the simulation was stopped. Such input avoided a sharp transient phase. The saturation value in the different simulations was varied from 10.2 ml s\(^{-1}\) to 14.2 ml s\(^{-1}\).

When the input flow reaches a sufficient entity the flow in the vessel segment starts to oscillate (Fig. 1). The self-sustained flow oscillation persists in the whole vessel segment even after the input flow becomes steady (simulation time equal to 1.8 s). Actually, the transient along the vessel segment is not completely ended when the input flow becomes steady, and only after 4 s the flow oscillation can be considered stationary (Fig. 1).

![Figure 1: Flow oscillation from the onset to the stationary condition.](image)

The flow oscillation appears at first in the post-stenotic region (i.e. that part of the stenosis downstream of the vessel section with the maximum constriction), then it propagates along the vessel segment in a few milliseconds (about 8 ms). It is worth noting that the flow oscillations in different vessel sections are not in phase, with the maximum flow that occurs at different moments, thereby implying a propagation phenomenon (Fig. 2).

In the simulation with the steady input flow equal to 10.2 ml s\(^{-1}\) the flow oscillation is periodic. The period of flow oscillation depends on the axial position along the vessel segment (Fig. 2). In particular, in the post-
stenotic region the oscillation is doubled, in the sense that during one period of the oscillation (3.6 ms) two different maxima and two different minima are present (Fig. 2, thick curve). Conversely, in sections outside of the post-stenotic region the flow oscillation is simple and the difference in two consecutive maxima or minima is not distinguishable (Fig. 2, thin curve).

The flow power spectrum confirms the periodicity of the oscillatory phenomenon (Fig. 2). In particular, the flow power spectrum relative to a vessel section in the post-stenotic region shows two spectral lines (Fig. 2, top right), one at the frequency value of 275 Hz corresponding to the actual period of the oscillation (fundamental harmonic), and the other one with the highest power at the frequency value of 550 Hz, corresponding to half the flow oscillation period (first superharmonic). When the flow power spectrum is computed outside the post-stenotic region the fundamental harmonic vanishes and all the power moves to the first superharmonic (Fig. 2, bottom right). In keeping with this observation, in a section outside the post-stenotic region the flow oscillation has a simple sinusoidal time pattern with doubled frequency (550 Hz). The spectrum magnitude is normalized to the peak value.

4 Quasi-periodic flow oscillations

Flow oscillation significantly depends on the amplitude of the input flow (Fig. 3-5): changing the steady level from 10.2 ml s\(^{-1}\) to 14.2 ml s\(^{-1}\), different patterns of flow oscillations were found. In the case of input flow equal to 12.7 ml s\(^{-1}\), a third harmonic (3 Hz) appears in the flow power spectrum (Fig. 3), thus proving the existence of a new oscillatory component with incommensurate frequency. In this condition the flow oscillation is
quasi-periodic, with a low frequency sinusoidal component that modulates the amplitude of the high frequency flow oscillation (Fig. 3).

Figure 3: Global flow oscillation pattern for input flow equal to 12.7 ml s$^{-1}$.

When the input flow increases (13.2 ml s$^{-1}$), the quasi-periodic nature of the flow oscillation becomes more evident, and in the flow power spectrum new spectral lines appear in the low frequency band, which represent the spectral content of the signal modulating the amplitude of the high frequency flow oscillation (Fig. 4).

It is worth noting that a further increase in the input flow (14.2 ml s$^{-1}$) causes the oscillation to exhibit evident heterogeneity, with the high-frequency flow oscillation periodically damped (Fig. 5). However, although the oscillatory pattern is sensitive to the steady entry blood flow level, the high frequency component persists and can be considered a marker of the system oscillatory behaviour.

5 Discussion

The self-sustained flow oscillations were reproduced by a computer model of arterial hemodynamics in a flexible-walled tube with a stenotic-like geometry (42% of area reduction) and steady input flow.
The most interesting simulation result is that, when the flow oscillation occurs, it is characterized by a high frequency component (275 Hz). Even by applying constant input flow lower than 10.2 ml s$^{-1}$, it was not possible to obtain flow oscillations with a significant lower frequency. Actually, at a sufficiently low input flow, the flow oscillation disappears completely. Conversely, when the steady input flow level raises a low frequency component modulating the high frequency one appears: only in this condition were low frequency oscillations found. Simulations also showed that the flow oscillation becomes progressively more complex by increasing the steady input flow. As a consequence of the sensitive dependence on the value of the steady inflow the oscillatory pattern changes from periodic to quasi-periodic. In any case, the high frequency component persists.

The existence of oscillations with low and high frequencies was confirmed in several studies on collapsible pipes or channels, which present similarities to our system for their elastic properties and for the presence of a divergent region. Bertram et al. [6] observed unstable equilibrium behaviour with oscillation frequencies ranging from about 3 to 300 Hz.

Oscillations with frequency content similar to the one mentioned above were also reported in theoretical studies [7].

It must be underlined that a constant input flow, like those used in
this study, is far from the physiological condition. Because of the pulsatile nature of the cardiac pump, the arterial flow is unsteady, and an oscillatory component at frequency corresponding to the heart rate is present. We imposed a steady inflow in order to distinguish between the oscillatory phenomena due to the pulsatile nature of the cardiac pump and self-sustained flow oscillations, which were of interest in this study. On the other hand, the high frequency oscillatory component requires about 30 ms to develop completely (Fig. 1). The systolic phase of the cardiac cycle is sufficiently larger than that short time (ten times larger), and the systolic flow peak is comparable to the steady inflows used in our simulations. Therefore, the high frequency oscillatory component should occur at least during the systolic phase of the cardiac cycle. Conversely, the low frequency component could be suppressed, because its period is comparable with that of the cardiac cycle, and hence the low frequency component could have no relevant implications from a physiological point of view.

Our interest for the investigation of self-sustained flow oscillations in a stenotic artery is not purely theoretical, because these oscillations may be involved in the generation of vascular sounds, which are observed in the clinical contest when the vessel is partially occluded. Therefore, a deeper knowledge of the genesis of these sounds may be of some clinical relevance in the diagnosis of the stenotic sites. In the past years, an attempt was
made to obtain specific diagnosis of the stenotic condition by listening to the sounds produced by blood flowing through the stenosis itself. Lees and Myer [8] analysed the sound in patients with internal carotid artery stenosis, and found a break frequency of about 200 Hz in patients with mild stenosis and about 400 Hz in the case of severe stenosis. Therefore, the results of our study support the hypothesis that vascular sounds may be generated by high frequency flow oscillations.

References


