In praise of John Katsikadelis
A well-deserved eulogy

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It is always a difficult task for any scientist to review and comment on the career of a friend and colleague, particularly one who has been so creative and productive as John Katsikadelis. It is thus with trepidation that I write this eulogy of John’s achievements trying to focus on what I consider to be the most original aspects of his work and apologising at the outset for any omissions undoubtedly due to the great quantity of material that John has contributed to engineering sciences and, in particular, to the development of advanced computational techniques including, but by no means exclusively, boundary elements.

The magnitude of my task can be judged by the over 200 high-quality papers published by John, many of them on boundary elements, integral equations and other mesh reduction methods.

John introduced, at an early stage, the study of boundary elements to the School of Civil Engineering at the National University of Athens in the form of graduate and undergraduate courses, as well as setting up a research group that has achieved international recognition. His efforts in this regard, culminating in the publication of his book *Boundary Elements, Theory and Applications* [1], which has been published not only in Greek and English but also in Japanese (2004) and Russian (2007), and reaching in this manner a vast audience and establishing his group amongst the most active boundary element method (BEM) research centres in the world.

John’s early training as a Civil Engineer at the National Technical University of Athens (NTUA) and in mathematics at the University of Athens was followed by two PhD degrees, one at NTUA and the other at the renowned Polytechnic University of New York at Brooklyn (majored in Continuum Mechanics, Applied Mathematics and Advanced Dynamics); both of them were excellent preparation for a life dedicated to education and research.
Recent Developments in Boundary Element Methods

His firm grasp of the more theoretical aspects of engineering has not detracted from his emphasis on solving practical applications – a fact reinforced by the period that he spent (for an academic) working as a professional civil engineer, specialising in structural design. This crucial period in between his two PhD degrees reflects in the focus of his research in solving real, rather than academic, engineering problems.

There are many honours and distinctions that John has accumulated in his intensive professional and academic life; he has served on important committees, has been a member of the Editorial Board of prestigious journals, has been a Committee Member or Chairman of important conferences and has served the community and science in numerous other ways. Those activities include some in which I have participated, such as being a member of the Editorial Board of the International Journal of Engineering Analysis with Boundary Elements and Chairman of several international Boundary Element Conferences. His involvement in all those activities has been, as is always the case with John, most thorough and included, among others, being Editor of Several Conference Proceedings [2,3] and three times Guest Editor of special issues prepared for the Engineering Analysis with Boundary Elements Journal [4–6].

It is my intention, in these few pages, to concentrate on the originality of John’s output which covers topics related to computational mechanics, in the area of BEM and meshless methods applied to solving linear and non-linear problems, under static and dynamic loads. He has made significant contributions in the fields of plate bending, structural shape optimisation, stability of structures, inverse problems and response of structures to non-conservative loads. More recently, he has been investigating the numerical solution of fractional differential equations and studying the response of structures under fractional-type inertia and damping forces; topics which serve to indicate the continuous and uninterrupted evolution of John’s scientific thoughts.

His interest in boundary elements started when reading a paper that was seminal to the development of several groups that were to contribute to the development of the method. This was the paper by M. Jaswon and R. Ponter on Integral Equation Solutions of Torsion Problems [7], where the basis of the direct BIEM formulation for potential problems was first established. Maurice Jaswon’s interpretation of Green’s formulae for those cases later led to the development of the direct boundary integral formulations in terms of Somigliana’s identity for the stress analysis case. A few people around the world – including our own UK school and another in the USA – realised the importance of this work and it does John great credit that he also understood that the basis had been set up for a promising computational method. John applied the new ideas to the solution of the biharmonic equation for stress functions in plane elastostatic problems in preference to the more popular Muskhelisvili’s complex variable formulation. This resulted in his early (1977) paper in Mechanics Research Communications [8]. In this paper, he presented for the first
time the derivation and use of the integral representation of the normal derivative in the form of a boundary integral equation.

The formulation was later on applied by different authors after John’s pioneering development. This was at a time when the interest in finite elements precluded further research or rested importance to any work done on other types of numerical methods.

John, nevertheless, saw the potential of boundary integral formulations and continued to develop his ideas further in his second Doctoral dissertation submitted at Brooklyn Polytechnic [9]. In this thesis, he presented the boundary integral equation method for plates on Winkler’s Foundation, deriving the corresponding fundamental solution and obtaining accurate numerical results.

As boundary elements became better known, John’s work on plate bending started to receive the recognition that it was due. His Brooklyn thesis produced two important papers, one dealing with clamped plate analysis on elastic formulations, published in ASME Transactions [10], and the other on plates with different boundary conditions published in the ASCE Journal of Engineering Mechanics [11]. During that period, those journals were the most prominent publications for mechanical sciences.

These early papers were followed by others dealing with the applications of boundary integral equations to plates resting on other types of elastic subgrade. He derived the fundamental solution for the case of two parameter soil model as well as the corresponding boundary integral solution, resulting in another two important papers [12,13].

John’s continuous interest in plate bending led to him proposing new formulations, including one based on the use of Reissner’s plate model [14,15]. The solution in this case was expressed in terms of two potentials, one biharmonic and the other, Bessel’s, resulting in an original approach which produced accurate numerical results. It also demonstrated that Reissner’s theory could be applied for a wide range of plate thicknesses, ranging from very small values to large ones without apparent loss of accuracy.

John contributed to the solution of many other plate bending problems. For instance, he published the first integral equations paper dealing with large deformation analysis of plates of uniform thickness with arbitrary geometry and boundary conditions [16, 17].

A complete review of John’s work on plate bending would require considerable space as his work in this field has been most productive. His contribution is described in more detail in the Chapter on “Special Methods for Plate Bending” that has been published in reference [18]. This plate-bending work precludes some of his more recent highly original contributions to be shortly described. John’s contribution to our current understanding of boundary integral solutions for plate bending needs to be stressed and given proper recognition.

In the years that followed, John applied the BEM to solve a variety of problems, static and dynamic, whose fundamental solution could not be easily established (such as is the case of governing equations with variable coefficients);
problems for which that solution may have been difficult to compute (such as dynamic problems) or others for which no solution existed (most non-linear cases). John’s approach was to use a simple fundamental solution in all cases, i.e., Laplace for second-order equations and that of the biharmonic operator for fourth-order equations.

I met John for the first time in a series of lectures organised at Centre for the Study of Mechanical Sciences (CISM) in Udine in 1983 and there we discussed the importance of using simple fundamental solutions at the same time as allowing for all domain terms to be taken to the boundary, without the need to carry out domain integrations. This idea was the basis of the dual reciprocity method (DRM) which I published in 1982 [19]. John promptly realised the need to use simple fundamental solutions if we were to extend the range of applications of BEM and, in his characteristic manner, he went a step further and developed a more general version of the idea.

To better understand the importance of John’s contribution, I will briefly explain the fundamentals of the DRM. The method has two important steps, the first of which is the splitting of the governing equations of the problem into two parts, one of which represents the terms for which a fundamental solution can be postulated, while the other groups those terms that are not part of that solution. Those terms represent fictitious boundary effects or domain sources and may result from non-linear or time-dependent effects which cannot be dealt with by the fundamental solution. The second important step of the DRM is to express those terms approximately, expanding them in terms of localised functions. The localised functions can be interpreted as defining the non-homogeneous terms of the same known (and usually simple) operators used in the first step. This results in the possibility of finding a series of localised particular solutions through which the domain sources can be taken to the boundary using the same integral identities applied when dealing with the fundamental solution used for the first step.

The DRM is quite general and produces boundary-only solutions for those cases for which a linear operator with a well-known fundamental solution could be extracted for the full governing equations. This, John realised, is not always possible, say for the case of partial differential equations with variable coefficients for instance.

Hence, John developed the concept of the analogue equation according to which a problem governed by linear or non-linear differential equations of any type (elliptic, parabolic or hyperbolic) can be converted into an analogue problem described by an equivalent linear equation with a simple known fundamental solution of the same order as the original equation subjected to fictitious sources, unknown in the first instance. The value of these sources can be established using BEM. By applying this idea, coupled linear or non-linear equations can be converted into uncoupled linear ones for instance. The analogue equation method (AEM) only requires that the derivatives in the new equations are of the same degree as the original equations. If the higher derivatives are
fourth order as in the case of plate bending, the same degree ought to apply to the proposed equation in the AEM. John’s idea which was truly original has a wide range of important applications.

At first, John applied the AEM without reference to the possibilities of using the localised interpolation functions described in Step 2 of the DRM. Because of that, fictitious terms needed to be computed in the domain, using either the standard finite element method (FEM) technique or the domain-type cells appearing in some forms of classical BEM.

John’s AEM idea was published for the first time in the 1993 Boundary Element Conference [20] and fully developed in his keynote address at the next meeting – 1994 – in that series of conferences [21]. AEM was then fully explained and I cannot do better than quote his words from that seminal paper:

*The unknown source density function is established numerically by adhering to the following steps.*

a. The integral representation of the field function is established from the equivalent fictitious linear problem which involves the unknown source density in the domain integral.
b. Direct differentiation of this integral representation yields the derivatives involved in the operator of the real problem.
c. Use of BEM technique for the boundary integrals and FEM technique for the domain integrals yields the discretized expressions for the field function and its derivatives.
d. Collocation of the field function at the boundary and domain nodal points, collocation of the derivatives at the domain nodal points and elimination of the boundary quantities making use of the boundary conditions, yield the nodal values of the field function and its derivatives in terms of the values of the fictitious source density function at the nodal points inside the domain.
e. Application of the governing equation of the real problem at the nodal points inside the domain and substitution of the relevant values of the field function and its derivatives yields a system of algebraic equations (linear or non-linear, depending on the operator of the real problem) from which the nodal values of the fictitious source density function are established.
f. The field function and its derivative at any point inside the domain are obtained from their integral representation of the fictitious problem.

In differentiating the integral representation of the field function singular and hypersingular domain integrals arise which are evaluated efficiently by converting them to regular boundary integrals.

The method has the best features of the established computational methods, finite difference method (FDM), FEM, BEM and DRM. It combines their merits and circumvents their drawbacks [22].
Recent Developments in Boundary Element Methods

The concept of the analogue equation in conjunction with integral techniques rendered the BEM a more efficient and versatile computational tool for solving different linear and non-linear engineering problems using simple and well-known fundamental solutions.

An interesting application of the method was first presented in reference [23] in which the AEM was employed for system identification. In this case, AEM was used to identify constitutive material laws, including constant or varying parameters, i.e. those depending non-linearly on the unknown field functions and its derivatives. The examples presented in the paper included temperature distribution problems in non-homogeneous bodies, cases of temperature-dependent thermal conductivity as well as non-linear steady-state Burger’s equation type flow. This application of the AEM may prove to have important implications, because it opens the way to formulate mathematically, i.e. to establish the governing differential equations (deterministically or stochastically), the response of physical systems that are described by unknown physical principles and constitutive laws (e.g., composite materials [24], air pollution, wave propagation in bodies with unknown physical structure such as seismic waves) or systems that are not governed by physical laws at all (e.g., economic or other social sciences).

We all know that during the last three centuries the effort was given to solve the differential equations resulting from rather simplified physical laws. Efficient solution methods of the established equations have been already developed. However, a question arises: “Do these equations approximate the actual response of the physical system reliably and realistically?” Therefore, the problem of establishing the actual differential operator that models a system is apropos and a subject for future research. The AEM can give an answer to this problem.

Another provocative rather but interesting application of the AEM is the solution of “Equationless Problems Using Only Boundary data” [25, 26], that is problems whose equation is unknown but all boundary data are known, imposed and resulting from the response, that is both Dirichlet and Neumann BCs at each point on the boundary.

An important special issue dealing with Plate Analysis, edited by John, was published in 1996 in the Engineering Analysis with Boundary Elements Journal. There he has a paper extending the AEM to the dynamic analysis of plates with variable thickness [27]. He demonstrated that the fourth-order partial differential equation with variable coefficients giving the dynamic response of the plate could be substituted by an equivalent quasi-static plate bending problem with constant thickness subjected to a fictitious time-dependent load. In this case, singular and hypersingular integrals ought to be evaluated on internal cells; but John simplified the problem by transforming the domain singular integrals into regular integrals on the boundary of each cell using Green’s reciprocal identity.

In 1997, John extended his AEM to solve a case previously never attempted, using BEM, i.e., the buckling of a plate with variable thickness [28]. The original eigenvalue problem for the differential buckling equation was substituted by a classical linear eigenvalue problem with discrete value of the fictitious load,
from which the buckling loads were established numerically. Furthermore, also in 1997, the AEM was applied to study vibrations of plates with variable thickness subjected to in-plane force, giving also excellent results [29].

Until then, the domain integrals in AEM were computed using either finite elements or the domain cells of classical BEM. In 1998 [30], John presented the first paper in which the use of the radial basis functions (RBFs) of DRM were applied in his method together with the concept of localised particular solutions. These concepts have been described in a general way in 1992 [31] but without reporting the wide range of application that were offered by the AEM. John was the first to combine the AEM with the use of localised particular solutions. The methodology was generalised in a subsequent paper that appeared in the special issue on non-linear BEM that he edited in 1999 for the Engineering Analysis with Boundary Elements Journal [32].

In this work, the distribution of the fictitious domain sources of AEM was approximated by the type of radial basis functions used in DRM. The solution of the analogue equation was obtained as the sum of the homogeneous and a particular solution. Then the non-homogeneous terms, i.e., the field function and its derivatives were expanded in terms of unknown series coefficients which were found by collocating the equation at a series of discrete points in the domain. The AEM, hence, became a truly boundary-only method in the sense that only boundary discretisations are required.

This latest version of AEM has been successfully applied by John to solving several complex engineering problems, such as static and dynamic large deflection analysis of non-homogeneous anisotropic membranes [33, 34], non-linear dynamic analysis of heterogeneous orthotropic membranes [35], space membranes [36], membranes subjected to ponding loads free [37] and floating in a liquid [38], static and dynamic analysis of rib-reinforced plates [39–41], the optimum design of structures subjected to follower loads [42] and other equally novel applications, including papers on linear and non-linear flutter instability of damped plates [43, 44], plate thickness optimisation problems [45] and a generalised Ritz Method in domains of arbitrary shape using global shape functions [46] (Table 1).

The importance of the AEM is its generality and that it opens up new possibilities and a better understanding of how to apply numerical methods. It also reveals a touch of genius in John’s work.

To understand its implications, nothing is more appropriate than to revisit the basic principles and, in particular, the work of another famous Greek author, Aristotle, who can be regarded as the originator of the idea of virtual work. In his renowned book, Physics, the philosopher stated that the behaviour of physical systems could be expressed in terms of ‘potentialities’ and ‘actualities’. In other words, he set up the basis of the principle of virtual ‘potentialities’ or what we now call principle of virtual work. While the ‘actual’ field functions are to satisfy the equations governing the problem; the ‘virtual’ function can be much
Table 1: Comparison of the different methods can be presented in tabular form following a classification suggested by John [22].

<table>
<thead>
<tr>
<th></th>
<th>FDM</th>
<th>AEM</th>
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<tbody>
<tr>
<td>Collocation of the equation at domain nodal points</td>
<td>Requires regular mesh</td>
<td>No regular mesh</td>
</tr>
<tr>
<td>Substitution of the derivatives</td>
<td>Numerical differentiation reducing accuracy. Only values in the neighborhood of the collocation point contribute to the derivative.</td>
<td>Analytic differentiation of the integral representation provides a stable and smoothing process. Very good approximation. All domain and boundary values contribute to the derivative</td>
</tr>
<tr>
<td>Application of boundary conditions</td>
<td>Difficult task or practically impossible for irregular boundaries</td>
<td>It applies to boundaries of arbitrary shape</td>
</tr>
<tr>
<td>The solution and its derivatives are evaluated</td>
<td>Only at nodal points</td>
<td>At any point using the integral representation of the solution</td>
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<tr>
<th></th>
<th>FEM</th>
<th>AEM</th>
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<tr>
<td>Domain discretisation is used</td>
<td>To approximate the continuum. Inter-element continuity is required.</td>
<td>Only required to approximate domain integrals, in case they are not transformed into boundary integral. Inter-element continuity relaxed</td>
</tr>
<tr>
<td>Solution</td>
<td>Only the solution is evaluated at nodal points.</td>
<td>Both the solution and its derivatives can be evaluated at any point using the integral representation of the solution</td>
</tr>
<tr>
<td>Linear and non-linear, static, dynamic and diffusion problems</td>
<td>Applies to all problems</td>
<td>Applies to all problems</td>
</tr>
<tr>
<td>Fractional Differential equations</td>
<td>Has not been applied as yet</td>
<td>It applies</td>
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<tr>
<td></td>
<td>BEM</td>
<td>AEM</td>
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<tr>
<td>Applicability</td>
<td>Applies, in principle, only to linear problems with known fundamental solution</td>
<td>Applies to linear and non-linear problems</td>
</tr>
<tr>
<td>Dynamic and diffusion problems</td>
<td>Employs the fundamental solution of the hyperbolic equation and parabolic equation, respectively</td>
<td>Employs the simple static fundamental solution for all problems</td>
</tr>
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</table>
| Problem dependency | It is problem dependant  
Each problem requires special numerical solution and computer programming | It depends only on the order of the equation  
The numerical solution and the computer program is the same for elliptic, hyperbolic and parabolic problems, linear or non-linear, for differential equations of the same degree |

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<th></th>
<th>DRM</th>
<th>AEM</th>
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<tr>
<td>Applicability</td>
<td>Applies to linear and non-linear problems</td>
<td>Applies to linear and non-linear problems</td>
</tr>
<tr>
<td>Dynamic and diffusion problems</td>
<td>Can use simple static fundamental solution</td>
<td>Can use simple static fundamental solution</td>
</tr>
</tbody>
</table>
| Problem dependency | Applies if a dominant linear operator with known fundamental solution can be extracted from the governing operator  
It is problem dependant  
Each problem requires special numerical solution and computer programming | No limitations  
The numerical solution and the computer program is the same for elliptic, hyperbolic and parabolic problems linear or non-linear for differential equations of the same degree |
Recent Developments in Boundary Element Methods

more general. Usually, we assume that they also satisfy the same equations as the actual field, or in the case of DRM, some reduced version of these equations. John instead stated that they do not need to necessarily satisfy the same type of governing equations as the actual problem, provided they have the necessary degree of continuity (say order fourth for plate bending, etc).

More recently, John has extended his AEM idea by using the Multiquadric (MQ) type of functions proposed by Kansa [47, 48], but using them in a DRM type formulation. This avoids the primary disadvantage of the classical MQ scheme, i.e., that of being a global method and hence resulting in full coefficient matrices which suffer from ill-conditioning, particularly as their rank increases. This is a serious disadvantage that complicates the implementation of the MQ Method. Moreover, the performance of the classical MQ Method depends on the shape parameter of those functions which are chosen empirically, a process that makes the technique problem dependent.

Instead, John uses the MQ function in the same way as classical radial basis functions are applied in AEM and DRM. He called the new technique Meshless AEM (MAEM), which exhibits key advantages over other RBFs collocation methods in that the method is highly accurate and the matrix of the resulting system of equations is always invertible. The new RBFs resulting from the integration of MQs permit a strong formulation of the solution. Furthermore, it has a further advantage over Kansa’s Method in that the derivatives of the equation after collocation are at most MQs. The accuracy is increased when using optimal values of the shape parameters of the multiquadrics.

This optimisation is possible by minimising the functional that produces the partial differential equations governing the problem [49–51]. In theory, the optimisation could include the position of the collocation points as well but this is seldom necessary and would give rise to lengthier calculations.

The advantages of the MAEM Method as summarised by John are:

- Since the method allows the control of the condition number, an invertible coefficient matrix for the evaluation of the new RBFs expansion coefficients can always be established.
- The method gives good results, because the new type of RBF resulting from the integration of the MQ function approximates accurately not only the solution itself, but also its derivatives.
- Optimum values of the shape parameter can be established when minimising a functional that yields the particular differential equations governing the problem (the position of the collocation points could also be optimised, if necessary). Therefore, the uncertainty of choice of shape parameter is eliminated.
- As in the case of AEM, the MAEM Method depends only on satisfying the order of the differential operators and not on the operator for the specific problem.
The method can be employed for the solution of other types of problems as well as those already presented for the AEM. The method has been already employed for the solution of several problems described by second- and fourth-order partial differential equations, such as 3D analysis of thick shells [52], 3D elastostatic problem for inhomogeneous anisotropic bodies [53] and plate problems [54].

John’s ever-active mind is currently interested in the role of fractional derivatives in mechanics, and their importance in order to describe realistically the response of emerging materials and processes. The use of such concepts leads to fractional partial differential equations, which after discretising the continuum provides ordinary differential equations with fractional derivatives. John has developed a numerical method for solving linear and non-linear multi-term fractional differential equations by extending the AEM in conjunction with a novel integral equation solution [55].

He has applied the method to solve a whole range of problems, including the fractional wave-diffusion equations [56]; the post buckling response of viscoelastic plates; the non-linear vibrations of viscoelastic membranes [57]; the non-linear vibrations and resonance of viscoelastic plates [58]. In all these cases, the viscoelastic method is described using a fractional derivative model. This pioneering work opens the way for solving a whole range of new problems.

In summary, the range of interests and novel ideas developed by John over his scientific and academic career is truly outstanding and has secured him a place among the main computational mechanics scientists in the world. He is particularly prominent among those researchers who have been actively involved in finding new methods to replace the classical mesh-dependent techniques, most frequently used in engineering practice, such as FDM and FEM. This led to his early interest in BEM and more recently to his work on other mesh reduction and meshless methods.

John’s other great virtue has been his intellectual generosity in sharing his knowledge with colleagues and researchers, contributing to creating a unique School of Computational Mechanics in Greece. His group is now recognised throughout the world for the excellence of their work, and this is the best legacy that John could give to his country and the world.

References

Recent Developments in Boundary Element Methods


