CHAPTER 5

Modeling and prediction of the mechanical properties of woven laminates by the finite element method

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Abstract

The paper presents a review of the prediction methods for the mechanical properties of woven fabric laminates by the finite element method. Woven fabrics usually present orthogonal interlaced yarns according to different architectures: here plain-weave and twill weave are considered. A reference volume or unit cell and appropriate boundary conditions to enforce continuity and periodicity in stresses and strains are initially defined. Three-dimensional finite element models are developed and used to predict stiffness and damage evolution up to final rupture of the model laminate. The computational models include several parameters affecting stiffness and strength of woven composites, such as the crimp ratio, the weave architecture, the fiber volume fraction and the mechanical characteristics of the constituents. Results of the computational approach and of parallel experimental investigations on carbon fiber reinforced epoxy laminates are compared.

1 Introduction

The interest in textile composites has been growing in recent years. They are increasingly used in the fabrication of advanced structures in the aerospace, naval construction or automotive sectors. Woven-fiber composite materials represent a type of textile composite where strands are formed by the process of weaving (Naik (1994) and Bogdanovich et al. (1996)). These strands are interlaced in two mutually orthogonal (warp and fill) directions to one another and impregnated with a resin material. Composite materials reinforced with woven fabric have many attractive aspects like low fabrication costs, ease of handling, high adaptability, and better out of plane stiffness, strength and toughness properties than unidirectional laminate composites. However, the geometry of this composite class is complex and there is a wide range of possible architectures and constituents because it is possible to act on microstructure geometry, weave type, hybridization or choice of constituents (e.g. geometrical and mechanical parameters of strands and resin), Ko (1989). The geometrical variables of the reinforcement (the yarn spacing, the yarn thickness and the weave type) or the fiber and resin types, the packing density of the yarns and the fiber volume fractions may be varied to obtain the specific mechanical properties. Carbon, glass or aramid fibers may be used as reinforcement.
To select the best possible combination of weight, cost, stiffness and strength properties of a woven-fiber composite, an intimate understanding of the link between material structure and mechanical performance is required. Therefore development of predictive tools of the 3D elastic and failure properties of woven-fiber composite materials has been the subject of great research efforts. Three different types of mechanics models, elementary, laminate theory and numerical, are available in the literature and were reviewed by Byun and Chou (1989) and Naik (1994).

This paper is especially devoted to the review of the finite element approach to the prediction of the stiffness and strength of woven composite laminates. After an initial overview of the approaches for modeling the mechanical response of woven composites available in the literature, the procedure for the development of a representative volume (RV) to model the complex but repetitive geometry of this class of materials is presented. Other issues with special reference to the boundary conditions to apply to the RV, the material models and the strategies for damage and failure description are discussed next. The paper ends with a review of finite element modeling applications to stiffness and strength predictions of various woven composite laminates performed by the authors.

2 Approaches for the mechanics of woven composites

Analytical models for determination of the mechanical properties of woven composites provide a cost-effective tool to evaluate the effects of several parameters (fabric weight, constitute volume fraction, yarn undulation, weave style, and properties of the constituent material) on the mechanical properties of woven composites. In the 1980s, Ishikawa (1981), Ishikawa and Chou (1982a, b), proposed three analytical models based on the classical thin laminate theory for the prediction of the elastic stiffness of woven-fiber composites (plain and satin weaves). The fabric composite of the “Mosaic” model consists of an asymmetrical cross-ply laminate assemblage. The “Fiber undulation” model takes into account fiber continuity and undulation only in the fill direction, and not in both fill and warp directions. The “Bridging” model breaks up the unit-cell into interlaced regions, whose in-plane stiffness is predicted by the fiber undulation model, and non-interlaced regions, which are modeled as cross-ply laminates. An experimental verification of these models was carried out by Ishikawa et al. (1985).

Many other researchers attempted to define the two-dimensional (2D) orthogonal plain-weave fabric geometry mathematically. In the early 1990s, Naik and Shembekar (1992a, b, c) developed an analytical bi-dimensional model based on the one-dimensional models of Ishikawa and Chou by considering the undulation of both warp and fill yarns, the cross-sectional geometry of the yarns and the gap between adjacent yarns. They predicted the in-plane elastic properties for a single lamina on the basis of classical lamination theory and a mixed parallel-series arrangement of infinitesimal composite pieces. This method lacked simplicity as the fabric representative unit cell is divided into elements and pieces, and also involves substantial computation. Therefore Naik and Ganesh (1995) developed a closed-form analytical method to predict the thermo-elastic properties of 2D orthogonal plain weave fabric laminate. Naik and Ganesh (1995) considered strand continuity along both fill and warp directions and the presence of an inter-strand gap, and they also simulated in detail strand cross section and strand undulation. Scida et al. (1999) presented a model similar to that of Naik and Ganesh (1995) that took into consideration the strand undulation in the x and y directions and the possibility of superposing several layers in the right and wrong side with or without a relative translation. They adapted it to the hybridization principle, which is an advantageous solution to satisfy specific cost and performance requirements.

Vandeurzen et al. (1996a, b) proposed a 3D geometric description of several woven-fiber composite architectures. The full geometry of weave architecture is built from rectangular
An analytical model, called a combi-cell model, is developed. It is based on modeling each strand system with a matrix and a strand layer. The stiffness values predicted by applying the complementary variational principle are compared with finite-element models and show a good correlation, even in terms of shear modulus. Tabiei and Yi (2002) presented an analysis of a large range of woven-fiber composite materials and their hybrid equivalents. Their model is called MESOTEX (MEchanical Simulation Of TEXtile) and it is based on the application of the classical thin laminate theory to the woven structure. It predicts 3D elastic properties, damage initiation and progression and strength for several woven-fiber composite materials. Recently Lomov \textit{et al.} (2001) defined a hierarchical structure of a textile and implemented in the code WiseTex and the models serve as a base for meso-mechanical analysis. Analytical methods for the prediction of ultimate failure strength, stresses at different stages of failure and stress-strain history of 2D orthogonal plain weave fabric laminates under on-axis tensile loading have also been developed (Naik (1994) and (1995)).

Although cost-effective, the analytical approaches have a number of drawbacks especially in terms of modeling accuracy of weave complexity and strength response. In the last decade the finite element (FE) method has been increasingly used in the stiffness prediction but also in damage and strength analysis for these materials. Early work of a number of authors, Paumelle \textit{et al.} (1991) and Whitcomb (1991) examined the finite element method as a tool for obtaining insight in the stress state inside a fabric and for the prediction of the stiffness and strength of woven composites. Woo and Whitcomb (1994) analyzed the plain weave fabric composites with 2D finite element analysis by studying the internal stress distributions. To reduce the computational effort, they proposed a global/local methodology combined with special macro-elements as a valid alternative to conventional finite element analysis (Whitcomb and Woo (1994)). With this method a relatively coarse global mesh, characterized by single-field macro-elements, was used to determine the global response of the structure and the local fine meshes with conventional finite elements of the zones of interest were used to obtain accurate information. Woo and Whitcomb (1996) applied this method to three-dimensional FE models, by investigating the performance of the global/local procedure with macro-elements and by studying the stress state and the failure behavior of the plain weave composites. They found a good agreement between the results obtained by the global/local analysis and a conventional FE analysis and showed that the failure behavior of the infinite unit cell for plain weave composites under tension load changes with the curvature of the yarns. Blackketter \textit{et al.} (1993) presented a FE-based approach for the prediction of the damage initiation and evolution in a woven fabric composite under tensile and in-plane shear load. The elastic modulus was reduced once the failure stress was locally reached. The normal maximum stress criterion was adopted for failure prediction of the linear elastic isotropic matrix. The same failure theory, referred to the local material coordinate systems, was used for the transverse isotropic yarns. An extension of Blackketter’s method was developed by Whitcomb and Srirengan (1996). The authors demonstrated that the predicted numerical response of a plain weave composite is strongly sensitive to some computational parameters, i.e. the quadrature order, the number of elements, the damage method. In addition, the influence of the yarn shape and yarn curvature was studied (Chapman (1994), Chapman and Whitcomb (1995), Whitcomb and Tang (1999)).

### 3 Application of the finite element method to woven composites

Through the years this approach has been consolidated in a series of steps to be followed in order to properly exploit the numerical tools for the prediction of material behavior. These steps will be reviewed here so that they can provide guidance to the newcomer in the field.
The steps of the approach are defined as follows:

- definition of a representative volume (RV) of the woven composite
- finite element modeling of the RV
- definition of boundary conditions for the RV
- selection and application of material and damage models.

### 3.1 Identification of a representative volume (RV)

A brief description of the woven laminate production route is preliminarily given. Woven-fiber composite laminas (or mats) are obtained by interlacing yarns (or strands) in two mutually orthogonal (warp and fill) directions and impregnated with a resin. A yarn is obtained by weaving together thousands of reinforcing fibers (i.e. carbon, glass, aramide, etc). Yarns are woven according to different interlacing schemes thus obtaining different textures such as plain-weave, twill weave, satin weave, etc. (see fig. 1).

The first two textures are of special interest to this paper. While the plain-weave texture was extensively studied, the twill weave architecture has been relatively less investigated (Ng et al. (1998), Baruffaldi and Riva (2003)). A structural laminate is then obtained by laying up a number of laminas, each with its own specific orientation in a mold to obtain a desired 3D shape. The laminate is then processed, i.e. cured in an oven. After polymerization the laminate is cut to the desired form and finished.
Figure 2: Idealized cross-sections and definition of architectural parameters: (a) plain-weave lamina, (b) twill-weave lamina.

From the finite element modeling point of view, the repetitive nature of the textures suggests the identification of a so-called representative volume, RV (also termed unit cell). It can be thought of as the building block of the lamina and of the laminate by multiplication in the different spatial directions of the RV. Different textures are associated to different RV as shown in fig. 1. The definition of suitable boundary conditions on the RV will be discussed in a subsequent section. The cross-section of the woven laminas (see fig. 2) shows the idealized fill and warp yarn geometry and its dependence on texture. In the case of plain weave it is defined by a sequence of curved portions, while the twill weave texture is characterized by both straight and curved portions. The epoxy matrix envelops the yarn fibers and fills the voids at the edges of the yarn intersections.

A number of geometrical parameters related to the material structure need to be preliminary defined. With reference to the cross-sectional view of a lamina shown in fig. 2, the following variables are identified: the yarn thickness $b$, the yarn width $a$, the yarn-to-yarn gap $g$. Geometrical parameters such as radius of curvature $R_L$ and the radius of curvature of the yarn cross-section $R_T$ are also to be defined. Cutting, polishing and microscopic inspection can be used to obtain the realistic values of these parameters in actual laminates. Chapman (1994), Chapman and Whitcomb (1995), and Thom (1999) investigated the influence of the assumed yarn shape comparing the sinusoidal and the elliptical shape based on models with the same crimp angle, (i.e. different aspect ratios). The difference in modulus in the longitudinal direction was less than 20%, but more significant for the Poisson’s ratio.

### 3.2 Finite element modeling of RV

The finite element method requires the discretization into finite elements of the material geometry. Different strategies have been used depending on the objectives of the analysis. 2D plane strain models of lamina cross-section have been used to evaluate a number of geometric variables dependent on manufacturing (Medri et al. (1998)). Detailed 3D models have been developed...
using 3D brick and wedge finite elements to describe in detail the three-dimensional architecture of the woven composite laminates (Guagliano et al. (1997), Riva (1999), and Guagliano and Riva (2001)).

In the initial mesh generation phase, several simplifications of the geometry related to the shape of the yarn curvature and the shape and the constancy of the cross section are required in finite element model development. Several authors have advanced different descriptions of the yarn cross-sections. For example Naik (1994) proposed the sinusoidal shape. Quadrilaterals, circle, ellipse, compressed hexagon and lenticular areas (formed by two arcs) have also been proposed (Ng et al. (1998)). Thom (1999) introduced a new shape for the cross section with a blunt yarn edge thus improving the geometry and numerical stability of the model. The constant cross-section leads to the problem in modeling the gaps in the fabric (see fig. 2), which can be solved in different ways. Whitcomb (1991) and Whitcomb and Srirengan (1995) removed the gap but that works only for certain aspect ratios (wavelength/amplitude) as long as the shape of the cross section is kept constant.

3D finite element models of the RV normally involve more than a thousand elements. Commercial solid modeling software can be advantageously used to develop a parametric finite element mesh of the unit cell with solid finite elements according to the mapped meshing format. An example of a 3D mesh of the twill weave RV is shown in fig. 3 (Nicoletto and Riva (2004)). It consists of 73728 eight-noded-linear brick elements and 75595 nodes. When the damage initiation and evolution has to be modeled, the finite element models of the RV need to be extremely detailed. An example of a mesh refinement for the plain weave texture is shown in fig. 4.

Alternatively the submodeling technique whereby two separate meshes, i.e. the global and the local meshes, can be used to overcome the huge amount of CPU time and computer memory required. An example is shown in fig. 5, where the global response of the plain weave lamina (fig. 5(a)) can be determined by the local mesh, which described only 1/16 of the unit cell (i.e. 2014
3.3 Boundary conditions on RV

The finite element analysis of an RV is aimed at providing the macroscopic (global) response of a laminate, i.e. an ideal infinite medium subjected to uniform boundary conditions, either in stresses or displacements. Therefore, once the RV is identified and the mesh is generated, the continuity and the periodicity have to be enforced by appropriate boundary conditions on the boundary of the RV. Finite element modeling with a displacement formulation requires the specification of elements and 2379 nodes) (fig. 5(b)). The agreement between the results obtained with the global coarse mesh and those determined by the local fine mesh, for any configuration and crimp ratio, is excellent with differences of 3% (Riva (1999)).
boundary conditions that generate local (microscopic) stress and strain fulfilling the periodicity of the heterogeneous material.

Following Suquet (1985), the displacement fields $\mathbf{u}(x)$ inside the RV is assumed to be related to strain fields by the following general equation:

$$
\mathbf{u}(x) = \mathbf{u}_o + \sum_i \mathbf{E}_i x + \tilde{\mathbf{u}}(x),
$$

where the first term $\mathbf{u}_o$ and $\sum_i \mathbf{E}_i x$ are a rigid displacement and a rigid rotation of the RV, respectively. The other two terms are, respectively, $\mathbf{E}_i$ i.e. the macroscopic homogenous strain on RV and $\tilde{\mathbf{u}}(x)$ i.e. a periodic strain term with zero average value, which is associated to periodic part of the displacement field.

To apply boundary conditions in accordance to eqn 1, the following approach proposed by Carvelli and Poggi (2001) can be used. Four reference points are identified in the RV, A, B, C and D in fig. 6, corresponding to the mid nodes of the lateral sides $S_A$, $S_B$, $S_C$ and $S_D$. These nodes are used to enforce the repetitivity conditions on the boundaries of the RV. Suitable equations are defined so that all corresponding nodes on the opposite sides have to behave like the mid nodes.

Similarly, the lateral edges of the RV are to be constrained. Two node pairs (A, C) and (B, D), and the respective opposing sides ($S_A$, $S_B$) and ($S_C$, $S_D$) are identified and the following boundary conditions in the three directions are applied:

$$
u_i(S_A) - \nu_i(S_B) = \nu_i(A) - \nu_i(B) \quad \text{with} \quad i = 1, 2, 3,
$$

$$
u_i(sp_1) - \nu_i(sp_2) = \nu_i(A) - \nu_i(B) \quad \text{with} \quad i = 1, 2, 3.
$$

Furthermore, the following displacement conditions of the four reference nodes are enforced:

$$
\nu_2(A) = \nu_2(B) = \nu_2(C) = \nu_2(D).
$$

Analogous conditions are applied to the other node pairs (C, D), sides $S_C$ and $S_D$ and edges $sp_3$ and $sp_4$ (see fig. 6). If the top and bottom faces are not restrained the single lamina behavior is simulated. Alternatively, the response of a thick laminate can be studied by enforcing uniform displacement conditions to the top and bottom faces.

The finite element analysis of the RV provides the macroscopic response of the laminate that describes the relation between macroscopic stresses and strains defined as volumetric averages of the relevant microscopic stresses and strains variables. Therefore, the uni-axial tensile load case is obtained by imposing boundary displacements to the RV along the direction 3 (see fig. 6).

Figure 6: Reference points, sides and edges of the RV to be used in boundary condition specification.
The load level is controlled by the magnitude of the constant displacement $u_0$ applied to the face of the model normal to the loading direction while the corresponding point on the opposite face is constrained.

Macroscopic stresses $\Sigma_{ij}$ are determined with the following equation

$$\Sigma_{ij} = \frac{1}{A} \sum_{n} F_i^{(n)},$$

(5)

where $F_i$ are the $n$ nodal forces acting on the appropriate reference surface of area $A$. The macroscopic strain $E_{ij}$ are determined dividing the reference face displacement $u_0$ by the initial length of the model.

### 3.4 Constitutive laws

The global mechanics of the woven laminate is obtained in the finite element approach starting from the mechanical properties of the constituent materials. Here the main interest is for the graphite fiber yarns and an epoxy polymer matrix. The matrix, if made of epoxy resin, can be assumed as homogenous, isotropic and linear elastic. The yarn is a transversely isotropic linear elastic material, whose mechanical properties are available from experiments or are obtained with a preliminary modeling step (tables 1 and 2). In the latter case each yarn being constituted by fibers and matrix can be analyzed using a homogenization approach and the mechanical properties of the single phases, graphite fiber and epoxy. If the fibers are regularly distributed in the yarn, it can be considered a unidirectional fiber reinforced composites with fiber volume fraction equal to the packing density. Depending on the assumed fiber pattern (i.e. hexagonal) the RV of this UD composite can be easily identified and meshed with 3D finite elements.

### 3.5 Modeling damage evolution

When the strength response of a woven fiber reinforced laminate is sought, the simple elastic models for yarn and matrix are not adequate. The failure has to be simulated introducing appropriate material models for the matrix and the yarns into the finite element model of the RV.

| Table 1: Elastic properties of the carbon fiber yarns (Blackketter et al. (1993)). |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $E_{11}$ (GPa)  | $E_{22}, E_{33}$ (GPa) | $G_{12}, G_{13}$ (GPa) | $G_{23}$ (GPa) | $\nu_{12}, \nu_{13}$ | $\nu_{23}$ |
| 150             | 10              | 5.7             | 3.4             | 0.3             | 0.5            |

| Table 2: Directional strengths of a carbon fiber reinforced yarn (Blackketter et al. (1995)). |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| $S_{11}$ (MPa)  | $S_{22}, S_{33}$ (MPa) | $S_{12}, S_{13}$ (MPa) | $S_{23}$ (MPa) |
| 2550            | 152             | 97              | 55              |
The determination of the ultimate strength of textile-fabric composites is often based on macro-mechanical strength theories, such as maximum stress theory, maximum strain theory or Tsai–Wu’s theory. In Ko (1989) a maximum strain energy criterion is proposed to predict the yarn failure and the ultimate strength of a composite, while application of the maximum stress theory to plain-weave composites is presented in Ito and Chou (1998). Among the limitations in applying the macro mechanical strength theories to the failure analysis of woven composites there is the fact that the applicable strength parameters may not be determined only by means of experiments on UD specimens (Carvelli and Poggi (2001)).

The phenomenon of the progressive failure of woven composites can be simulated with the finite element method on the basis of the damage mechanisms for the matrix and the yarns. The matrix is assumed to be isotropically brittle and the maximum principal stress criterion is applied. When, during the loading phase, the matrix strength is locally exceeded, the elastic properties of the matrix are modified, i.e. reduced, element by element, of a predefined amount. In the yarns it is assumed that fiber and matrix are perfectly bonded.

With reference to the material coordinates, four different damage mechanisms are considered for a yarn and classified in table 3 (Zako et al. (2003)). Referring to the principal material directions (i.e. longitudinal L, and transverse T and S), the mechanism L represents fiber rupture, the other mechanisms represent transverse and shear fractures of the epoxy within the yarn. Mechanisms L and T are controlled by normal longitudinal and transverse stresses, while mechanisms LT, SL and TS are mainly controlled by the shear stresses (table 3). During the loading phase, the local stresses in the yarns are determined at each integration point of every finite element, referred to the material directions and compared to the directional strengths (table 3).

When the directional strength is exceeded, the corresponding elastic modulus \( q_{ij} \) is discounted of a prescribed amount \( D_{ij} \), according to the relationship:

\[
q_{ij}^D = q_{ij}(1 - D_{ij}), \quad \text{where } i, j = L, T, S.
\]  

The parameters \( D_{ij} \), associated to the material degradation, are defined in table 4, according to the Blackketter method (1993). Similarly, see table 2 for the strength data. The damage parameters are based on experimental considerations (Blackketter et al. (1993)). The epoxy matrix was assumed linear, elastic and isotropic with Young’s modulus \( E = 4.4 \) GPa; Poisson’s ratio \( \nu = 0.34 \); tensile strength \( S = 159 \) MPa.

A FORTRAN routine was developed to implement the failure conditions in the ABAQUS input file. An assessment of the different computational features embedded in the ABAQUS code for damage evolution modeling, such as element integration, geometric nonlinearity and time increment, was reported in Guagliano et al. (1997).

4 Prediction of the stiffness response

The finite element analysis of the RV of the woven composite provides the global constitutive law that describes the relation between macroscopic stresses and strains defined as volumetric averages of the relevant microscopic stresses and strains variables. This section presents several applications of the FE approach to the prediction of stiffness response and compared with experimental data. The effects of a number of modeling parameters on the 3D elastic properties of weave architectures are also presented.

It is also stressed at this point that a parallel extensive experimentation of different woven architectures was carried out to gain the necessary background information for model set-up and verification. Shape and dimensions of yarn cross-section, actual crimp ratio, ply thickness, lay-up...
Table 3: Parameters of the fiber yarn damage model.

<table>
<thead>
<tr>
<th>Damage mode</th>
<th>Critical condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode L</td>
<td>$\sigma_L = X_L$</td>
</tr>
<tr>
<td>Modes T &amp; LT</td>
<td>$\sigma_T = X_T$</td>
</tr>
<tr>
<td></td>
<td>or $\tau_{LT} = Y_{LT}$</td>
</tr>
<tr>
<td>Modes S &amp; ST</td>
<td>$\sigma_S = X_S$</td>
</tr>
<tr>
<td></td>
<td>or $\tau_{SL} = Y_{SL}$</td>
</tr>
<tr>
<td>Mode TS</td>
<td>$\tau_{TS} = Y_{TS}$</td>
</tr>
</tbody>
</table>

Table 4: Degradation coefficients of the elastic properties (Blackketter et al. (1993)).

<table>
<thead>
<tr>
<th>Degradation coefficient $D_{ij}$</th>
<th>$Q_{ii}$</th>
<th>$Q_{ii}$</th>
<th>$Q_{ix}$</th>
<th>$Q_{ix}$</th>
<th>$Q_{ix}$</th>
<th>$Q_{ix}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MECHANISM L</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>MECHANISM T</td>
<td>1.0</td>
<td>1.0</td>
<td>0.01</td>
<td>1.0</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>MECHANISM S</td>
<td>1.0</td>
<td>0.01</td>
<td>1.0</td>
<td>1.0</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>MECHANISM LT</td>
<td>1.0</td>
<td>1.0</td>
<td>0.01</td>
<td>1.0</td>
<td>0.01</td>
<td>1.0</td>
</tr>
<tr>
<td>MECHANISM LS</td>
<td>1.0</td>
<td>0.01</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>0.01</td>
</tr>
<tr>
<td>MECHANISM TS</td>
<td>1.0</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>
arrangement were determined by a optical investigation of sectioned specimens. The FE models to be used in the correlation with the experiments accurately represented the material architecture. When material properties of the constituents had to be estimated, direct comparison of tensile test and FE predictions provided a valuable calibration and/or verification of the assumptions.

From the modeling point of view, the yarn cross-section (see fig. 2), was assumed to be lenticular in shape, the yarn-to-yarn gap was assumed equal to zero and the tow and fill yarns had the same cross-section. Furthermore, for the twill weave texture, the lengths of the straight and curved portions were both equal to $a$. If the only strain component in the global direction 3 is applied, then the homogeneized stiffness $Q_{33}$ of the woven composite is obtained

$$Q_{33} = \frac{\Sigma_{33}}{E_{33}}, \quad (7)$$

where $\Sigma_{ij}$ and $E_{ij}$ are the homogenized stress and strain. Figure 7 shows the in-plane displacement elastic components in the global directions superposed to the deformed mesh (amplification factor of 3). It is noted that the tow yarns are stretched upon tension loading while the fill yarns are further bent.

The microscopic stresses referred to the local reference system are shown in fig. 8. The longitudinal stress in the tow and fill yarns is nonuniform and periodic due to the repetitive nature of the textile. A comparison of the predicted global longitudinal stiffness for the plain-weave and the twill-weave textures as a function of the crimp ratio is shown in fig. 9, where upper and lower bounds given by the single lamina and the infinite-ply (thick) laminate. Inspection of fig. 9 shows that the plain weave texture is stiffer than the twill weave texture at low crimp ratios in both the single lamina and the thick laminate. But as the crimp ratio ($b/a$ in fig. 2) increases the opposite is true. These analytical results correlate favorably with previous results found in the literature and obtained experimentally (Riva (1999), Nicoletto and Riva (2004)).

The plot of fig. 10 shows that the single twill weave lamina is significantly less stiff (more than 20%) than that of a thick laminate because it lacks the constraint in out-of-plane displacements provided by adjacent plies. This is especially true for low crimp ratios as it is often the case...
Figure 8: Microscopic stress distributions in the local yarn direction L. Macroscopic deformation $E_{33} = 1.2\%$, twill-weave lamina (Baruffaldi et al. (2003)).

Figure 9: Role of texture on longitudinal stiffness (TW – twill weave; PW – plain weave).

in practice. Figure 10 shows also that the addition of only another lamina has a considerable stiffening effect.

The curvature of the yarns, described by the crimp ratio $b/a$ strongly affects the global stiffness as shown in figs 9 and 10. If the longitudinal elastic modulus of the yarn is considered (i.e. 150 MPa, table 1), the stiffness of the woven laminas and laminates is considerably reduced (i.e. from 1/3 to 1/5 of the yarn reference elastic modulus). This effect is due to the induced flexural and torsional deformations of the yarns when used in textiles form. An increase in crimp ratio increases the yarn curvature with a further reduction in stiffness.

When an actual laminate cross-section is inspected (see fig. 11), a number of deviations from the idealized lamina model are observed. For example lamina stacking is inevitably irregular
with longitudinal and lateral shifts. Therefore finite element models with shifted configurations were developed, Riva (1999), and are presented in fig. 12. The global stiffness of the two configurations, shifted and symmetric, vs. crimp ratio and for different number $N$ of plies in the laminates is given in fig. 13. It can be noticed that the two configurations have different trends for the intermediate number of layers. The results were obviously equal for the single and for
5 Prediction of damage evolution and strength response

This section provides an overview of results obtained by the finite element method as applied to the strength prediction of woven composites subjected to tensile load. Several factors make the prediction of strength more complicated than the prediction of global stiffness. First of all, the number of failure modes that can cause the composite failure (fiber, matrix or interface failure (see fig. 14)): matrix cracks in fill yarns and delamination between orthogonal yarns precedes final failure of longitudinal fibers. Furthermore, the random nature of failure (i.e. statistical effects), the localized nature of failure initiation and the influence of the associated stress field are also important.

The yarn can be considered as a unidirectional fiber reinforced composite. A reasonable prediction of its tensile strength is given by the fiber strength and the fiber volume fraction of the yarn. Fiber strength, however, is expected to vary statistically among fibers and to depend on the measurement length.

A parametric FE model of a woven composite provides a useful tool for analyzing the impact on strength of specific architectural features. The crimp ratio, for example, considerably influences the mechanical response of the composite because it is related to yarn curvature and therefore stress non uniformity in the yarns. An increase in crimp ratio, not only reduces laminate stiffness, reduces also the laminate strength and strain to failure as shown in fig. 15.

These results are for a plain weave architecture, but similar conclusions can be reached when the twill weave texture is examined as shown in fig. 16. The simulated tensile tests of figs 15 and 16 show that, when the tow yarns are aligned with load, the response is linear up to failure, which is associated to an abrupt stress drop. However, an increase in crimp ratio apparently favors some damage development, because the non uniform stress of the tow yarns is more significant.
Figure 14: Damaged woven laminate.

Figure 15: Stress-strain curves for different crimp ratios CR (number of plies of the model \( N = 4 \), experiment \( N = 8 \)).

Test data available for the one texture configuration are also introduced in fig. 15 demonstrating the remarkable correlation that can be achieved. Aspects, such as different constituents, different architecture, number of plies, etc., can be addressed with adequate accuracy using the present FEM-based method. However, a close match between experiments and prediction may need sometimes fine-tuning of the numerical procedures and of the input properties of the constituents.

The role of the inevitable irregularity in ply stacking (fig. 11) on the strength response is shown in fig. 17. The predicted stress-strain curves obtained for the two configurations, shifted and symmetric (see fig. 12), for a different number of plies are presented along with experimental observations, Riva (1999). The number of plies does not influence significantly the mechanical response in the case of the symmetric configuration. The peak stress obtained with the two configurations and for the \( N = \infty \) are nearly the same. The experimental response (for an eight-ply laminate) correctly falls between the predicted curves for \( N = 4 \) and \( N = \infty \).
Fig. 16: Influence of the crimp ratio on the tensile response of twill-weave laminates.

Fig. 17: Stress-strain curves for the (a) shifted and (b) symmetric configurations (Crimp ratio = 0.07).

Damage evolution during tensile loading of the twill weave laminate is discussed in terms of global stress-strain curve up to ultimate failure and damage mechanism activation. The material architecture and the nature of constituent materials are such that failure of a woven composite is characterized by a number of different mechanisms and modes (fiber, matrix or interface failure). The stress-strain curve of a laminate having a low crimp ratio ($b/a = 0.084$ as in experiments) is presented in fig. 18 and it is used to identify (with letters) the different stages of the damage process. The time increments were optimized with a first step up to conditions of damage initiation followed by many very small time increments up to complete failure. The evolution of damage...
Figure 18: Stress-strain curve of a twill-weave lamina and sequence of damage mechanisms (Crimp ratio $b/a = 0.084$).

was obtained by monitoring continuously the value of the state variables at each integration point. Referring to fig. 18 and to table 3 the following sequence of events is predicted:

(a) initially damage develops in the tow yarns according to the mechanism T in table 3, that is in the transverse direction where there is yarn overlapping and maximum out-of-plane deformation;

(b) and (c) with increasing deformation, damage spreads along the entire yarn length. As a consequence the damage transfers to the resin-rich matrix. Damage of the matrix develops on the top and bottom surfaces of the model where yarn are superposed;

(d) finally, when the macroscopic strain reaches values around 1%, fiber fracture in the tow yarns (mechanism L in table 3), is predicted with a subsequent sudden catastrophic rupture of the model.

6 Conclusions

A predictive method of the mechanical properties of woven fabric laminates based the finite element method has been presented. It is based on the definition of a representative volume RV of the texture and on the application of suitable boundary conditions to investigate the macroscopic
mechanical behavior. Three-dimensional finite element models were shown to predict both the stiffness and the strength of woven fabric laminate. This approach makes it possible to include in the model all the important parameters that influence the mechanical behavior, such as lamina thickness, the yarn shape and orientation, texture, fiber volume fraction and mechanical characteristics of the constituent materials. The predictive capability of the proposed method was verified using experimentally determined elastic behavior and ultimate strength of woven fabric laminates.

References

References are sorted according to year and divided in (i) general, (ii) analytical (when the analytical approach is adopted) and (iii) FEM (when the finite element method is used).

General


Analytical

Fracture and Damage of Composites


**FEM**


Fracture and Damage of Composites


