Chapter 15

Kelvin in the twenty-first century

D.H. Saxon

*Kelvin Professor of Physics (1990–2008), University of Glasgow, United Kingdom.*

Abstract

Aspects of Kelvin’s work that are a subject of study today are outlined focusing on five topics: The Kelvin (foam) problem, Kelvin Waves and their role in the El Nino phenomenon, the application of Stirling engines and the possibility of reliable earthquake prediction.

1 ‘The Kelvin Problem’: space-filling foam

Kelvin’s interest in everything has given him an influence that far outlasted his time. One could mention the first and second Thomson relations in thermoelectricity, or the Kelvin angle, which describes the wake of anything (e.g. duck, boat, aircraft carrier) moving through deep water. The tidal predictor (the ‘great brass brain’) was used for almost a century. Two particular points of interest are the Kelvin foam and Kelvin waves, and there is the renewed interest in Stirling engines for generating electricity.

In 1887, Kelvin was much occupied in trying to understand the nature of the æther, the medium which he saw as supporting electromagnetic waves and which pervades all space and all matter. Because of the extraordinarily high value of the speed of light it needed to be very low density but very strong. This led him to consider a foam rather than a continuous medium, and thus to formulate the “Kelvin problem”: How can one partition space into equal-sized cells with a minimum surface area?

Kelvin’s proposed solution (the so-called Kelvin Structure) was a tetrakaidekahedron (14-faced solid,) structure, constructed from a regular octahedron by cutting off each of its points, leaving a cell with six regular hexagonal faces and eight square faces [1]. Figure 1 (left) shows the stacking of these cells to make a three-dimensional structure.

For such a structure to be stable it must have minimal energy associated with it. If the energy is proportional to the area (as in surface tension), it obeys Plateau’s rules for stable (minimum energy) soap films [2]. Plateau’s rules dictate the following. Films meet at edges: always three films meeting at an angle of 120°. Sets of four films meet at corners at the tetrahedral angle...
(109.4°). There is a pressure difference across any face proportional to the curvature of the face—strictly the sum of the two principal curvatures. In the Kelvin-foam case the cells are identical and so the pressure difference is zero. The faces in fact turn out to be not totally flat but with the principal curvatures adding to zero.

Is the Kelvin tetrakaidekahedron the answer to the Kelvin problem? Is it the way of partitioning space using equal-sized units with the minimum surface area? For 106 years, it was unchallenged, until Denis Weaire and Robert Phelan (W–P) found a solution with 0.3% less area [3]. Since then, a large effort has produced other structures which do better than Kelvin but not Weaire and Phelan.

The W–P solution uses two different cells (but with the same volume). One is an irregular dodecahedron with pentagonal faces. The second is a 14-faced structure with two hexagonal and 12 pentagonal faces. Again, the faces are slightly curved. Figure 1 (right) illustrates how these stack in three dimensions. Experiments have shown that actual foams with equal volume cells can be made to form a W–P structure.

When the Beijing Olympics’ organisers were looking for a structure to symbolise water, they chose to make the ‘water cube’ in which the swimming and diving events took place, as a Weaire–Phelan structure. This lightness of structure and strength of this structure were remarkable [4], see Fig. 2.

Today the Kelvin problem remains unsolved. The W–P structure has not been bettered, but neither has it been proved to be the best of all possible solutions. As for the aether, in the same year as Kelvin was perfecting his foam, the Michelson-Morley experiment showed that the motion of the earth, or any other body, through the aether is undetectable [5]. This led to the theory of Relativity and the demise of the aether. Yet, today after the discovery of a likely Higgs Boson a more challenging æther is hinted at [6]. But now we seem to have three fields pervading all space: the Higgs field which gives mass; dark matter which may be things we would recognise
or may be something else; but dark energy, which makes the universe expand at an accelerating rate, is not yet understood.

2 Kelvin Waves and the El Nino effect

In 1879 Kelvin published a paper ‘on the gravitational oscillations of rotating water’ [7]. In this work he predicted that the earth’s rotation constructs a waveguide that transports (hot) water from the East Indies in an Easterly direction along the equator to South America, forming the basis of the El Nino climate phenomenon. We briefly outline the mechanism here.

An inertial frame is a frame of reference (the ‘fixed stars’ as we used to say) in which Newton’s first law of motion applies. In the absence of an external force, a body moves at a uniform speed in a straight line. If we move to a rotating frame of reference, things are more tricky. The most important rotating frame of reference is the earth itself, which rotates on its axis once in every 23 h and 56 min. For everyday purposes, it is most sensible to describe things securely attached to the earth as at rest in a frame of reference that rotates with the earth.

Such a body of mass \( m \) fixed at a point on the surface of the earth experiences a downward force, directed at the centre of the earth of \( mg \), where \( g \) is the magnitude of the acceleration due to gravity, and a much smaller centripetal force of magnitude \( nrw^2 \cos \lambda \), directed towards the axis of rotation of the earth, where \( r \) is the radius of the earth (\( = 6400 \text{ km} \) approximately), \( \omega \) is its rate of rotation (1 turn per 24 hours = \( 1.1 \times 10^{-5} \text{ rad s}^{-1} \)) and \( \lambda \) is the latitude.

There is an additional force that comes into play when a body is moving relative to the earth. This is known as the Coriolis force and was of much concern in the nineteenth century to artillery officers, who found that the trajectories of shells were deflected to left or to right as they travelled towards distant targets. The most celebrated use of Coriolis force was the Foucault
pendulum, which shows (without needing any reference to the fixed stars) that the earth is rotating. If a pendulum is allowed to swing freely, the plane in which it moves rotates around a vertical axis in time $t$, relative to local buildings and so on, by $\omega t \sin \lambda$. The Coriolis force which achieves this is directed at right angles to the motion of the body and also at right angles to the earth’s rotation.

Consider a small volume of water near the surface of the ocean and being heated by the sun. It will expand as it is heated and this will cause the surface of the ocean to rise a little: this height difference gives rise to a pressure difference $\rho gh$ (where $\rho$ is the water density, and $h$ is the difference in height) that will make the water travel across the surface of the ocean away from its starting point. As it moves, it therefore experiences a Coriolis force deflecting its motion to the right in the Northern hemisphere and to the left in the Southern hemisphere. Let the velocity have Easterly and Northerly components of $(v_e, v_n)$. Then, the Coriolis acceleration has components $(fv_n, -fv_e)$ where the Coriolis factor, $f = 2\omega \sin \lambda$, is zero at the equator (where $\lambda = 0$) and has opposite signs in the two hemispheres.

For Eastbound motion, $v_e > 0$. If this motion is deflected Northward ($v_n > 0, \lambda > 0$), it experiences a Southward Coriolis acceleration of magnitude $2\omega v_e \sin \lambda$, which reverses the Northward velocity. Similarly, if it is deflected Southward, its Southward velocity is reversed. Thus, an Eastbound-moving object at the equator is trapped in a waveguide that directs it to move around the equator.

Conversely, for Westbound motion $v_e$ is negative. The Coriolis acceleration serves to increase any Northward motion in the Northern hemisphere and Southward motion in the Southern hemisphere. Thus Westbound motion around the equator is lost as moving water is transported away from the equator to larger latitudes.

The water in the ocean obeys the wave equation, and the Eastbound motion of water is known is a ‘Kelvin Equatorial Wave’ [7,8]. Recall, this was driven by heating water via the sun. This warming extends to a depth of water ($H$) called the thermocline, with much colder water below. The Kelvin Wave speed ($c$) is given by $c^2 = gH$. The phase and group velocities are identical so the wave travels right across the Pacific Ocean without changing its shape. The speed $c$ is about $3$ ms$^{-1}$, so that it may take a couple of months for the hot water to be transferred to the American face of the Pacific. This is the mechanism that creates the el Nino phenomenon which has such drastic effect on the weather, see Fig. 3 (left).

Figure 3: Left: schematic side view of Kelvin equatorial wave and Kelvin coastal wave production. Right: schematic top view of Kelvin equatorial wave and Kelvin coastal wave production. (Both images from Department of Oceanography, Naval Postgraduate School, US Navy.)
When the Kelvin equatorial wave hits the Americas, it will be deflected to run parallel to the coast, Northbound in the Northern hemisphere and Southbound in the Southern. In this case the Coriolis force traps the water in a waveguide, one side being given by the land, and the other by the Coriolis force. This is called a ‘Kelvin Coastal Wave’, see Fig. 3 (right).

3 The Stirling engine

The Stirling engine has a vital position in history, as it was by making studies on this from 1847 that Kelvin formulated the first and second laws of thermodynamics [9]. The story of this, and of the essential contributions of his brother James, is told in considerable detail in Chapter 3 of this volume: James and William Thomson: The Creation of Thermodynamics. The Stirling engine is an external-combustion heat engine with a permanently enclosed gaseous working fluid, see Fig. 4. It has a hot (red) and a cold (blue) reservoir, joined by a pipe that allows the gas to flow between them [10].

The Stirling thermodynamic cycle has four elements, from left to right in Fig. 4:

1. Gas is heated in the hot cylinder, forcing pressure build up and doing useful work pushing the hot piston to the left.
2. The hot piston moves to the right (making the hot cylinder smaller) while the cold cylinder gets larger. Gas is forced down the pipe into the cold cylinder where it is cooled.
3. The piston in the cold cylinder compresses the gas, generating heat which is removed by the cooling source.
4. Gas is forced from the cold cylinder into the hot cylinder where it heats up rapidly and the cycle repeats.

The transfer pipe greatly improves the thermal efficiency. Its thermal capacity acts as a ‘regenerator’, heating the gas as it flows toward the hot cylinder, and cooling it as it goes the other way. The efficiency approaches that of the Carnot cycle, the most efficient possible. But in real-life systems it comes out to be less efficient than the diesel (Otto) engine. Interest in the Stirling engine languished until recently. There is now renewed interest for a number of reasons.

Domestic heating, hot water and electricity can be produced more efficiently than in a conventional gas-burning heating system and separate mains electricity. This helps in achieving low carbon emissions. Typical systems produce of order 5 kWe of electrical power.

Figure 4: Four steps through the thermodynamic cycle of the Stirling engine. (Images courtesy of the author, Zephyris (Richard Wheeler).)
Because the Stirling engine is an external-combustion engine it is easy to change the fuel. Systems are under development to burn waste previously destined for landfill and to produce combined heat and power.

In sunny place (such as Arizona) large steerable parabolic mirrors are used to focus the sun’s rays on to a receiver, which transmits the heat to a Stirling engine. The engine is a sealed system filled with hydrogen. As the gas heats and cools, its pressure rises and falls. The change in pressure drives the piston inside the engine, producing mechanical power, which in turn drives a generator and makes electricity. Systems produce up to 25 kWe each during daylight hours.

Energy-conscious engineering is now revisiting the thermodynamics of heat engines, giving the Natural Philosophy of 1850 renewed life in the 2010s.

4 The Stirling engine in outer space

Space-craft instrumentation is usually powered by solar cells, laid out in large ‘sails’ to capture the sun’s radiation. As one passes into the outer solar system the intensity of solar radiation falls and a new approach is needed. The Advanced Stirling Radioactive Generator (ASRG) uses the heat from $^{238}$Pu decay to drive a Stirling engine to generate electricity. $^{238}$Pu is very scarce, and so the maximum efficiency is needed. The half-life (14 years) is well matched to the target of 15 years operation. Each generator unit is designed to produce over 100 We from less than 1 kg of Plutonium.

Systems built around the Advanced Stirling Converter (ASC) are currently under development to support future space missions on the Martian surface or in the vacuum of space [11]. The device must be ruggedised to withstand the vibration of the launch and subsequent operation. The device consists of a free-piston Stirling engine and an integral linear alternator that converts the piston reciprocating motion to electrical output. The ASC weighs only about 1.3 kg, has demonstrated a convertor efficiency of 38% at 850°C hot-end temperature and 90°C cold-end temperature. The internal moving components are supported by hydrostatic gas bearings, which allow movement without contact or rubbing. Two ASCs are used in each ASRG, mounted opposite each other and electrically synchronised so that their pistons move in opposite directions, eliminating most of the vibration, see Fig. 5.

The variety of innovative applications of Stirling engines shows that the intellectual vigour Kelvin brought to them in 1847 is by no means exhausted. The search for efficiency, for carbon-neutral electricity generation and waste disposal leaves us knocking at the door of opportunity at the ‘final frontier’.

5 Atmospheric electricity

The formulation of the first and second laws of thermodynamics around 1851 enabled one to re-cast physics in terms of energy, and motivated the effort to study (electric) potentials in all possible situations. During the period 1859–1861, Kelvin studied the electrostatic field near the surface of the earth, which is of order 100 V/m, varying according to the weather and other factors.
Kelvin was responsible for two devices for measuring this field, notably the water drop-per equaliser sensor. A narrow column of water is slowly poured. The electric field is found from the change in height at the point where the column breaks up into drops. He added photographic recording of the result. He expressed the view that at some date in the future, electrical measurements would form important additions to our ‘means for prognosticating the weather’ [12]. It is interesting to note that Kelvin was recycling a technology he had just (1858) invented - the syphon recorder (the ancestor of the ink-jet printer.) Both rely on an understanding of the behaviour of ionised fluid droplets [13].

Various observatories were set up around the world, including Kew in 1908 – UK measurements continued to the 1980s. Kakioka in Japan continues to make measurements after over a century [14]. There the matter may perhaps rest, but two events have unexpectedly renewed interest. It appears that earthquakes are preceded over a few days by anomalously high electric fields. Many lives could be saved if this could form the basis of accurate predictions. The answer to the question ‘should I stay out of doors through the coming night, or is it safe to sleep indoors’, really matters [15]. The heavy prison sentences handed out to scientists who did not predict the 2009 earthquake at L’Aquila in Italy shocked many people for different reasons. The Fukujima earthquake in Japan would have avoided much loss of life if such predictions had been available. A satellite proposal wishes to make a step-change in our knowledge. Over a 3-year period, some 400 earthquakes of magnitude 6–6.9 are expected, and these will form the training data set for quake predictability tests.
6 From the nineteenth to the twenty-first century

In the cover text to the very recent Kelvin: Life, Labours and Legacy [16] we read that ‘the waxing and waning of the twentieth century and the supplanting of classical physics have eroded his reputation, so that for many scientists he is remembered as little more than a unit of temperature’. This concise study demonstrates that Kelvin’s ‘sure touch’ has resonated in five fields which epitomise our current century: the Olympic movement, the apparent need for fields that fill all space, climate change and environmental disaster and space travel.

References