Optimum design of tuned mass damper systems for seismic structures

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Abstract

Tuned mass dampers are well known devices for the passive control of vibrations in buildings subjected to earthquake loadings. Various methods have been proposed for the design of tuned mass damper (TMD) systems. In the present work, a method is suggested for obtaining the values of the parameters required for designing an efficient TMD system when attached to a SDOF system. In this method, the values of the optimum frequency ratio and optimum damping ratio for the TMD system are defined as the values that will reduce the maximum displacement of the structure to a minimum value when subjected to a specific earthquake time-acceleration history. For this purpose, a MATLAB computer program is developed. The program consists of a dynamic analysis subroutine embedded in a nonlinear constrained optimization program. The suggested method is used in selected case studies showing its efficiency when compared to other methods for designing TMD systems attached to SDOF systems.

Keywords: tuned mass damper, control of structures, optimum design, earthquake time-history, optimum design.

1 Introduction

The tuned mass damper (TMD) system represents an important type of passive control device for structures subjected to dynamic loads. It can be installed to new or existing structures to improve their resistance to earthquakes and winds. A TMD system consists of a mass, a spring and a damper. If these properties are properly designed and selected, then the TMD device can be effective in suppressing undesirable vibrations induced by earthquake or wind loads.
Obtaining the optimal design for a TMD system has been the goal of many researchers for many decades. In these researches, various assumptions have been made regarding the simulation of the acting dynamic force, its location and the criteria used in defining the optimal design parameters. Den Hartog [1] has derived the formula for the optimum values of the TMD parameters for a SDOF structure when subjected to a harmonic load. An extension has been made by Warburton and Ayorinde [2] and Tsai and Lin [3], where damping in the main mass was considered and several types of harmonic excitations were examined. Extensive research was also made by Warburton [4] and Rana and Soong [5], where formulas for several types of excitations were developed. In this case the harmonic and random excitations were applied either on the main system or at the base of the structure. Sadek et al. [6] suggested a method for estimating the design parameters of TMDs for seismic applications; the criterion used to obtain the optimum parameters was to select, for a given mass ratio, the frequency and damping ratios that would result in equal and large modal damping in the first two modes of vibration.

As can be noticed from the brief literature review, various assumptions have been made regarding the earthquake loading (harmonic or random), and about the location of the acting force (on the structure or at its base). In the present work, a method is suggested to obtain the optimum TMD parameters. In this method, earthquake records are used to obtain the optimal TMD properties when subjected to the earthquake forces at its base.

2 Description of the suggested method

Consider the TMD system with mass \( m_d \), spring stiffness \( k_d \) and damping coefficient \( c_d \) shown in fig. 1, attached to a SDOF structure with a stiffness \( K \), mass \( M \) and a structure damping coefficient \( C \). The resulting overall structure will have two degrees of freedom. The basic features of the method suggested for obtaining the optimum design parameters of a TMD system are explained in the following sections.

![Image](image1.png)

Figure 1: SDOF structures with the TMD system.

2.1 Excitation force

In order to obtain the optimum design parameters of the TMD system, certain assumptions regarding the excitation force should be made. To simulate actual
behavior, it is assumed in this study that the structure is subjected to base excitation. The excitation force vector is computed from the time acceleration history for a given earthquake. In the present work, all time acceleration histories are taken from actual earthquake records. It should be noted that research is carried out on generating ground earthquake time histories for design (Fengxin et al. [7], Abdalla and Hag-Elhassam [8] and Varpasuo and Gelder [9]).

2.2 Optimization criterion and optimization parameters

In the present work, the optimization criterion and parameters used by many authors is adopted (Den Hartog [1], Rana and Soong [5] and Tsai and Lin [3]). According to this criterion, the optimum design parameters $k_d$, $c_d$, for a given $m_d$ are defined as those values that minimize the maximum relative displacement of the structure when subjected to an excitation. The maximum relative displacement of a regular SDOF shear building frame usually occurs at the top.

2.3 Statement of the problem as a constrained nonlinear optimization problem

Fig. 2 shows a shear building structure provided with a TMD system at the top floor. When the structure is subjected to a given earthquake excitation (acceleration-time history $\dot{x}_g$), then $u_t$ is the relative displacement occurring at the top of the frame; $u_{max}$ is defined as the maximum value of $u_t$ occurring during the earthquake duration.

For given total structure properties ($[K]$, $[C]$, $[M]$), TMD mass $m_d$ and earthquake excitation $\dot{x}_g$, $u_{max}$ will be a function of $c_d$, $k_d$ only. This problem can be stated as the following optimization problem: Find $c_d$, $k_d$ that minimizes the following objective function:

$$u_{max} = f(c_d, k_d),$$ subjected to the inequality constrains $c_d > 0$ and $k_d > 0$. 

Figure 2: Shear building under earthquake excitation with the TMD system.
This problem can be classified as multivariable, nonlinear constrained minimization problem. For the treatment of such problem, one of the functions available in the MATLAB [10] optimization toolbox is used.

2.4 Developed computer program for optimum design of a TMD system

As a first step, a MATLAB computer program is developed for the analysis of SDOF structure with TMD system when subjected to earthquake excitation. This program is based on Newmark’s method. As a second step, the above mentioned program is embedded in another nonlinear constrained optimization MATLAB program. This last program is used to obtain the optimum design parameters of the TMD system as mentioned in previous section. Details of the MATLAB software are given in [11].

2.5 Convergence of the proposed method to the optimum solution

To demonstrate the capability of the proposed method to identify the optimum design parameters for a TMD system when attached to a structure, many problems are examined (Al-Taweel [11]). One of these verification problems is discussed herein. A single story shear building with properties shown in fig. 3 is considered. A TMD is attached to the top with mass \( m_d = 1.5 \) ton equal to 3\% of the total mass of the structure.

\[
\begin{align*}
C &= 6.283 \text{ kN} \cdot \text{s/m} \\
K &= 1973.9 \text{ kN/m} \\
M &= 50 \text{ ton} \\
m_d &= 1.5 \text{ ton}
\end{align*}
\]

Figure 3: Structure studied as verification problem.

The objective is to determine the optimum value of TMD stiffness \( k_d \) and damping \( c_d \) that will minimize displacement \( u_{\text{max}} \) at the top when the structure is subjected to El-Centro earthquake excitation. To understand the variation of \( u_{\text{max}} \) with various values of \( k_d \) and \( c_d \), the first stage software is used to compute \( u_{\text{max}} \) for \( k_d \) in the range from 40 to 70 kN/m in steps of 2 and for \( c_d \) in the range from 0 to 1.9 kN-s/m in steps of 0.1. The results are plotted as three-dimensional function surface \( [u_{\text{max}} = f(c_d, k_d)] \) in fig. 4 and as contour lines in fig. 5. Next, the second stage software is used to obtain the optimum design parameters through a minimization process, as described in previous sections. For the minimization process, it is given that for this problem, the upper bound and the lower bound value of the stiffness \( k_d \) are 0 and 1000 kN/m, respectively, while the upper bound and lower bound of the damping are 0 and 100 kN-s/m, respectively. After running the program, it is found that the optimum values of stiffness and damping of the TMD are \( k_d = 54.08 \) kN/m and \( c_d = 0.643 \) kN-s/m. The
corresponding value of $u_{\text{max}}$ is 0.1179 m. When projecting these results on the contour plot of fig. 5, it can be clearly noticed that the solution given by the second stage optimization software represents the minimum value for the surface or contour plot shown in figs. 5 and 6. This proves the capability of the software to capture the minimum value of $u_{\text{max}}$ and the corresponding optimum values of $k_d$ and $c_d$.

![Figure 4: Variation of $u_{\text{max}}$ with $k_d$ and $c_d$ for the example of fig. 3 as a three dimensional surface.](image)

3 Case study 1: efficiency of the present method in reducing SDOF structure vibrations under earthquake excitations

In this case, a SDOF structure with $M = 30$ ton, $K = 2700$ kN/m ($f = 1.5$ Hz) is studied. TMD systems with 3% mass ratio are tuned to control vibrations in the structure when subjected to 18 earthquake records. These earthquake records cover a wide range of earthquake dominant frequency (from 0.3 to 4.82 Hz). The properties of TMD systems $k_d$, $c_d$ are computed using three methods (as shown in fig. 6) and the present study method using the MATLAB program developed in this work. This was repeated for each of the 18 earthquakes excitations. Fig. 6 shows the variation of percentage reduction in maximum displacement $u_{\text{max}}$ with earthquake dominant frequency for the investigated structure. The results showed that for all 18 earthquake records, the method proposed in this study gave the maximum reduction in $u_{\text{max}}$. In contrast to other methods, the present method gave different values for the optimums $k_d$, $c_d$ because it accounts for the earthquake characteristics in obtaining these optimum values. The figure also
shows that the efficiency of the TMD system for all methods may vary with the characteristics of earthquakes. The maximum reduction in displacement observed is about 80% in this case study.

\[ u_{\text{max}} = 0.1179 \text{ m} \]
\[ k_d = 54.08 \text{ kN/m} \]
\[ c_d = 0.643 \text{ kN s/m} \]

Figure 5: Variation of \( u_{\text{max}} \) with \( k_d \) and \( c_d \) for the example in fig. 3 as a contour line.

Figure 6: Percentage of reduction in \( U_{\text{max}} \) for various earthquake records for \( f = 1.5 \text{ Hz} \).
Case study 2: effect of TMD mass ratio on the present study results

The main purpose here is to investigate the effect of the variation of TMD mass ratio on the optimum values of TMD frequency ratio \( f_{d_{\text{opt}}} \) and TMD damping ratio \( \xi_{d_{\text{opt}}} \) when computed according to the present method. Fig. 7 shows the SDOF structure used in the present investigation. A TMD system with mass ratio ranging from 0 to 0.1 is attached to the structure. Also, three earthquake records are used in the study. The developed MATLAB software is used to compute the optimum \( \xi_{d_{\text{opt}}} \) and \( f_{d_{\text{opt}}} \).

![Structure studied in the case study.](image)

The various parameters are represented by

\[
\omega = \sqrt{\frac{K}{M}}, \quad \omega_d = \sqrt{\frac{k_d}{m_d}}
\]

\[
\xi = \frac{C}{2\omega M}, \quad \xi_d = \frac{c_d}{2\omega_d m_d}
\]

\[
\mu = \frac{m_d}{M}, \quad f_d = \frac{\omega_d}{\omega}
\]

Fig. 8 shows the effect of TMD mass ratio variation on the optimum TMD frequency ratio \( f_{d_{\text{opt}}} \) for three earthquake excitations. The figure also shows the variation of \( f_{d_{\text{opt}}} \) with \( \mu \) as computed by the Den Hartog method. The results show that this relation is greatly affected by the type earthquake excitation to the extent that there is no unique shape for this relation as given by the Den Hartog method. Similar behavior is also noticed for the relation between \( \xi_{d_{\text{opt}}} \) and the mass ratio \( \mu \) as shown in fig. 9. The important conclusion from the above mentioned figures is that the relation of \( f_{d_{\text{opt}}} \) or \( \xi_{d_{\text{opt}}} \) to \( \mu \) is highly affected by the earthquake type and cannot be predicted by simple equations as those in the Den Hartog or other similar methods.

Fig. 10 shows the variation of maximum displacement \( u_{\text{max}} \) of the structure (when controlled with \( \xi_{d_{\text{opt}}} \) and \( f_{d_{\text{opt}}} \)) with mass ratio \( \mu \). The figure shows that for some earthquakes, the maximum displacement decreases as the mass ratio \( \mu \)
Figure 8: Variation of $\xi_{d\text{opt}}$ with $\mu$ for various earthquakes.

Figure 9: Variation of $f_{d\text{opt}}$ with $\mu$ for various earthquakes.
increases, however there are other cases which do not show the same trend [11], and in some cases the displacement may increase with the increase of $\mu$.

The conclusion here is that the relation between maximum displacement $u_{\text{max}}$ and the mass ratio $\mu$ is also affected by the earthquake type and there is no general trend for this relation as determined by Den Hartog’s or other similar methods.

5 Conclusions

The following main conclusions can be drawn from the present study:

- For a given SDOF structure, earthquake excitation and TMD mass, the present method and software developed are capable to trace and compute the optimum values for $k_d$ and $c_d$.
- The TMD system designed according to the present method is more effective in reducing the maximum structure displacement than other methods, such as Deng Hartog’s. This is also found true for a wide range of earthquake excitations and structure frequencies.
- The efficiency of the TMD system designed according to the present method as well as other methods are generally affected by the earthquake excitation. This means that the earthquake characteristics have an important effect on the TMD behavior and should be considered in the design process.
- With the increase of research on the prediction of earthquake time-acceleration histories, the importance of the present study will increase as an efficient method for designing TMD systems.
For SDOF structures with a TMD system designed according to the present method, the relationship between $f_{d_{opt}}$ (optimum tuning frequency) or $\xi_{d_{opt}}$ (optimum TMD damping ratio) and $\mu$ (TMD mass ratio) is affected by the earthquake type and cannot be predicted by a simple equation as in Den Hartog’s theory or similar methods as shown.

For some earthquakes, the maximum structure displacements decrease with the increase of $\mu$ (as in Den Hartog’s method). However, there are cases of other earthquakes that showed different trends. This implies that the earthquake type may affect the trend of the relationship between the maximum structure displacement and mass ratio $\mu$.

References