Boundary element modelling of concrete slabs

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Abstract

The present paper demonstrates the use of a developed software package called the PLPAK for modelling of structural concrete slabs. Practical building floor is demonstrated and results are compared with those obtained based on the finite element method.

Keywords: Boundary element method, Plate bending, The PLPAK software, Building slabs, Structural analysis.

1 Introduction

Boundary element applications to plate bending problems started several years ago. Some researchers such as Stern \cite{1}, Bezine \cite{2} and Tottenham \cite{3} used the classical plate bending theory (the Kirchhoff theory \cite{4}) in their formulations. Others employed the shear deformable plate bending theory (the Reissner theory \cite{5}). It is believed among the authors that employing the latter in a boundary element formulation is much better for the following reasons:

1- The simplification, made by Kirchhoff, of ignoring the shear deformation led to extra differentiation in the governing differential equation to eliminate shear forces. This appeared in the corresponding boundary integral formulation in extra need for additional integral equation for the slope; which is hyper-singular. Consequently, it needs
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special numerical treatments (higher order elements and difficulty of modelling curved boundaries).

2- Condensation of shear forces in the governing equations leads to unreliable shear values, especially in zones of stress concentrations (e.g. near columns).

3- It was proven [6] that the Reissner plate bending theory (as a refined plate bending theory) is capable of modelling both thin and thick plates.

4- The use of the Reissner plate bending theory is more suitable in modelling building raft foundations or pile caps [7].

A few researchers have applied their formulations to model building slabs in real practical applications. Among them are de Paiva and Venturini [8,9], de Paiva and co-workers [10,11], Hartly and Abdel-Akher [12] and Hartmann [13]. All of the trials (with no exception) conducted by them use the thin plate bending theory.

Other researchers like van der Weeën [14] have used the plate bending theory according to Reissner [5] in a boundary integral formulation. Rashed [15] extended the formulation of van der Weeën [14] to model slab over columns. Hence, the formulations in [7] and [15] are implemented into a software package called the PLPAK.

The purpose of this paper is to demonstrate the components of the PLPAK. Critical notes on modelling of concrete floors using this software are discussed. Finally, a practical concrete building slab is modelled. Results are discussed and compared with corresponding values obtained from finite element analysis.

2 The PLPAK software

The PLPAK software is an integrated development environment that helps the user to generate his model using the PLGen module. It allows the user to view or/and edit the used boundary element model via the PLView module, so that the user can solve it using the PLCoreMan module. The PLPost module serves as a post-processing module for the obtained results. The package could be linked to
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other packages such as the PTPAK which provide the capability of adding post-tensioned cables to the model. Figure 1 demonstrates the operation diagram for the PLPAK different modules.

2.1 The PLGen module

The PLGen module stands for the virtual model generator or the pre-processor of the PLPAK. It mainly changes any structural drawing to what is called the “virtual model”. Modeller can import structural drawings from DXF CAD files or can draw them directly using the CAD capabilities of the PLGen. The virtual model looks very similar to the original structural drawing. The PLGen mainly defines the structure using series of objects. These objects are classified into one of three categories: the geometrical objects (single slab and openings), the loading objects (column load, wall load, load patch and load assembly) and the supporting objects (column, wall, wall assembly, soil support and beam). Using such objects, the modeller can define all elements in the building slab. It has to be noted that the virtual model represents the actual slab shape (not like the centre-line model of the finite element method). Columns and walls are represented by the actual cross-section shape. The PLGen can also input the numerical models of each object such as the number of boundary elements for each segments, the discretisation of the beam–slab contact areas, etc. Load cases only (i.e. no load combinations) are defined in the PLGen module. Material properties are also defined herein. Figure 2 demonstrates the graphical user interface of the PLGen module.

Figure 2: The PLGen module interface.
2.2 The PLView module

The PLView module is an MDI (multiple document interface) graphical environment that allows the modeller to view or/and to edit the boundary element numerical model of the considered problem. Boundary element discretisation and internal loading or supporting patches are viewed. The PLView could be launched directly from the PLGen module. Practical engineers or beginners can skip this module. It has to be noted that the expert modellers can write their own input text file for the considered numerical model and input it directly to the PLView model without passing through the PLGen module. Figure 3 demonstrates the graphical user interface of the PLView module.

2.3 The PLCoreMan module

The PLCoreMan module serves as a link between all the PLPAK modules. It allows solution for multiple load cases. It also allows adding any additional loading (such as pre-stressing loading) or sophisticated supporting elements (such as any substructure). It also acts as solution tracer. Figure 4 demonstrates the graphical user interface of the PLCoreMan module.

2.4 The PLPost module

The PLPost module allows the user to display the results in forms of strips, contours and colour patches. It has to be noted that all results obtained in the
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PLPost environment are computed using analytical integral equations. In other words, no approximation is involved. Load combinations are defined herein. Results of any form could be exported easily to text files or to spreadsheets programs. Figure 5 demonstrates the graphical user interface of the PLPost module.
2.5 The PTPAK module

The PTPAK is an additional package that allows adding pre-stressing cables to slab or to beams. The description of this package is out of the scope of this paper.

2.6 Other modules

Many other modules could work with the PLPAK. Some of these modules are basics and others are added by the user. Among the basic modules is the “Column head” module, which computes the stiffness matrix in case of having tapered columns. The PLCTRL module or the PL Controls allow the user to print any intermediate matrices in the boundary element solution.

3 Structural modelling aspects

The following section summarises some of the numerical modelling aspects. Most of these aspects are based on the authors’ personal experiences.

3.1 Discretisation

- Usually, boundary element solution requires very small number of boundary elements to obtain a fairly accurate solution. Four to six elements (quadratic elements) could be used on the longest sides and one element could be used along internal opening sides. In some cases, where stress resultant gradient is high, additional elements could be used.
- Beam discretisation could start from three per beam for the shortest beam. Hence, other beams discretisation could be obtained rationally. Additional discretisation is recommended for beams with high stress resultant gradients.
- It is very important that the user has to carry out simple adaptive analysis to check that results will not be affected by changing the number of used boundary elements or the beam or/and support discretisations.

3.2 Gauss points

- The solution for the boundary values are done mainly with Four Gauss points. This is usually defined in the PLGen module.
- Internal solutions (in the PLPost module) could be carried out using two Gauss points. However, in few cases it needs at least four points. Again, it is recommended that the user has to carry out simple adaptive analysis to check that results will not be affected by changing Gauss points.
3.3 Beam-end conditions

- The beams in the PLPAK can be modelled using the actual contact area to the slab. Such a capability allows modelling the actual beam shapes. Also, beams connected to columns as hinged supports could be modelled by releasing the column stiffness and embedding the beam by a distance equal to one-tenth of the beam width. This will simulate the partial hinge condition. This is usually recommended by the building codes [16].
- Intersecting beams could be placed without intersecting node.

3.4 Notes on results

- Results could be obtained easily anywhere in the domain. Straining actions, especially shear forces, show some disturbances near the boundaries. In such cases, additional boundary elements or/and increasing the number of Gauss integration points is recommended.
- If oscillations appeared in beam straining actions, additional beam discretisation is recommended.

4 PLPAK unique capabilities

The following are some unique capabilities offered by the PLPAK:

- The PLPAK provides a possible way to check the obtained finite element results in companies.
- The easy drawing capabilities of the PLPAK make life simple for practical engineers, especially in case of having changes. Moreover, the boundary-only discretisation minimise the errors generated by engineer in the numerical models.
- The PLPAK multiple DXF import and the multiple pre-stressing cable imports allow engineers to work in parallel in a single project.
- The PLPAK has the capability of zooming in for local result details.
- The PLPAK can generalise soiled stress map to check the bearing capacity violation under rafts.
- The PLPAK models beams with its actual interaction area to the slab which make it unique in case of modelling slab–beam type bridges.
- Unlike the centre-line modelling of finite elements, the PLPAK considers the actual slab geometry. This makes the PLPAK more suitable for design and detailing software. Moreover, it will be a perfect numerical environment for what is called the Building Information Modelling (such as REVIT) [17].
5 Practical building slab

The slab shown in Figure 6 is considered under its own weight and an additional live load of 0.5 t/m². The slab thickness is 0.3 m and is made of concrete having $E = 2,210,000$ t/m² and $v = 0.2$. The column IDs are shown in Figure 7. The used boundary element mesh is presented in Figure 8.

Slab bending moments are plotted along two strips as shown in Figure 9. The results of the bending moment along strip 1 are compared, in Figure 10, with those obtained from finite element analysis based on two models. The first model treats columns as single frame elements and the second one treats columns as 3-D solid elements. In both models, core is modelled using the shell element. It can be seen from Figure 10 that the present boundary element model agrees with those obtained from finite element analyses, especially with the second finite element model in the vicinity of column.

Figure 6: The considered practical slab.
Table 1 demonstrates a comparison of some of the support reactions (bending moments in the two directions and the vertical axial load) between the present boundary element model and the finite element first model; agreement between the results can be found. It has to be noted that some of the column bending moments are a bit different from the finite element analysis. This is mainly due to the consideration of the column actual shape in the boundary element analysis. This confirms results obtained by the first author in Ref. [15].

6 Conclusions

In this paper, an integrated development environment called the PLPAK is presented. This package employs the Reissner plate bending theory to solve practical building applications. The components of the package are discussed. Finally, a practical example considering building slab is solved using the PLPAK.
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Figure 8: The used boundary and internal discretisations.

Figure 9: Bending moment results along two strips.
Table 1: Column reactions.

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<th>PLPAK (Mx)</th>
<th>FEM (Mx)</th>
<th>PLPAK (My)</th>
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Figure 10: The bending moment results along strip 1 for different numerical models.
References


[16] ACI Committee, *Building code requirement for structural concrete (ACI 318-08) and Commentary (ACI 318R-08)*. American Concrete Institute: Detroit, 2008.