Time-varying failure rate for system reliability analysis in large-scale railway risk assessment simulation

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Abstract

For large-scale dynamic railway traffic control systems, the reliability analysis and risk assessment are important for system evaluation, decision making and enforcement of performance-based safety-critical standards. The Axiomatic Safety-Critical Assessment Process (ASCAP) developed at the University of Virginia’s Center of Rail Safety-Critical Excellence provides a dynamic Monte Carlo simulation methodology for the quantitative risk assessment of large-scale rail systems. An appropriate reliability model is critical for effective and valid ASCAP simulation results.

In reliability engineering, it is known that the electrical and mechanical equipment, such as switch machines, track circuits and trip-stops in railway infrastructure, usually manifest deterioration and/or improvement in reliability over time. A constant failure rate, which entails an exponential distribution of the object’s lifetime, may not be sufficient and appropriate. A time-varying failure rate is adopted in our reliability probabilistic model. The Weibull distribution is one of the most widely used distributions for modeling lifetimes and the Weibull process is particularly suitable for modeling repairable systems due to their flexibility in shaping and scaling time-varying failure rates over time. The Weibull shape parameter determines the trend of reliability variation, however, the repair and scheduled maintenance may also contribute to a rise or fall of the failure rate at the repair time. To represent the physical failure characteristics, a so-called piecewise Weibull process model is proposed. The likelihood function is derived and used to estimate the Weibull parameters with maximum likelihood. We demonstrate the use of the piecewise Weibull model by applying it to evaluate the reliability of transit signaling devices in a large-scale railway system. The algorithm of the model implementation and Monte Carlo simulation is presented. The numerical results from applying the method are also provided.

Keywords: failure rate, piecewise Weibull, reliability, ASCAP, repairable system, time-to-failure.
1 Introduction

From an engineering point of view, every system or object will eventually fail in some fashion, and the failure can lead to critical damage or loss. Thus, estimation and analysis of system failures, reliability and maintenance have always been one of the primary concerns in system design and risk assessment. Reliability theory has developed rapidly over the last several decades, and reliability engineering plays an important role in various engineering applications, such as railway systems, nuclear power plants, aerospace technologies, military systems, etc. [1]. In this paper, we investigate a railway transit signaling system reliability/failure mechanism and its impact and application in large-scale railway risk assessment simulation.

Our railway risk assessment is based on a simulation-based tool-set called the Axiomatic Safety-Critical Assessment Process (ASCAP), which has been developed at the University of Virginia Center of Rail Safety-Critical Excellence. The risk assessment is performed per the US Federal Railroad Administration (FRA) Rule 49 CFR Part 209/234/235 (Standards for the Use of Processor-Based Signal and Train Control Systems). This novel Monte Carlo-based simulation methodology (ASCAP) is used to explicitly simulate the execution of the large-scale dynamic railway traffic control systems and quantitatively assess risk and predict societal cost of the system operations. Within ASCAP, the railway equipment, signaling and traffic control system devices are all characterized as objects. The detailed physical operations of the system are explicitly modeled. In addition to the object’s normal operations and interactions with other objects, trains or humans, the reliability property of the object plays a significant role in the overall system simulation and risk assessment. Object failures are one of the primary causes leading to incidents or accidents of the railway system. The object reliability is highly correlated with the quantitative system risk. Thus, an appropriate reliability model and proper time-to-failure (TTF) estimation are fundamental for the ASCAP risk assessment.

The reliability modeling and simulation of the objects are based on the reliability analysis of their failure history. We analyzed the historical data to quantitatively estimate and predict the object reliability properties. There are two primary procedures in our reliability analysis: mathematical reliability modeling and statistical parameter estimation. The repair process executed after an object failure also plays an important role in our reliability modeling methodology. Based on the repair type assumption, different probabilistic or statistical models are used for our object reliability analysis. For renewal (or perfect) repair assumption, which means that the object is brought to a like new state after the repair, the TTFs are independent and identically distributed (iid). We use the Weibull distribution to model the TTF random variable. For imperfect repair, which is a more reasonable assumption for our transit traffic control devices, we proposed a new mathematical modeling methodology, termed the piecewise Weibull process.

Our object failure records are derived mainly from the maintenance logs of the transit signaling and control system in a large-scale railway infrastructure. Compared to the operational time of the device, which is usually hundreds of days, the repair time (usually several hours) is simply insignificant. So we assume instanta-
neous repair in our reliability analysis, since the availability is not an issue here. However, the repair time is taken into consideration in the ASCAP simulation and has an explicit influence on the system risk assessment.

2 Perfect repair modeling

In the ASCAP simulation, the operational state of the object means the object is fully functioning. The time-to-failure (TTF) is a random variable which denotes the operational time period from its installation until its failure, also known as lifetime. For perfect repair, the time between failures for one device can be considered as different samples of the lifetime random variable for this type of device. The assumption is that the repair brings the device back to perfect state and for the same type of devices, the lifetime data (TTFs) share the same distribution and are iid.

The failure rate (hazard function) is commonly used to represent the reliability property of the object. It is defined as the object’s tendency to fail, i.e., the failure rate function

\[ h(t) = \lim_{\Delta t \to 0} \frac{P(t + \Delta t \geq T > t | T > t)}{\Delta t} , \]

and is the probability of the object failing at time \( t \) given the object is operational up to time \( t \). It can be shown that the cumulative distribution function (CDF) of the lifetime \( T \)

\[ F_T(t) = 1 - e^{-\lambda t} , \quad t \geq 0 , \]

where \( \lambda \) is the failure rate. One appealing property is \( \lambda = E(T) \), where \( E(T) \) is the mean-time-to-failure (MTTF). The parameter of the exponential distribution \( \lambda \) can be easily derived from the sampling data. Another property is the “memoryless” characteristic, i.e., any failure incidence is independent, regardless of how long it has been in operational state.

However, in reality, these properties or assumptions may not always hold true, especially for electrical and mechanical equipment, such as switch machines, track circuits and trip-stops in the railway infrastructure, which usually manifest deterioration and/or improvement in reliability over time. For example, the earlier failure of general devices may contribute to a decreasing failure rate, while aging or wear out may result in an increasing failure rate, and maintenance and regular inspection may also affect the failure rate. A constant failure rate model can underestimate the system reliability for some time periods and overstate it for some others [2]. In our case, the constant failure rate model for the objects may lead to somewhat misleading final risk assessment results. So we try to formulate it with time-varying failure rate so that the distribution function of the TTF \( T \) can be better approximated and the reliability characteristics can be explicitly expressed.
One class of distribution functions widely used in reliability model is the Weibull distribution \([3]\), where the general form of the failure rate function is given by

\[
h(t) = \frac{\beta}{\lambda} \left( \frac{t}{\lambda} \right)^{\beta - 1}, \quad t \geq 0.
\]

The form of the time-varying failure rate function \(h(t)\) is the major reason for the popularity of the Weibull distribution in system reliability analysis. There are two parameters in the model, whereas \(\beta\) determines the shape of \(h(t)\), usually referred to as the “shape” parameter, and \(\lambda\) scales variable \(t\), usually referred to as the “scale” parameter. As we can see, when \(0 < \beta < 1\), the failure rate is decreasing, implying improvement in reliability over time; when \(\beta > 1\), the failure rate is increasing, implying deterioration in reliability over time. When \(\beta = 1\), the failure rate is constant, in which case the TTF becomes an exponential distribution random variable with \(MTTF = \lambda\).

There are two widely used methods to estimate Weibull parameters: median rank regression (MRR) and maximum likelihood estimation (MLE). We use both methods to analyze the failure data of the switch machines in our transit signaling system using Weibull lifetime distribution model. The TTFs data are extracted from the raw failure records. Figure 1 is the histogram of all the lifetime samples.

As we can see, the switch machine has a higher tendency to fail at the early phase of its lifetime. The estimated Weibull parameters (\(\hat{\beta}\) and \(\hat{\lambda}\)) are shown in Table 1, and both MRR and MLE methods are used for comparison.
Table 1: TTF Weibull model parameters.

<table>
<thead>
<tr>
<th></th>
<th>$\beta$</th>
<th>$\lambda$</th>
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<tbody>
<tr>
<td>MRR</td>
<td>0.9785</td>
<td>291.8117</td>
</tr>
<tr>
<td>MLE</td>
<td>0.9214</td>
<td>295.0706</td>
</tr>
</tbody>
</table>

As we can see, the shape parameter $\hat{\beta} < 1$, which indicates a tendency of reliability improvement over the lifetime. It reflects the real sample data pattern. Figure 2 is the probability distribution comparison between our Weibull model and the actual TTF data. The reliability model fits the data samples fairly well.

![Comparison of MRR and MLE methods](image)

Figure 2: Comparison of the Weibull model with actual data.

3 Imperfect repair modeling

For repairable systems, a stochastic point process is commonly used to model the occurrences of failures over time. For a repairable system observed from time $t = 0$, let $T_1, T_2, \cdots$ denote the successive failure time, and let $N(t)$ denote the number of failures in the interval $(0, t]$. The Poisson process is ideal for depicting the counting process $N(t)$ [4]. The intensity function is defined as

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{P(N(t, t + \Delta t] \geq 1)}{\Delta t}.$$
Compared with the failure rate that indicates the tendency of the system’s failure during its lifetime, the intensity function measures the unconditional probability that the system will fail in a small interval. A non-constant intensity function entails a non-homogeneous Poisson process (NHPP), which is suitable for modeling the variations in system reliability over time. For example, a deteriorating system entails an increasing $\lambda(t)$, and an improving system entails a decreasing $\lambda(t)$. A widely used NHPP called power law process, or Weibull process, specifies an intensity function in the form of

$$\lambda(t) = \frac{\beta}{\vartheta} \left( \frac{t}{\vartheta} \right)^{\beta - 1},$$

which is exactly the same form of the failure rate function of a Weibull distribution. However, care should be taken to differentiate the two cases: the power law process depicts the successive failure sequences for a repairable system, while the Weibull distribution specifies the lifetime of a non-repairable system or the TTF of a repairable system with perfect repair. Actually, the time to the first failure for a power law process has a Weibull distribution with shape parameter $\beta$ and scalar parameter $\vartheta$. Corresponding to a constant failure rate in the exponential distribution of lifetime, a constant intensity function entails a homogeneous Poisson process (HPP). For minimal repair assumption, i.e., the repair done on a system leaves the system in exactly the same condition as it was just before the failure, and both HPP and NHPP model have been extensively studied. The MLE method can be effectively used for parameter estimation in both cases. However, for many applications, the assumption of minimal repair is not reasonable, neither is perfect repair. A better than minimal or less than perfect repair assumption is needed. There is no general or prevalent mathematical approach for such repairable systems due to its complexity in reliability modeling and statistical inference. Brown and Proschan suggest a model that randomly selects either perfect repair action or minimal repair action after a failure [5]. Statistical inference is studied in [6]. An extension of Brown and Proschan’s model is proposed in [7]. In regard to relaxing the iid assumption on the TTFs between failures, an approach called piecewise exponential (PEXP) model is proposed and studied in some literatures (e.g. [8]). The TTFs are independent, but not identical exponential random variables. This modeling methodology allows the specification of changes in reliability characteristics immediately after each repair. However, the exponential distribution limits the flexibility and generality of the application. For example, a complex system is usually composed of interconnected several parts or subsystems. Failure of one part can lead to the replacement of the dysfunctional part as well as several other related parts. This repair action will abruptly change the reliability characteristic of the system. But in the long run, the system usually manifests deterioration or improvement in reliability. This is a reasonable assumption for most of the complex mechanical or electrical equipment, as in the case of our railway traffic control devices. To address this issue, we propose a new model called piecewise Weibull process.
4 Piecewise Weibull process

Assume a repairable system observed from time \( t = T_0 = 0 \), let \( T_1, T_2, \ldots \) denote the successive failure time. We propose the piecewise Weibull process intensity function in the following form

\[
\lambda(t) = \frac{\beta}{\theta_i \alpha} \left( \frac{t}{\theta_i \alpha} \right)^{\beta-1}, \quad t \in (T_{i-1}, T_i].
\]

From the model, we can say that the system’s intensity function will rise or fall at each repair time because the repair is supposed to have an immediate impact on the system reliability. The parameter \( \alpha \) determines the weight of the impact. On the other hand, the intensity function changes continuously at other time as the system ages. The overall reliability model is tuned by a Weibull process. This can be seen by setting \( \alpha = 0 \), then \( \lambda(t) \) becomes a regular Weibull process.

To study the properties of the piecewise Weibull process and use MLE (Maximum Likelihood Estimation) to estimate the parameters, we derive the joint PDF (probability density function) of the failure times \( T_1, T_2, \ldots, T_n \) as

\[
f(t_1, t_2, \ldots, t_n) = \left( \frac{\beta}{\theta^\beta} \right)^n \prod_{i=1}^{n} i^{-\alpha} \beta t_i^{\beta-1} e^{\frac{1}{\theta^\beta}} \frac{1}{\alpha^\beta} \left( t_i^\beta - t_{i-1}^\beta \right),
\]

where \( 0 = t_0 < t_1 < \cdots < t_n \).

Given a sequence of failure time \( 0 = t_0 < t_1 < \cdots < t_n \) from the historical failure record of a repairable system, we use the piecewise Weibull process model to fit the data, the likelihood of fitting is just the joint PDF \( f(t_1, t_2, \cdots, t_n) \).

Thus the best estimation of \( \beta, \theta \) and \( \alpha \), denoted as \( \hat{\beta}, \hat{\theta} \) and \( \hat{\alpha} \), satisfies the following maximization problem

\[
f(\hat{\beta}, \hat{\theta}, \hat{\alpha}|t_1, t_2, \cdots, t_n) = \max_{\beta, \theta, \alpha} f(\beta, \theta, \alpha|t_1, t_2, \cdots, t_n).
\]

Generally, the closed form solution for this problem is not tractable, however, the numerical solution can be evaluated by some iterative methods. We use the Newton-Raphson method in our simulation.

The application of this piecewise Weibull model and parameter estimation technique can be easily extended to time truncated data and multiple repairable systems. However, the form of intensity function and joint PDF needs to be modified.

5 Example

We use the failure data of one switch machine for example to illustrate the piecewise Weibull process modeling methodology. The failure data is shown in Figure 3. As we can see, the time between failure tends to increase over time. This may be due to early failures for typical mechanical/electrical systems. The repair and maintenance may also contribute to the improvement in reliability.

We apply the parameter estimation method described in Section 4. The results are in Table 2.
Failure data for one heavily used switch machine

Figure 3: Failure data record of a switch machine.

Table 2: Piecewise Weibull model parameters.

<table>
<thead>
<tr>
<th>$\hat{\beta}$</th>
<th>$\hat{\theta}$</th>
<th>$\hat{\alpha}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.86584</td>
<td>165.89735</td>
<td>0.00208</td>
</tr>
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</table>

Based on the model, we plot the intensity function as shown in Figure 4. As we can see, the repair action affects the reliability of the system abruptly (vertical drops right after the repairs). While the system reliability is improved continuously between system failures (decreasing intensity function).

6 Implementation in the ASCAP

The ASCAP involves an Monte Carlo-based simulation environment modeling real world railway operations. For the object reliability model, TTF implementation is one of the primary issues.

6.1 TTF generation

Given the CDF derived from our model, Monte Carlo method can be used for TTF realization. Assume $\mu$ is a uniform distributed random variable where $0 < \mu < 1$. We summarize the TTF generation results under the two repair model assumptions.

- The Weibull distribution for perfect repair assumption,

$$F_T(t) = 1 - e^{-(\frac{t}{\theta})^\beta}$$
Figure 4: Piecewise Weibull process intensity function.

\[ TTF = \lambda (-\ln \mu)^{\frac{1}{\beta}} \]

- The piecewise Weibull process for imperfect repair assumption,

\[ \lambda(t) = \frac{\beta}{\theta_i^{\alpha}} \left( \frac{t}{\theta_i^{\alpha}} \right)^{\beta - 1}, \ t \in (T_{i-1}, T_i] \]

\[ TTF_i = \theta_i^{\alpha} \left( -\ln \mu + \left( \frac{TTF_{i-1}}{\theta_i^{\alpha}} \right)^{\beta} \right)^{\frac{1}{\beta}} \]

Note that for the piecewise Weibull process model, TTF generation is not only dependent on \( \mu \), but also on the order of the failure \( i \) and the last TTF generation.

6.2 Usage based object differentiation

In the real world railway system, for the same type of objects, different individual objects may undergo different loads of traffic or usage. This fact is somewhat reflected in the historical failure record. Take switch machines for example, the ones installed in the major intersections which require more frequent actions usually show up more frequently than the others in the failure records. The piecewise Weibull process is a good candidate for their reliability model. We distinguish the same type of objects by their work load and use different models or parameters, thus producing a more faithful result.
7 Conclusions

We study the reliability characteristic and failure mechanism of the repairable system based on the transit signaling system failure records. Different assumptions on the repair action motivate different approaches for our reliability model and analysis. For the more reasonable imperfect repair system assumption, we propose the piecewise Weibull process model. Parameter estimation methods are given. The implementation in the ASCAP is discussed. The improvement of compliance of the reliability model is important in our future work.

References