Application of the modal expansion technique to improve the results of FEM/BEM acoustic radiation

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Abstract

The use of FEM/BEM models in the prediction of sound radiation by vibrating structures is a well-known method already available in different software packages. There are many papers dealing with the numerical precision of the acoustic BEM methods, but it is clear that the accuracy of their predictions is mainly affected by the correct description of the problem boundary conditions, especially the surface normal velocities.

In this work, a simple test case consisting of a vibrating thin metal plate is solved using different methods to describe the normal velocities. FE structural model, measured data assigned to different surfaces and a hybrid technique called “modal expansion” are used.

The aim of this paper is to evaluate the differences in the prediction using the different models and to check the possible errors coming from usual mistakes in real application (modelling errors and measuring errors). Finally an experimental work is carried out to enlighten the previous conclusions.

1 Introduction

In the last years, numerical methods based on the FEM (Finite Element Method) and BEM (Boundary Element Method) are available in commercial codes to simulate the acoustic field radiated from vibrating structures. This tool seems to be very interesting in the analysis of sound emissions of different kind of products like machinery, vehicles and white goods to name just a few.
162 Modelling and Experimental Measurements in Acoustics III

The precision of the simulation is conditioned to a series of requirements in the modelling process, but mainly to the accuracy in the structure surface movement description.

In order to obtain the surfaces normal velocities, results from finite element structural analysis provide a reliable source of information, but their accuracy is limited by the uncertainties about the material properties, the actual boundary conditions, the excitation and damping. To solve this problem, there are techniques where the use of experimental data improves the model accuracy by means of improvements in the description of the structure vibrations. In this work, a technique named modal expansion based on the projection of the real measurements in a finite element modal basis is checked. Another option, also used in this paper, is to remove the structural FE model and work directly in the acoustic BE model boundary conditions, assigning the measured vibration values to different areas of the problem boundary.

All these features as well as different FEM/BEM formulations are implemented in LMS/SYSNOISE software (see reference [1]) which is used in the whole investigation.

Within the work, the advantages and drawbacks of each method are evaluated, comparing the results obtained and checking the sensitivity to usual mistakes in real applications. This way, errors in sensor positioning, model mistakes and boundary conditions inaccuracy are simulated and evaluated. Finally, a test of the modelled problem in controlled conditions is carried out. With these experimental results the accuracy of all the methods is analysed in terms of errors in the acoustic prediction and the models deviations are discussed.

2 Summary of theoretical background

In this work, the acoustic problem is modelled by means of the boundary element method (BEM) in an indirect element formulation implemented in SYSNOISE. This formulation (see eqn.1) is suitable for open surfaces and the main variables are called the “single layer potential” $\sigma$ (jump of velocity across the surface) and the “double layer potential” $\mu$ (jump of pressure across the surface). There are many text and books related to this topic and it is not going to be shown in this paper. The reader interested in the formulation details and the numerical implementation is referred to Wu [2] or von Estorff [3].

$$p(x) = \int_S \left( \mu(y) \frac{\partial G(x,y)}{\partial n_y} - \sigma(y)G(x,y) \right) dS(y) \quad (1)$$

On the other hand, the structural part is modelled via the well-known finite element method, and the harmonic response is obtained in direct formulation (eqn.2) or applying normal mode superposition (eqn.3). Both methods are also implemented in SYSNOISE for thin shell elements.
Modelling and Experimental Measurements in Acoustics III  163

Here the reader is referred to Zienkiewicz [4] or Craig [5] for more details on the FE equations and the numerical issues of the implementation to dynamic analysis.

\[
\begin{align*}
K + i\omega C - \omega^2 M \{u\} &= \{F\} \\
\begin{bmatrix} \phi \end{bmatrix}^T \begin{bmatrix} K + i\omega C - \omega^2 M \end{bmatrix} \{\phi\} \{a\} &= \{\phi\}^T \{F\}
\end{align*}
\]

Regarding the introduction in the BE models of the vibration boundary conditions, the application of the structural finite element model is quite straightforward. The velocities are directly obtained in the nodes multiplying the displacements by the angular frequency. This feature is implemented in SYSNOISE and no special proceeding is needed.

The modal expansion technique uses the calculated (or measured) modes of vibration to extrapolate onto the whole mesh from values known at a limited number of nodes. Naturally, the idea is that the value is provided by real measurements. This technique is based on the fact that the displacements at any node \(i\) in the direction \(j\) can be expressed as a linear combination of \(m\) modes of vibration eqn.4.

\[
u_{ij} = \sum_{k=1}^{m} a_k \phi_{jk}
\]

where \(u_{ij}\) is the displacement at node \(i\) in the direction \(j\), \(a_k\) is the modal participation factor of mode \(k\) and \(\phi_{jk}\) is the displacement at node \(i\) in direction \(j\) for eigenvector \(k\).

For each node displacement known, we obtain an equation, and hence, the above relation yield a system of \(n\) equations (one per known displacement component) with \(m\) unknowns (one per modal participation factor). As it is obvious, the system created is different depending on the number of known displacement components and the number of modes involved in the expansion. This give us three possible cases:

- \(n > m\)  more equations than unknowns, overdetermined system.
- \(n = m\)  square matrix, determined system.
- \(n < m\)  more unknowns than equations, undetermined system.

This algebraic linear system of equations is solved by means of the widely used singular value decomposition technique (SVD). This decomposition is suitable when mapping one vector space in another vector space of different dimensions, as is our case. Once again, this paper is not aimed at discussing the mathematical details for which a number of excellent reference works exist (see Golub and Van Loan [6]). We only want to mention that this technique looks for a least-square solution in an overdetermined system and for a minimum solution of the
solution space if the problem is undetermined, that is, it seeks the most appropriate solution. Hence, it is clear that the solution found will be quite dependent on the matrix aspect, for instance a system with a high degree of undetermination will be probably solved poorly. Once the modal participation factors are obtained, the rest of the structural displacements can be calculated by eqn.4 and the normal velocity directly derived from the following equation (eqn.5).

\[ v_{n,j} = \sum_{k=1}^{m} (i\omega a_{k}) \phi_{n,ik} \]  

(5)

3 Work description and problem modelling

The example problem under analysis consists of a thin metal plate in free-free conditions excited in the centre with a pure sine (100 Hz, 200 Hz, 400 Hz and 1 kHz). The finite element model and the plate dimensions can be seen in the Figure 1.

To clarify the possible benefits that can be extracted from the application of the modal expansion technique and its robustness against input mistakes, some measurements and three different sets of calculations are carried out, each of them with a different target. The modelling process as well as the objectives of each activity is described in the following points.

![Figure 1: Plate model](image1)

![Figure 2: Experimental set-up](image2)

Experimental measurements

In order to have real data to check the accuracy of simulation models, different measurements are done in the real part:

- Experimental modal analysis in free-free conditions (18 points).
- Vibration level measurements in 16 points equally distributed in the plate surface (keeping one accelerometer as reference).
Modelling and Experimental Measurements in Acoustics III 165

- Sound pressure level measurements in 10 points (the acoustic environment is free field on a reflective surface).

A sketch of the experimental set up can be seen in the Figure 2. The excitation is applied to the plate by means of an electrodynamic shaker and the real force introduced is measured with a load cell located at the end of the drive rod.

Modal expansion sensitivity to measuring points and mode selection

The main idea of this part of the work is to check the mistakes that may be produced because of the incorrect combination of points and modes in the expansion step and, as a consequence, to give some guidelines in order to use an optimum selection.

To reduce the uncontrolled variables, all the data used in this part is extracted from finite element calculations. This means, a FE/BE calculation is run and the results in some specific points are considered as measured points in a real case. Hence the values obtained in that calculation are used to expand the modes with the following sequence of calculations:

- Mode range: 1-5,1-10,1-15,1-20,1-30 and1-50
- Measurement points: 2, 4, 8 and 16 (equally distributed).

- Number of modes around the calculated frequency: 4, 8, 16 and 24.
- Measurement points: 2, 4, 8 and 16 (equally distributed).

Modal expansion sensitivity to FE model and measurement errors

At this point, the aim is to check if usual FE model mistakes or measurement errors affects too much the expansion results. This way, two wrong structural FE models are built, one of them with lower material stiffness to get frequency mistakes and the other with springs in connected in three different points of the plate in order to shift the eigenfrequencies and change the mode shape.

Calculations with the erroneous models are run with a complete numerical procedure (FEM/BEM) and applying the modal expansion technique with the optimal projection obtained in the previous step.

After and using the correct structural model, the sensitivity to bad accelerometer positioning is also checked. This is, the values used for the modal expansion are obtained from points located close to the correct measurement points, simulating a random positioning mistake in the data acquisition. This way, also possible mistakes in the sensors are evaluated, because the differences in the input may be also caused by this fact.

Comparison with other boundary condition generation methods

Finally, the results obtained from real measurement expansion using the best combination of modes and measurement points will be compared with the results
obtained from other boundary velocity modelling: FE models and direct
generation from measurements. All of them will be also compared with the real
acoustic measurements performed in the experimental part of the work.

4 Results and observations

In the following points a summary of the results obtained as well as some
commentaries about will be presented. In order to help the reader, the results are
ordered in the same way and with the same title as they have been presented in
the previous point (except the experimental measurements that are included in
the last point).

![Graphs of structural response at 100, 200, 400, and 1 kHz](image)

Figure 3: Structural response of the plate and modal participation factors

Modal expansion sensitivity to measuring points and mode selection

In the graph 1 and table 1 a summary of the results obtained from the different
modal expansion cases (changing the modes and the measuring points used in the
projection) can be seen. It presents the average error in acoustic pressure
prediction in dB related to the FE/BE model (reference case).
<table>
<thead>
<tr>
<th>Point</th>
<th>1a5</th>
<th>1a10</th>
<th>1a15</th>
<th>1a20</th>
<th>1a30</th>
<th>1a50</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-0.6</td>
<td>-5.6</td>
<td>-10.6</td>
<td>-12.2</td>
<td>-17.9</td>
<td>-20.7</td>
</tr>
<tr>
<td>4</td>
<td>-5.5</td>
<td>-11.2</td>
<td>-13.8</td>
<td>-12.9</td>
<td>-15.9</td>
<td>-22.4</td>
</tr>
<tr>
<td>8</td>
<td>-0.7</td>
<td>-1.0</td>
<td>-3.7</td>
<td>-4.6</td>
<td>-7.9</td>
<td>-13.4</td>
</tr>
<tr>
<td>16</td>
<td>-0.7</td>
<td>-0.5</td>
<td>-2.8</td>
<td>-0.6</td>
<td>-1.3</td>
<td>-26.2</td>
</tr>
<tr>
<td>200 Hz</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Point</td>
<td>1a5</td>
<td>1a10</td>
<td>1a15</td>
<td>1a20</td>
<td>1a30</td>
<td>1a50</td>
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<td>2</td>
<td>8.9</td>
<td>3.1</td>
<td>-1.9</td>
<td>-4.2</td>
<td>-10.0</td>
<td>-13.2</td>
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<tr>
<td>4</td>
<td>6.5</td>
<td>1.5</td>
<td>-0.8</td>
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<td>-12.0</td>
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<td>-4.1</td>
<td>-2.0</td>
<td>-3.1</td>
<td>-12.3</td>
</tr>
<tr>
<td>16</td>
<td>3.5</td>
<td>2.2</td>
<td>12.2</td>
<td>-0.5</td>
<td>0.7</td>
<td>-20.9</td>
</tr>
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<td>400 Hz</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Point</td>
<td>1a5</td>
<td>1a10</td>
<td>1a15</td>
<td>1a20</td>
<td>1a30</td>
<td>1a50</td>
</tr>
<tr>
<td>2</td>
<td>-23.5</td>
<td>-21.6</td>
<td>-26.1</td>
<td>-24.7</td>
<td>-29.8</td>
<td>-31.0</td>
</tr>
<tr>
<td>4</td>
<td>-6.8</td>
<td>-12.7</td>
<td>-14.3</td>
<td>-14.5</td>
<td>-18.1</td>
<td>-23.7</td>
</tr>
<tr>
<td>8</td>
<td>-16.1</td>
<td>-24.0</td>
<td>-20.0</td>
<td>-26.6</td>
<td>-31.5</td>
<td>-30.9</td>
</tr>
<tr>
<td>16</td>
<td>-30.3</td>
<td>-24.3</td>
<td>5.8</td>
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<td></td>
<td></td>
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<td>Point</td>
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<td>1a10</td>
<td>1a15</td>
<td>1a20</td>
<td>1a30</td>
<td>1a50</td>
</tr>
<tr>
<td>2</td>
<td>7.4</td>
<td>7.4</td>
<td>7.3</td>
<td>6.6</td>
<td>8.6</td>
<td>10.6</td>
</tr>
<tr>
<td>4</td>
<td>8.1</td>
<td>8.7</td>
<td>7.7</td>
<td>8.7</td>
<td>8.2</td>
<td>9.6</td>
</tr>
<tr>
<td>8</td>
<td>8.0</td>
<td>12.7</td>
<td>7.6</td>
<td>5.4</td>
<td>4.0</td>
<td>2.5</td>
</tr>
<tr>
<td>16</td>
<td>7.3</td>
<td>6.6</td>
<td>21.8</td>
<td>15.6</td>
<td>8.1</td>
<td>10.8</td>
</tr>
</tbody>
</table>

Table 1 and Graph 1: dB differences with different combinations of points and mode shapes in the expansion step

It can be seen that at 100 Hz and 200 Hz there are several cases that offers an approximation good enough (errors < 3dB), even though there are cases where the average mistake reaches 25 dB. On the other hand, at 400 Hz and 1 kHz there are very few combinations of points and modes that yield a good approximation to the correct values.

These mistakes are due to different numerical aspects involved in the problem solution: the spatial complexity of the structural response, the condition of the matrix $[\Phi]$ and the grade of underdetermination of the problem.

The matrix condition is very important because the solution of the equation system implies the inversion of the matrix $[\Phi]$. The modes and measuring points selection should be done in a way that each point only have information about one or two modes and also every mode should be presented in at least a point. If this is not the case, the matrix will be ill-conditioned being rank deficient (close to singular) or some modes couldn’t be presented in the solution. This is likely to
produce a bad system solution. In this work, the selection for the plate problem was done in a equally distributed sense, without being concerned about the matrix aspect. The matrix $[\mathbf{D}]$ is shown in the figure 4 and it is clearly ill-conditioned (similar information in every row and column).

![Modal matrix, ill-conditioned](image)

Figure 4: Modal matrix, ill-conditioned

Other of the facts observed, the higher the frequency the bigger the error has to be related with the spatial complexity of the response. This probably affects because the information contained in each measuring point has more mode components, amplifying the errors caused by the matrix ill-conditioning. At last, the other fact derived from the use of the SVD is that the solution searched if the undetermination is quite big is a minimum of the solution space, and then, the radiation is underestimated. If the case is the opposite and the system is overdetermined, the trend of the solution is to share as much as possible the participation factors and that normally provoke the overestimation of the acoustic prediction.

The selection of the modes around the calculated frequency doesn’t give any improvement and the results are very similar to the ones already presented and they are not shown here.

In the rest of the work, every time the modal expansion technique is used, we work with the best combination of points and modes obtained here: 16 points and 20 modes at 100 Hz and 200 Hz, and 16 points and 30 modes at 400 Hz. No more results will be given at 1 kHz because it has been shown that the technique doesn’t work properly in that range.

**Modal expansion sensitivity to FE model and measurement errors**

In the table 2 the summarised results of the usual mistakes likely to appear are shown. Once again, the average acoustic pressure difference in dB related with the reference case is presented. In the table 3 and figure 5, the differences in the input data and the mode shapes can be seen.
Table 2: Results in the expansion application with different mistakes.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Without error</th>
<th>Input data error</th>
<th>FEM frequency error</th>
<th>FEM mode shape error</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 Hz</td>
<td>-0.6</td>
<td>2.3</td>
<td>-0.6</td>
<td>-1.1</td>
</tr>
<tr>
<td>200 Hz</td>
<td>-0.5</td>
<td>5.2</td>
<td>-0.5</td>
<td>-20.1</td>
</tr>
<tr>
<td>400 Hz</td>
<td>-1.5</td>
<td>-4.7</td>
<td>-1.5</td>
<td>-3.7</td>
</tr>
</tbody>
</table>

Table 3: Input data error

As it was expected because is an spatial projection, the modal expansion method is totally insensitive to frequency mistakes in the FE model, but the ill-condition of the system produces very big errors with every little deviations in the input values or in the mode shapes. The values at 100 Hz and 400 Hz aren’t affected very much in the mode modification case because there is no important changes in modes around that frequencies (see the MAC values in the figure 5).

Comparison with other boundary condition generation methods

Finally, the results of the prediction with different boundary condition generation methods and the experimental values are presented in the table 4.

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Measurements</th>
<th>FEM/BEM</th>
<th>BEM manual</th>
<th>BEM expansion</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 Hz</td>
<td>90.3</td>
<td>98.6</td>
<td>93.0</td>
<td>98.7</td>
</tr>
<tr>
<td>200 Hz</td>
<td>86.4</td>
<td>85.9</td>
<td>85.8</td>
<td>83.8</td>
</tr>
<tr>
<td>400 Hz</td>
<td>101.8</td>
<td>100.1</td>
<td>97.2</td>
<td>99.4</td>
</tr>
</tbody>
</table>

Table 4: Results from different methods (dB)
5 Summary and conclusions

In this paper the problems coming from the application of the modal expansion technique to generate the boundary conditions in FEM/BEM radiation models have been checked.

The advantage of models combined with this method compared with pure numerical models is that it is not necessary to know the excitation forces and the material damping (always a problem in FE models). Compared with pure experimental analysis, the predictive and analysis capabilities of having a validated model are a clear advantage.

In a real application, the simple idea of the method (real measurement projection in a modal base) becomes much more complex, because of the numerical aspects derived from the resolution of a system by means of the SVD pseudo-inverse. This fact, causes a big variability in the results as a function of the measuring points and mode shapes selected to perform the modal expansion and a high sensitivity to input mistakes in the measurements or in the modes. Then, special care must be taken using this method.

In spite of that, there are a lot of possible benefits coming from the application of an hybrid technique where the power of a numerical model is combined with real results, and then it seems worthy some attempts to cope with these problems.

A first idea to reduce the high sensitivity and the big mistakes that are likely to appear is the use of optimisation methods in the selection of the modes and measuring points to get a well conditioned matrix. Another possible improvement is the evaluation of the results including the measured values and/or the mode shapes as stochastic variables. Hence, the solution will be always accompanied with a confidence level.

These two improvement lines will be the aim of future works.

References