Role of hysteretic damping in seismic response of the ground under large earthquakes

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Abstract

Parametric studies are carried out to investigate the role of hysteretic damping in seismic response analysis. Peak ground acceleration is found to have an upper bound under very large earthquakes; the hysteretic damping does not affect the peak ground acceleration. On the other hand, the response acceleration becomes large in the high frequency region as the hysteretic damping increases. It can be explained that the stiffness after the unloading is large so as to increase the hysteretic damping ratio. This is a quite different feature from that of the viscous damping; the viscous damping and the hysteretic damping have been understood to have the same mechanical nature to suppress the vibration.

Keywords: earthquake, hysteretic damping, shear strength, upper bound acceleration.

1 Introduction

The nonlinear behaviour of soils for the seismic response analysis of the ground is usually characterised by a strain dependent shear modulus and a damping ratio. As such, the damping ratio is supposed to be an important mechanical property. It has been expected to suppress the earthquake response. In other words, earthquake motion becomes smaller as the damping ratio increases.

On the other hand, the authors showed a different point of view [1]. The peak ground acceleration (PGA) does not exceed a certain value, namely, the upper bound acceleration under large ground motions. It means that the damping ratio does not work to suppress the ground acceleration at the ground surface. Another aspect was also noted in addition to this observation; the response at high frequency is amplified as the damping ratio increases.
In the previous study, however, only one example was shown. A series of parametric studies is carried out in this paper in order to investigate what happens when the hysteretic damping changes.

2 Constitutive models

Three stress-strain models were used in this study, which are called Hyperbolic, H-D, and H-D/wE. All models use the same hyperbolic model for the skeleton or backbone curve,

\[ \tau = \frac{G_0 \gamma}{1 + \gamma / \gamma_r} \quad (1) \]

where \( \tau \) and \( \gamma \) denote shear stress and shear strain, respectively, \( G_0 \) denotes an elastic shear modulus, and \( \gamma_r \) denotes a reference strain. The shear strength \( \tau_f \) of this model is evaluated as

\[ \tau_f = G_0 \gamma_r \quad (2) \]

The hysteresis curves are defined differently as follows

1) Hyperbolic: ordinary hyperbolic model whose hysteresis loop is developed by applying the Masing's rule to the skeleton curve.

2) H-D: damping characteristics is evaluated according to the proposal by Hardin and Drnevich [2],

\[ h = h_{\text{max}} \left(1 - \frac{G}{G_0}\right) \quad (3) \]

where \( G \) denotes a secant shear modulus and \( h_{\text{max}} \) is a maximum damping ratio. The hysteresis curve that satisfies this equation can be made by using the method proposed by the authors [3]. The hyperbolic equation used for the hysteresis curve in this method is the same as eqn (1), but the two parameters \( G_0 \) and \( \gamma_r \) do not have mechanical meaning because they are automatically evaluated in order to get the damping ratio defined according to eqn (3).

3) H-D/wE: controlled stiffness at unload in addition to the damping characteristics according to eqn (3). Many constitutive models assume that stiffness at unload is the same as the initial or elastic modulus. The stiffness at unload is read from the hysteretic loop of the cyclic shear deformation characteristics test and is shown in fig. 1, where \( G_{\text{r0}} \) denotes stiffness at unload. As shown in fig. 1, the stiffness at unload is not a constant value, but decreases with strain. This behavior is simulated by the hyperbolic equation

\[ \frac{G_{\text{rh}}}{G_0} = \frac{1 - G_{\text{min}}/G_0}{1 + \gamma / \gamma_{r0}} + \frac{G_{\text{min}}}{G_0} \quad (4) \]

where \( G_{\text{min}} \) denotes minimum stiffness at unloading, \( \gamma_{r0} \) denotes a parameter similar to the reference strain. The hysteresis curve that satisfy eqn (4) is obtained by the previous method, but the Ramsberg-Osgood model is used in order to add the new condition (eqn (4)).
3 Brief review of previous calculations

3.1 Soil profiles and material

A ground in the Tokyo city area [4] shown in fig. 2 was analysed. Here, $V_s$ denotes S-wave velocity, $\rho$ denotes density, $c$ denotes cohesion, and $\phi$ denotes internal friction angle. Model parameters are set as in table 1.

Cyclic shear deformation characteristics are shown in fig. 3. Here, both H-D and H-D/wE (the H-D type models, hereafter) show the same damping characteristics and, as well known, the hyperbolic model shows a much larger damping ratio at large strains than those of the H-D models. The stress-strain curve is shown in fig. 4 for shear strain amplitudes of 0.6% and 4%. The curves are quite different for the hyperbolic model and the H-D type models, but those for the two H-D type models are similar to each other. This indicates that the effect of shear modulus at unload does not affect the seismic response analysis; this will be proved later in this paper.

Figure 1: Shear modulus $G$ and stiffness at unloading $G_{r0}$ as a function with respect to strain $\gamma$. 
Figure 2: Soil profiles.

Table 1: Model parameters.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\gamma_c$</th>
<th>$h_{max}$</th>
<th>$\gamma_0$</th>
<th>$G_{min}/G_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sand</td>
<td>$8.63 \times 10^{-4}$</td>
<td>0.22</td>
<td>0.002</td>
<td>0.4</td>
</tr>
<tr>
<td>Clay</td>
<td>$1.42 \times 10^{-3}$</td>
<td>0.22</td>
<td>0.013</td>
<td>0.1</td>
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</table>

Figure 3: Dynamic deformation characteristics of sand and clay.
3.2 Earthquake motions

Two earthquake motions are chosen among those shown in ref. [4]. One of them has a limited number of large amplitude waves while the other, a large number of large amplitude waves. They are called the shock wave and the vibration wave, respectively. Their waveforms are shown in fig. 5. Only the result obtained from using the shock wave is mainly shown in this paper since both results from the two earthquake records show a similar nature. Since large strain behavior is of interest, the acceleration is increased so that the peak acceleration becomes 8 m/s$^2$ at the outcrop base layer.

3.3 Seismic response and discussion

The maximum response is shown in fig. 6. The peak acceleration decreases rapidly from GL-5.8 m, resulting in about 2 m/s$^2$ at the ground surface. Since the layer between GL-2.8 and 5.8 m (clay layer) shows large strains up to several
### Figure 6: Maximum responses.

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Soil Type</th>
<th>$V_s$ (m/s)</th>
<th>Density (t/m$^3$)</th>
<th>Peak Acceleration (m/sec$^2$)</th>
<th>Peak Displacement (cm)</th>
<th>Max. Stress (kN/m$^2$)</th>
<th>Max. Strain (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>Sand</td>
<td>234</td>
<td>1.75</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td>1.9</td>
<td>Sand</td>
<td>234</td>
<td>1.95</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td>2.8</td>
<td>Clay</td>
<td>70</td>
<td>1.75</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>20</td>
</tr>
<tr>
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<td>Clay</td>
<td>70</td>
<td>1.75</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>20</td>
</tr>
<tr>
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<td>1.75</td>
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<td>1.75</td>
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<td>4</td>
<td>6</td>
<td>20</td>
</tr>
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<td>7.6</td>
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<td>261</td>
<td>2.00</td>
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<tr>
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<td>6</td>
<td>20</td>
</tr>
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<td>4</td>
<td>6</td>
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</tr>
<tr>
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<td>6</td>
<td>20</td>
</tr>
<tr>
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<td>6</td>
<td>20</td>
</tr>
<tr>
<td>13.8</td>
<td>Sand</td>
<td>231</td>
<td>1.95</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td>14.8</td>
<td>Sand</td>
<td>221</td>
<td>1.95</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>20</td>
</tr>
<tr>
<td>16.0</td>
<td>Base</td>
<td>400</td>
<td>2.10</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>20</td>
</tr>
</tbody>
</table>

- **Shock wave**
- **Vibration type wave**

**Note:** The table and diagram illustrate the distribution of various parameters at different depths, including $V_s$, density, peak acceleration, peak displacement, maximum stress, and maximum strain, across different soil types (Sand and Clay). The shear strength values are also indicated at various layers.
percent, shear stress in this layer reaches nearly the shear strength, which can be confirmed through the chained line (shear strength) in fig. 6. If a layer reaches shear strength, the acceleration above this layer reaches the limit acceleration $\alpha_{ult}$ [5]. Fig. 7 shows an equilibrium condition of a soil column. The left figure shows the equilibrium of an infinitesimally small element, which is used to develop a wave equation. On the other hand, the right figure shows a soil block above a weak layer and the equilibrium condition yields

$$\tau = \int_0^z \rho(z)\ddot{u}(z)dz \approx \ddot{u}_{ave} \sigma_v / g$$

(5)

where $\sigma_v$ denotes overburden stress, $\ddot{u}_{ave}$ denotes average acceleration of the soil block, and $g$ denotes acceleration of gravity. Then, if the shear stress reaches the shear strength $\tau_f$, the maximum value of the stress, the acceleration reaches the maximum value $\ddot{u}_{ult}$. In other words,

$$\ddot{u}_{ult} = \tau_f g / \sigma_v$$

(6)

Applying this equilibrium condition to the 4th layer (GL-2.8 to 3.8 m), the expected upper bound acceleration becomes 2.08 m/s, which agrees with the maximum acceleration at the ground surface in fig. 6. It is emphasized that the damping ratios are quite different in these three cases, but this does not affect the maximum acceleration.

Response spectra are computed from the acceleration at the ground surface; these are shown in fig. 8. There is no significant difference for periods longer than about 0.5 s, but the hyperbolic model shows much larger accelerations than the other two cases for shorter periods. It is emphasized that the damping ratio is largest in the hyperbolic model. If a large damping ratio suppresses the vibration, the response acceleration obtained from the hyperbolic model must be smaller than that from the other two cases.

It is also noted that the two H-D type models show almost similar response. This indicates that the stiffness at unload is not a big issue. The hysteresis loops essentially acquire a similar shape because they have a spindle shape with the same area.

Figure 7: Equilibrium of infinitesimally small element (left) and a block above a weak layer (right).
The stress-strain curves at the 6th layer where shear strain becomes largest are shown in fig. 9. The apparent stiffness after unloading seems much larger in the hyperbolic model than those in the H-D models. As seen in fig. 4, the stiffness after unloading should be kept large so that the damping ratio is kept large in the hyperbolic model. On the other hand, as the damping ratio is small in the two H-D models, the stiffness after the unloading is smaller than that of the hyperbolic model. Therefore, the apparent stiffness is larger in the hyperbolic model than in the two H-D models after unloading occurs in the skeleton curve. This is the
reason why the hyperbolic model shows larger response acceleration than the two H-D models.

4 Effect of input acceleration

If an upper bound acceleration associated with small soil shear strength exists, response amplification depends on the input acceleration. Thus, a parametric study is carried out to investigate the effect of the magnitude of the input motion on the seismic response. The same soil profile and input earthquake motion are used in the parametric study except that the maximum damping ratio $h_{max}$ is set at 25%. The magnitude of the input earthquake is scaled and applied. The input motion of the previous section is used as the standard value; the magnification factor is the ratio of the applied to this input motion.

![Figure 10: Maximum response.](image)

Fig. 10 shows the maximum response for magnification factors 0.01, 0.1, 0.5, 1.0, 1.5, and 2.0. While the input acceleration remains very small, the maximum acceleration increases upward. As the input acceleration increases, however, the maximum acceleration distribution shows constant values in certain layers. For example, the maximum acceleration is nearly constant between GL and GL-3.8 m and between GL-5.8 m and 12.8 m. Just below these layers, the maximum acceleration decreases rapidly. Shear strains in these layers are more than 1%, which indicates that shear stresses nearly reach the shear strength (see...
This means that these constant maximum accelerations are upper bound accelerations. There are two key layers defining the upper bound acceleration, both of which control the maximum accelerations above these layers. The shear strength of the upper layer is smaller than that of the lower layer.

Fig. 11 shows the variation of the maximum acceleration at the ground surface under earthquakes with variable magnitude and fig. 12 shows the corresponding amplification factor (maximum acceleration at the ground surface divided by maximum acceleration of the input motion). The maximum acceleration increases as the input motion becomes large, but the rate of increase gradually decreases, resulting in constant maximum acceleration or upper bound acceleration. This feature is the same as the one mentioned by Idriss [6] and Suetomi and Yoshida [7]. In the same manner, the amplification factor is 2.35 under the very small input or for elastic response, but it decreases quickly as nonlinear behaviour becomes significant. It is noted that the upper bound acceleration depends on the shear strength and depth of the weakest layer, thus the relationship in fig. 11 is not a unique curve but depends on site.

Figure 11: Maximum acceleration vs. input motion.

Figure 12: Amplification ratio vs. input motion.
As described above, this behaviour was first noted by Idriss [6], but the mechanism was not known at that time. Through this study, it becomes clear that it is caused because there is upper bound acceleration.

5 Effect of hysteretic damping

The maximum damping ratio $h_{\text{max}}$ is chosen as a parameter in this section. The theoretical maximum value of $h_{\text{max}}$ is $2/\pi$. Then $h_{\text{max}}$ is varied from 10% to 60%.

Fig. 13 shows the maximum response. The maximum acceleration at the ground surface is nearly constant regardless of the $h_{\text{max}}$. Therefore, it is clear that the damping ratio does not work to suppress the earthquake motion.

Fig. 14 shows the relationships of maximum acceleration and velocity to $h_{\text{max}}$. The maximum acceleration at the ground surface seems constant in fig. 13, but it decreases with $h_{\text{max}}$. Then, it seems that $h_{\text{max}}$ works to suppress the response of the ground. However, when looking at fig. 13 in detail, a different feature can be identified.

The maximum strains below GL-2.8 m increase as $h_{\text{max}}$ increases. This indicates that the key layer that controls the upper bound acceleration moves downward. As can be seen from eqn (6), the upper bound acceleration decreases as the depth of the key layer increases or $\sigma_v$ increases.

![Figure 13: Maximum response.](image-url)
The maximum damping ratio $h_{\text{max}}$ is usually between 15 and 25 degrees in the actual soil. For these damping ratios, the change of the maximum acceleration is not large. In this context, the damping ratio is said not to affect the maximum acceleration at the ground surface.

Fig. 15 shows the acceleration response spectra for a damping ratio of 5%. The response acceleration becomes larger in shorter period as $h_{\text{max}}$ increases. On the other hand, it becomes smaller in longer period. The boundary is around 0.5 s. It is noted that a shorter period, less than 0.5 s, is a very important period for many buildings or other civil engineering structures. From the discussion above, it is clear that this large response acceleration occurs as the stiffness after the unloading becomes larger as $h_{\text{max}}$ increases. This is quite a different feature that is associated with the word “damping”.

![Figure 14: Maximum responses vs. Maximum damping ratio.](image1)

![Figure 15: Acceleration response spectra.](image2)
6 Concluding remarks

A parametric study was carried out in order to investigate the how hysteretic damping mechanism works in the case of earthquake response of the ground. The following conclusions were reached.

1) Large hysteretic damping does not imply small response or amplification under large earthquake motion because the maximum acceleration has an upper bound associated with the failure of weak layers.
2) The stiffness after the unloading becomes large as the damping ratio increases because the area of the hysteresis loop must be large. This large stiffness excites ground shaking at high frequency or small period, resulting in large response accelerations within the small period range.

The second conclusion concerns the quite different feature that the term “damping” has. In this sense, hysteresis damping is not the same mechanical property as viscous damping.

References