Chapter 11

Entropy as thermal charge: an application of bond graphs inspired by Carnot and his cycle

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1 Sadi Carnot

Sadi Carnot (1796–1832) is recognised as the founder of thermodynamics. He formalised the Carnot cycle which we now call the second law of thermodynamics, the principle of the irreversibility of physical phenomena. This invention was published at his own expense in 1824 in his book *Reflections on the Motive Power of Fire and on Machines Fitted to Develop That Power* [1].

A precursor of Lord Kelvin (1824–1907), he also discovered towards the end of his life the first law of thermodynamics, that of the conservation of energy, as shown in his posthumous articles published in 1878. This principle was rediscovered and stated in 1842 by Mayer (1814–1878) following in the footsteps of Clapeyron who had already published his 'Memoir on the Motive Power of Heat' in 1834, in which he developed the ideas of Carnot who had died 2 years before.

Carnot's small book is at the root of all thermodynamics, the science of energy exchanges, thermal ones in particular, in all mediums whatsoever, particularly steam and gases.

It was published well after:

- the invention of the first cylinder and piston steam engine (1690) by Denis Papin (1647–1712),

- the separation of boiler and piston (1712) by Thomas Newcomen (1664–1729), and

- the introduction of the separate condenser (1763) by James Watt (1736–1819),

at a time, therefore, when steam technology was available to everyone, and after the invention of the first internal combustion engine (1804) by brothers Claude and especially Nicephore Niepce (1765–1833).

Although the main principles of work in a heat engine had been identified and applied, it still remained to produce a theory of thermal machines, which Carnot did. Carnot's book has been reprinted several times both in French, the original, and English. As a reference for this study, we use principally Mendoza's English edition (1960), reprinted in 2005 [2]. All things considered, Carnot's little book contains many new ideas despite some inconsistencies in terminology: Carnot often refers to 'calorique', what we would now call 'entropy', and to 'chaleur' which

today we still call 'chaleur' or heat. The book, however, must be considered in the context of its times when all thermal effects were thought of in terms of 'calorique', a substance without mass that enters matter when the latter is heated, increasing its temperature.

Then, the theory of calorique was replaced by the kinetic theory of heat by Clausius (1822–1888) and Joule (1818–1889), a theory in which the notion of entropy was completely absent. It was only introduced afterwards as a factor of integration: a somewhat obscure procedure. Carnot's idea is much better: heat flow is equal to the product of entropy flow times the absolute temperature, just as in electricity the flow of power is the product of the flow of charge times the voltage. In this sense, entropy is a very simple concept, a kind of thermal charge. Moreover, entropy is a simpler concept than electric charge as it does not generate a field as electric charges do. In short, entropy has numerous properties of electric charges but differs from them in other respects. The idea of considering entropy as a thermal charge is by no means a new one. Apart from Carnot himself, authors such as Callendar [3], Thoma [4], Job [5], Falk [6,7] and Fuchs [8] all use the same point of view.

Carnot's book contains other new ideas for the time, such as that of integrating in a repetitive process the various phenomena at work in a heat engine, in particular those occurring in a cylinder closed by a piston (although in reality steam was replaced at the end of each cycle in the steam engines of the day). In addition, Carnot compares the power of steam engines to that of hydraulic machines, in which the pressure of water is the intensive variable and the quantity of water the extensive variable, the product of one times the flow of the other giving the available power. Water is not affected by the process of extracting power, just as entropy is not itself affected in a thermal process. In other words, it is the transport of 'calorique' from the heat source to the cold source that produces motive power, just like the movement of water in a hydroelectric power station. In modern hydraulic terms, pressure is the intensive variable and the flow of water is the extensive flow variable. Transcribed to the field of heat engines, temperature difference is the intensive variable and entropy flow is the extensive flow variable. Carnot explicitly said that a cold source is needed to eliminate 'calorique' once it has performed its task of producing motive power.

On the other hand, he also mentions that zones of different levels of 'calorique', that is, of different temperatures, can balance each other out without producing motive power and that to obtain good heat engines such contacts should be avoided. Moreover, one of Carnot's recommendations is that a change in temperature must only take place by a change in the volume or quantity of gas and not by conduction, which he calls the pointless establishing a balance of 'calorique'. To sum up, entropy must not be exchanged by conduction, which is of no use, but by a change in the volume of gas, which allows an adjustment to high and low temperatures, and vice versa. He further adds that 'calorique' is not consumed, but transported from a warm body to a cold body, which produces motive power. Finally, Carnot mentions an inverse cycle: by providing motive power, we can pump 'calorique' from a cold source to a warm source.

2 Bond graphs

Bond graphs are a tool for studying dynamic systems, thermodynamic systems in particular, developed by Paynter [9] at MIT in the 1960s and still appearing (2009) in the MIT course by Hogan [10]. This tool, which provides an easy method to show Carnot's ideas, is now documented in a large body of literature, developed in particular by Thoma [4,11–13], a former team from MIT [14], and extended into French by Dauphin-Tanguy [15].

One of the main ideas of bond graphs is to show the reality studied as a network of elements in energy interaction, that is to say, to show by arrows the flows of energy (therefore, of power) circulating between the elements making up the dynamic reality that one seeks to understand.

According to the maximum abstraction principle that Lazare Carnot (1753–1823) had instilled in his son Sadi when teaching him differential equations, another fundamental idea of bond graphs is to represent simply by letters the constituent elements of the reality studied that can produce, consume, accumulate or dissipate energy. The consequence of this idea is that the elements of representation are selected on the basis of the constraints limiting the observation and action of the scientist studying his problem. Using this principle, the particle of matter and the environment of Fig. 1 are reduced to simple letters in bond graphs, as shown in Fig. 2.

Matter

Mechanical power

Energy

Thermal power

$\dot{E}$ [Watt = J s^{-1}]
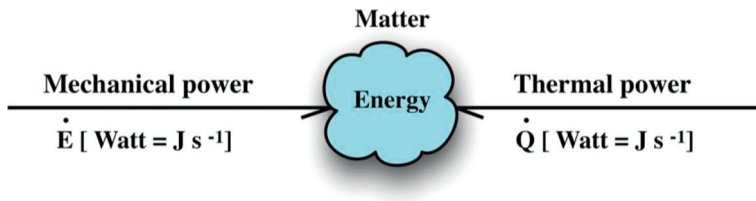
$\dot{Q}$ [Watt = J s^{-1}]

Figure 1: Graphical representation of a domain of study consisting of a particle of matter interacting mechanically and thermally with its environment. (i) The two modes of energy interaction (mechanical and thermal) are represented by thick semi-headed arrows to indicate clearly that these arrows represent an energy flow, i.e. power. (ii) The position of the arrowhead, determined by the researcher, indicates to him the conventional positive sign for energy circulation in the bond considered.

SF —— p [Jm^{-3}] —— **C** —— T [K] —— **SF**

$\dot{V}$ [m^3 s^{-1}]

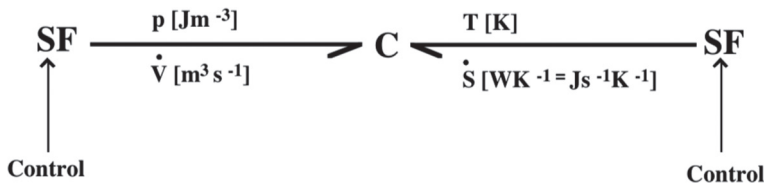$\dot{S}$ [WK^{-1} = Js^{-1}K^{-1}]

Control

Control

Figure 2: Representation of a particle of matter C interacting hydraulically and thermally with its environment represented by the two sources of flow SF. (i) The letter C represents a particle of matter and, specifically, its capacity to accumulate the extensive magnitudes of the energy domains considered: volume for the hydraulic domain (left) and entropy for the thermal domain (right). (ii) The letters SF (Source of Flow) represent sources (or sinks) of extensive flow variable. Indeed, it represents the conditions imposed at the boundaries of the system considered, the so-called conditions at the limits of partial differential equations. (iii) These conditions at the limits can be modulated, which is indicated by the narrower vertical arrows terminated by a small arrowhead. These arrows, which differ from the thick ones of the horizontal energy bonds, represent signals and thus enable the representation of reality in the form of bond graphs to be used immediately to make automatic adjustments. When the sources are not modulated by a control signal, they are simply considered as constants. (iv) The names of the conjugate variables of the energy domain considered appear on the energy bonds: pressure and volume variation for the hydraulic domain and absolute temperature T and entropy flow rate for the thermal domain.

We can show here the power of representation of bond graphs, which can be used to describe, in a highly compact and calculable manner (as they can be directly implemented as computer programmes to carry out simulations) phenomena encompassing a multitude of energy domains. This arises from the fact that each energy bond represents a power considered as the product of an effort (the conventional word in bond graphs for a potential) times a flow:

$$Power = Effort \times Flow$$

in which the efforts (potentials) and flows can belong to different scientific disciplines. In classic thermodynamics, the conservation of the power that represents the variations of energy in a system is expressed by the equation:

The relation between a representation of thermodynamics in the form of equations and one in the form of a bond graph becomes apparent in Fig. 3.

For the purpose of synthesis, we can construct a table of the principal domains of energy interaction:
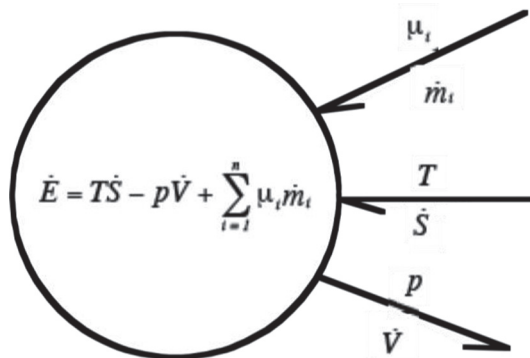

Figure 3: Representation of a thermodynamic reality in the form of a formula and a graph. The sign of the powers in the formula is represented by the direction of the arrows, the heads of which correspond to the signs of the equation.

Table 1: Efforts and flows of different domains of thermodynamics.

Domain	Effort variable	Flow variable
Mechanical translation	Linear speed (m s^{-1})	Force (N)
Mechanical rotation	Rotation frequency (rad s^{-1})	Torque (N m)
Fluidics	Pressure (N m^{-2}) or (J m^{-3})	Volume deformation (m^3 s^{-1})
Electricity	Electric field D (V)	Electric current D (A)
Magnetism	Magnetic field H (A)	Magnetic flow B (Wb s^{-1})
Thermal domain	Temperature (K)	Entropy flow (W K^{-1}) or (J K^{-1} s^{-1})
Chemistry	Chemical potential (J kg^{-1})	Mass flow (kg s^{-1})

In Table 1, it is interesting to note that:

- Chemical energy appears there as stored in matter in the form of a force field between the particles of which it is composed, effort, called chemical potential, being measured in Joules per particle.

- Bond graphs can be used to represent tensor magnitudes such as are found in continuum mechanics [16].

Another great idea of bond graphs is to represent causality, that is, to distinguish permanently the variables that are the causes of the phenomena from those that are its effects, independent variables from dependent variables as it were, Fig. 4.

We can sum up the principles of bond graphs in diagrams showing the relations between the various concepts used to represent dynamic systems, linear or otherwise, since bond graphs can represent non-linear systems. We can start with the circular relation that exists between the variables, Fig. 5.

The diagram above is a modified version of the circular diagram that Thoma has used in his courses, the so-called 'Thoma Carrousel' [4], Fig. 6.

Observing the Thoma Carrousel, we note that there is still a direct relation R both ways between effort and flow. We can represent it by the diagram in Fig. 7.

To be complete it would still be necessary to introduce memristance, an element that connects accumulations of effort and flow (practical applications of which are beginning to emerge in 2009), Fig. 8.

To use bond graphs in thermodynamics, Thoma has introduced a new element, resistive source, called RS, whereby thermal phenomena can be represented by a production of entropy, Fig. 9.

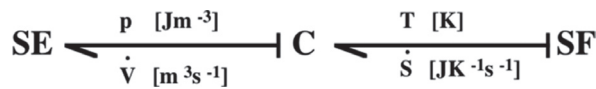We can show element RS by the diagram in Fig. 10.

Figure 4: Representation of a particle of matter interacting hydraulically and thermally with its environment. (i) Here, the small vertical line at the right end of the left arrow, representing the hydraulic link, called a causality line, indicates that the matter C considered is subjected to a fixed pressure p, possibly constant, by the source of effort SE (source of pressure in hydraulics). (ii) The small causality line on the right of the right arrow, representing the thermal bond, indicates that the entropy flow is fixed by the source of flow SF (i.e. by the environment).

Figure 5: Circular arrangement of variables to represent dynamic systems. (i) Elements representing system parameters are surrounded by a thick black line and those representing variables are surrounded by a double line. (ii) Energy originating from changes in flow f is stored in a form p in the reality considered (element C in bond graphs). (iii) Energy from changes in effort e is stored in a form q in the reality considered (element I in bond graphs). (iv) Effort e is fixed by a source of effort (element SE in bond graphs) and flow f is fixed by a source of flow (element SF in bond graphs). (v) A double arrow appears in the relation between energy and elements C and I because these can accumulate and return energy. (vi) Letters C and I represent relations that are not necessarily linear.

Figure 6: The Thoma Carrousel in its integral causality version. (i) The variables of a dynamic system are organised circularly in bond graphs: if one starts with a flow f, one integrates it temporarily in a displacement q, which gives an effort e at the output of capacitor C which is integrated temporarily in a momentum p which returns a flow f at the output of inductor I. (ii) The reasoning reciprocal to integration would be valid for derivation if it is turned the other way round.

Figure 7: Direct relation between effort and flow (element R in bond graphs): an effort e multiplied by a resistance R gives a flow f, and a flow f divided by a resistance R gives an effort e.

q (accumulation)	← /	**M (Memristance)**	. →	p (accumulation)

Figure 8: Direct relation between accumulation of effort and accumulation of flow: an accumulation of effort q divided by a memristance M gives an accumulation of flow p, and an accumulation of flow p multiplied by a memristance M gives an accumulation of effort q.
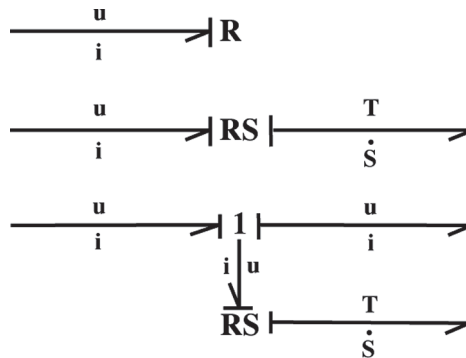
Figure 9: Introduction of the element RS ('an R set by an S' as Thoma calls it). (i) Element R at the top of the figure becomes element RS in the middle because all dissipation, by friction in particular, generates entropy. The irreversibility of this production appears clearly from the fact that the energy arrow producing entropy emerges from element RS, which implies that power always emerges from element RS into the environment, according to the laws of nature. (ii) The causal sign, which distinguishes the dependent variables from the independent ones, is arbitrary on the left bond of element RS. Conversely, the arrow representing thermal power emerging from the right of that RS has a causality line at its left end, which indicates that the input of RS is the temperature of the environment and the output is entropy flow, in the sense of systems theory. (iii) With element RS, it becomes very easy to introduce a thermal part in the study of a complex system: one needs to couple only this thermal part by a link (represented in the figure by the symbol 1) to another energy domain, to an electrical part for example, as shown in the third bond graph at the bottom of the figure.

e (effort)	← /	**RS (Resistive Source)**	. →	f (flow)
		↓		
		Entropy		
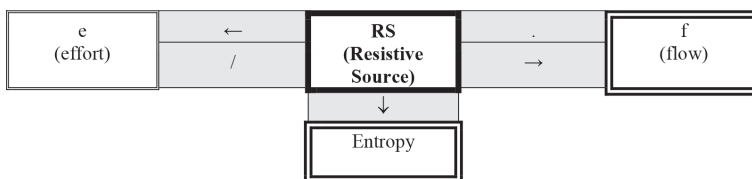
Figure 10: Schematic representation of element RS (resistive source). (i) Here the arrow linking element RS to Entropy only points in one direction, contrary to those making a connection between elements that can store and return energy in the previous diagrams, this being a clear representation of the irreversibility of nature. (ii) After the introduction of element RS, it becomes very easy to represent the production of entropy from the various friction phenomena acting in nature, including chemical friction, Fig. 11.
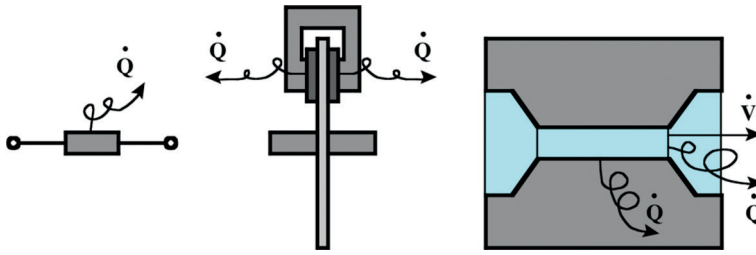
Figure 11: Various modes of entropy-generating friction: electrical on the left, mechanical in the centre and hydraulic on the right.

3 Systemics

In some discontinuous representations of reality, such as bond graphs, for example, reality is considered as made up of a network of elements in interaction. The lines connecting the elements have no other meaning than to show a conceptual relationship between these elements. At first sight, the spatial positions and dimensions of the elements have no effect on the study of the reality considered. Such representations, although approximate, are more than sufficient for some studies.

3.1 Structure

Once the elements and their interactions are defined we still need to be able to describe, in a systematic manner, the structure itself of the interactions occurring in the reality studied, that is to say, to be able to connect multiple elements to each other to constitute a whole in interaction.

To do this, bond graphs arbitrarily use the symbol '0' to represent a point of potential (effort) known in relation to a reference: all energy arrows attached to this link have an identical effort, which amounts to saying that the elements attached to it are topologically in parallel (see Fig. 12).

To be systematically complete, one still needs to be able to the show points of reality at which the flow is common to all energy arrows connected to them. As we have seen in Fig. 13, bond graphs conventionally use the symbol '1' to represent such links in series. This type of link provides, in particular, a very easy way of including the thermal domain in the analysis of electric or other circuits, the R elements, a first approximation of reality, being replaced by RS elements, which include the fact that all types of friction produces entropy conservatively. And often, as experience shows, the thermal part should be included, since power lost by friction appears as heat (or entropy × absolute temperature) and heats the installation, often leading to dangerous levels of overheating that must be taken into account. To repeat, a pure R element does not exist and must be replaced by the RS field as soon as one wants to include thermal effects in the bond graph. In this sense, entropy can be considered as a kind of substance that is always linked to the transmission of heat and thereby the RS field becomes power conserving.

Other productions of entropy arise from mixing chemicals or from chemical friction ([13, p. 23], and with reversible mixing with selective pistons, p. 74). These modes, easily represented by bond graphs, are interesting in principle, but not relevant for this study. Although bond graphs
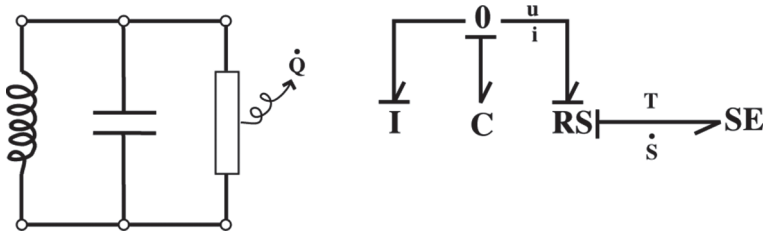
Figure 12: Left: schematic representation of a parallel electric circuit generating entropy. Right: a bond graph representation of the same circuit. In the bond graph, it can be noted that the energy arrow emerging on the right of element RS represents dissipation in the form of entropy production and is orientated positively towards the environment, itself represented by source of effort SE which sets the temperature.
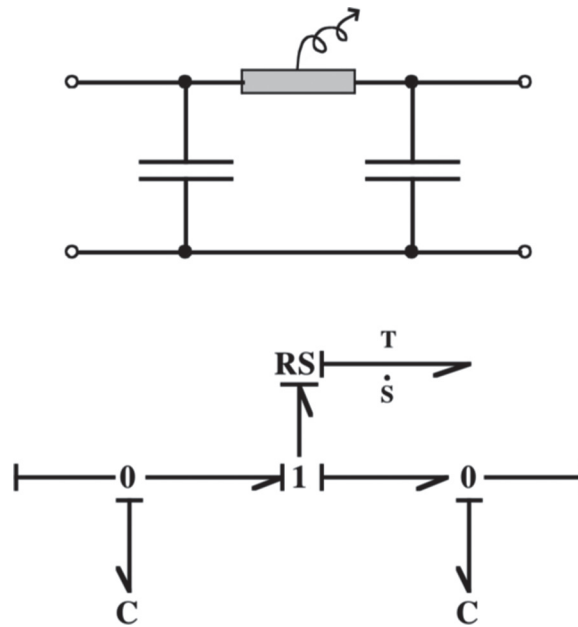

Figure 13: Representation of a link in series by bond graph symbol 1. (i) In the bond graph, junction '1' compels the flow to be identical in the three energy bonds connected to it. Resistive source RS situated between the two capacitors C produces a flow of entropy as long as there is a voltage imbalance between the two capacitors. (ii) The new entropy is indestructible and must be eliminated, for example, in an inverse source on the right of the bond graph. As the causal sign on the RS element shows, the resistive part can depend on temperature.

can account immediately for the thermal aspects of an electric system, it still remains to state where the entropy generated by the resistance is going to go, that is, to show in a calculable manner how the thermal effects propagate in the circuit. For example, if a resistor is surrounded by two capacitors, the one to which the entropy is going to propagate must be determined, and bond graphs can do this.

3.2 Dynamic systems

We show how the problem can be solved graphically and also how a bond graph allows the derivation of the various flows of entropy acting in an electric circuit: just as electric capacitors are said to be electrically charged, thermal capacitors can be said to be thermally charged, the thermal charge being the entropy.

We note that the electric flux in the circuit is

$$i = \frac{(u_1 - u_2)}{R}$$

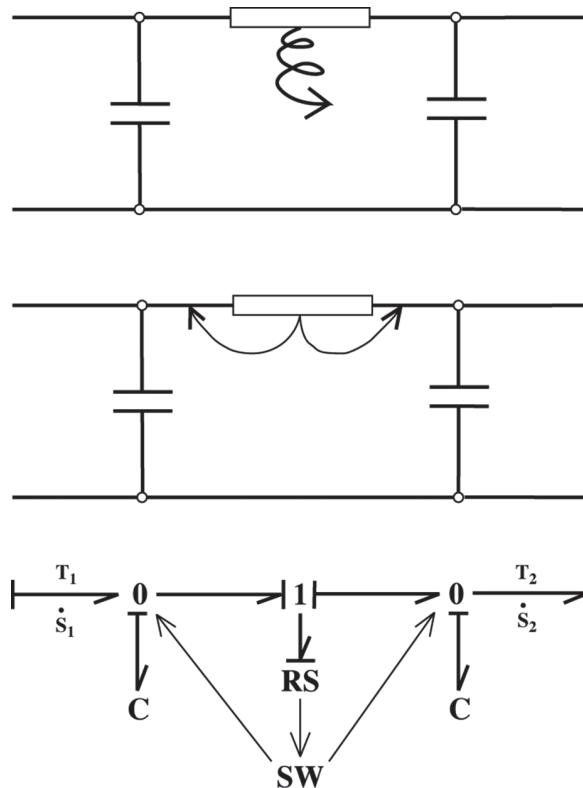and the dissipated power:

$$\Phi = (u_1 - u_2) \cdot i = \frac{(u_1 - u_2)^2}{R}$$

Figure 14: Representation of entropy distribution in an electric circuit. (i) The power dissipated in the resistor is always positive whatever the sign of the electric current. (ii) The entropy generated by the resistor is injected into capacitors C around it. (iii) The introduction of a switch SW allows the entropy created in the centre to be transferred to the left or right and enables us to understand how the entropy is injected into the capacitor C that is at the lowest temperature.

We see in particular that the dissipated power is always positive whatever the direction of flow, as indicated by the formula.

3.2.1 Multiport C elements

We can continue to explore the electric domain by considering networks of capacitors, known as multiports C or, in the bond graph community, simply C-fields. An example of a multiport is given in Fig. 15 in which two capacitors, one on the left and one on the right, are connected by a coupling capacitor, in the centre; this has a radical effect on the behaviour of the circuit. The capacitive reactance that can be measured on energy bond 2 depends on what arrives from side 1. If the sources of flow are replaced by sources of effort, a change in side 2 sends a small amount of power to side 1, which affects the capacitance of side 2.

Multiports are very important for representing standard thermodynamics. They can easily show situations in which hydraulic and thermal phenomena appear simultaneously. For example, Fig. 16 shows a bond graph that enables us to calculate the change in entropy at constant volume and constant pressure and, after multiplication by the absolute temperature, the change in specific heat values, since that change simply comes from the cross terms of the multiport C [13].
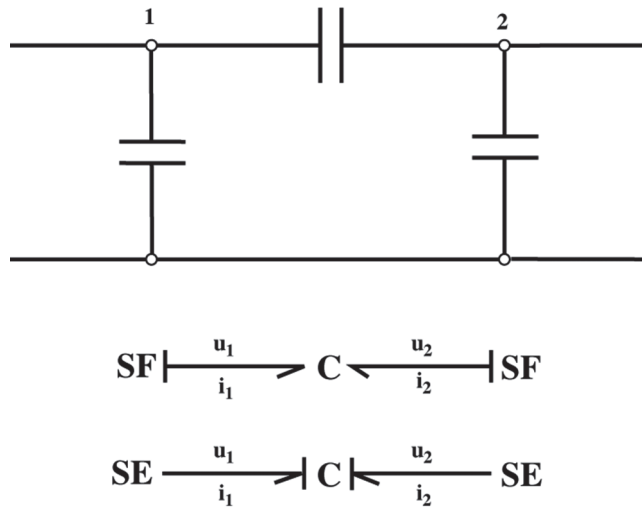
Figure 15: Example of an electric circuit, here a network of capacitors, represented at the top in schematic form and at the bottom in bond graph form by a multiport C. (i) In the top circuit, the capacitor in the centre couples the two left and right parts. (ii) At the bottom are two examples of a multiport C, one with flow sources and the other with effort sources, a circumstance which imposes different causalities on the multiport C.

Figure 16: Hydraulic and thermal multiport C with sources of effort.

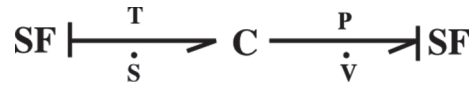$$SF \longmapsto \frac{T}{\dot{S}} \longrightarrow C \xrightarrow{\frac{P}{\dot{V}}} SF$$

Figure 17: Multiport C between two sources of flow. If the sources of flow are set to zero, the power circulating in the two energy bonds disappears and multiport C conserves the energy.

Accumulation equation: Let us consider a particle of matter that presents coupled thermal and hydraulic phenomena. If both entropy and volume are injected into this particle, it accumulates both thermal power and hydraulic power. We can show such a phenomenon by the bond graph in Fig. 17, which presents two sources of flow SF, one thermal and the other hydraulic, and a multiport C representing matter:

The power circulating in the bonds is:

$$\dot{E} = T \cdot \dot{S}$$

$$\dot{E} = -p \cdot \dot{V}$$

Multiplying by an interval of time and integrating for a short time, one obtains the familiar equation:

$$dE = T \cdot dS - V \cdot dP$$

which represents the accumulation, in the form of energy, of the power supplied to multiport C by the two energy bonds.

Power in hydraulic circuits: Bond graph representation reveals its full power when we try to understand what happens in pipe circuits carrying fluids. The total power transported in a pipe is

$$\dot{E} = \dot{U} + p \cdot \dot{V}$$

This power is made up of the sum of the variation in internal energy and in hydraulic power, itself represented by the product of the pressure times the volume flow of the fluid considered, that pressure being the only measurement that really matters in practice in hydraulic circuits.

3.2.2 Heat conduction and entropy conduction

The power of representation of bond graphs can also be demonstrated by taking as examples systems in which we only seek to understand the thermal phenomena that develop in them.

Heat conduction through a wall is the most typical example of this. We have seen how bond graphs have drawn our attention to the fact that all resistive phenomena generate entropy, as shown in the top circuit in Fig. 18. If this production of entropy is not accounted for, we have what we call a pseudo-bond graph, with variables T and Q as conjugate variables instead of and for a true bond graph. The representation of the wall then becomes a simple RC network as in the lower circuit in Fig. 18 because the heat flux is conserved in conduction.

If we make a bond graph representation of the wall, we obtain a pseudo-bond graph as shown at the top of Fig. 19 and a true bond graph as shown in the lower diagram in Fig. 19:

We see that for problems of heat conduction, the true bond graph is more complex than the pseudo-bond graph. This explains why pseudo-bond graphs are used almost universally in this domain: many people even try to avoid the notion of entropy altogether.
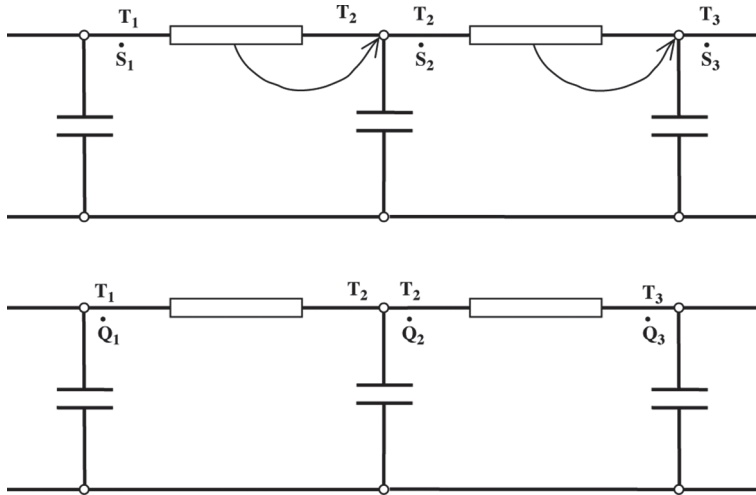
Figure 18: Thermal circuit of a house wall. The thermal circuit at the bottom is simple because it does not show the entropy flows generated by the resistors as shown in the top circuit: the heat flux is conserved.
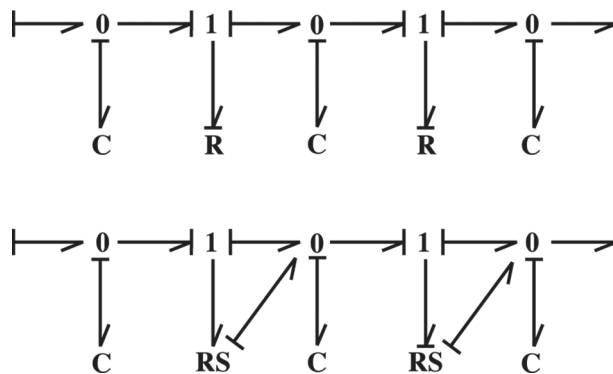
Figure 19: Bond graph representation of a wall with a pseudo bond graph (top) and a true bond graph (below). (i) The true bond graph at the bottom is more complex because the new entropy generated by the RS element, which has replaced the R element, is injected into the multiport C on its right. (ii) Note that since in a house the entropy and heat flows always from the interior to the exterior, we do not need the switch SW of Fig. 10.

To sum up, in true bond graphs the product of effort times flow gives a power, that is, a flow of energy, while in pseudo-bond graphs this is not the case: the product of effort times flow does not have the meaning of a flow of energy. The classic example of this is the transmission of heat in which one considers heat flow and temperature.

3.2.3 Perfect gas equation

A perfect gas is a very good approximation of the behaviour of fluids at high temperatures and low densities, as well as matter in general in a very diluted form.

Its behaviour equation is as follows:

$$p = \frac{m}{V} \cdot R \cdot T$$

For pseudo-bond graphs, two others, usually called calorific equations, are added to it:

$$\dot{U} = c_v \cdot T \cdot \dot{m}$$

and

$$\dot{S} = c_v \cdot \dot{m}$$

3.2.4 Origin of heat and enthalpy

Normally, heat and enthalpy are used in preference to entropy. With water and steam, the origin of temperatures is defined as 0°C for the liquid state and all values of energy variables are taken from there. Thus, one writes:

$$U = c_v \cdot m \cdot T$$

Entropy is treated differently and one writes

$$dS = \frac{dQ}{T}$$

$$dQ = c_v \cdot dT$$

and

$$S = S_1 - S_2 = \int c_v \cdot \frac{dT}{T}$$

such that one obtains, with the usual approximation $c_v = Cte$:

$$S_1 - S_2 = c_v \cdot \ln \frac{T_1}{T_2}$$

According to Nernst's theorem, entropy is zero at null absolute temperature, a purely theoretical value as the entropic content at low temperatures is not known. Moreover, one

can also have freezing at low temperatures [13, p. 78], which is why one always starts with intermediate values.

3.2.5 Conduction and convection

So far, we have spoken of the step-by-step conduction of entropy in matter. It still remains to represent the convection of entropy, which is the transport of entropy by the movement of the fluid itself: in this case, the power transmitted is given by the flow of enthalpy.

To represent the phenomenon of convection, we can use vector bond graphs in which the energy bonds no longer represent a single pair of energy variables (effort and flow), but a vector in the algebraic sense, that is, an ordered set of pairs of these variables (efforts and flows). In fact, for convection, we use a pseudo vector bond graph containing two pairs of variables, that is, two efforts, pressure and temperature, and two flows, mass flow and enthalpy flow. The advantage of using enthalpy flow is that the change it undergoes is equal to the power injected in the fluid by the solid walls. We can now construct new thermodynamic elements and add them to the basic elements of bond graphs.

3.3 Bond graphs of pipe circuits with hot gases

It is often necessary to use bond graphs to represent installations including fluid pipe circuits. This can be done immediately using pseudo vector bond graphs with temperature and enthalpy flow as the first pair of variables and pressure and mass flow as the second pair of variables. One can thus develop progressively all of the important components for understanding such circuits, starting with the simplest, resistance to convection which we call RECO, Fig. 20.
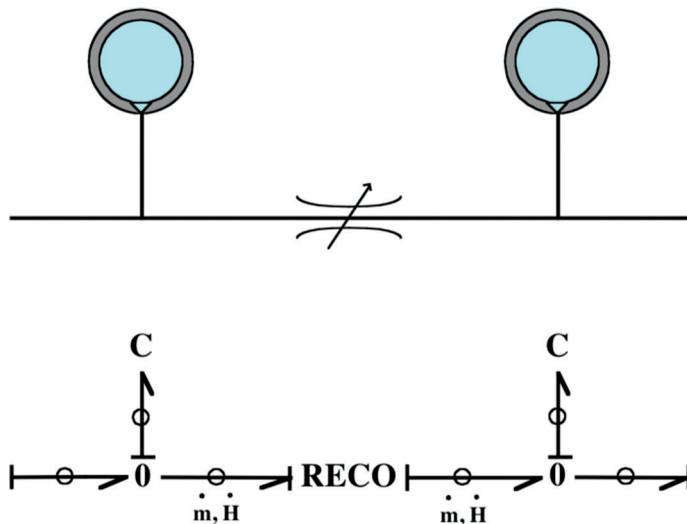

Figure 20: Bond graph of a resistance to convection (RECO). The fact that the bonds are vectors (in the algebraic sense) is indicated by the small circles in the middle of the bonds.

In the RECO of Fig. 20, enthalpy flow is conserved as is mass flow, and we have the equations:

$$m = K_d \cdot A \cdot \frac{\sqrt{(p_1 - p_2)p_2}}{T_1} \; if \; p_2 > 0.5 \cdot p_1$$

$$m = \frac{K_d}{2} \cdot \frac{A \cdot p_1}{\sqrt{T_1}} \; if \; p_2 < 0.5 \cdot p_1$$

Remarkably, the temperature at the bottom of the flow does not influence the flow. The next component to be introduced is the heat exchanger, which we call HEXA, in which a flow of thermal power is simply added to the flow of enthalpy:

$$H_{Sortie} = H_{Entrée} + \dot{Q}$$

We can show such a heat exchanger by Fig. 21:

A resistor R can be added to this bond graph to adjust the causalities, Fig. 22:

The next element is a thermofluid machine, named TEFMA, which allows us to show a mechanical energy bond in which the flowing power is , where is the torque and is the angular frequency of rotation of the shaft. In Fig. 23, in addition to the bond supplying power to the environment, an inertia I has been added in series (by link '1' on the right) to make the causalities consistent.

The equation for such a thermo-fluid machine becomes

$$\dot{H}_{Sortie} = \dot{H}_{Entree} + \dot{Q} + M \cdot \dot{\varphi}$$
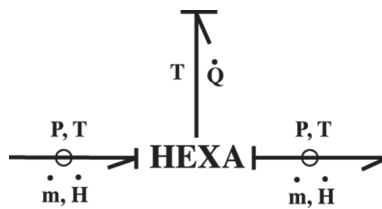

Figure 21: Bond graph of a heat exchanger HEXA, in which we have a balance of power.
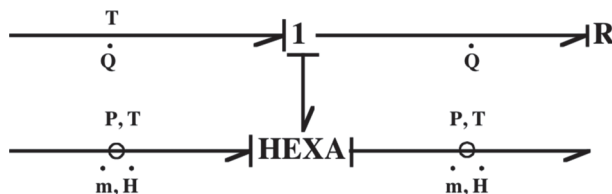

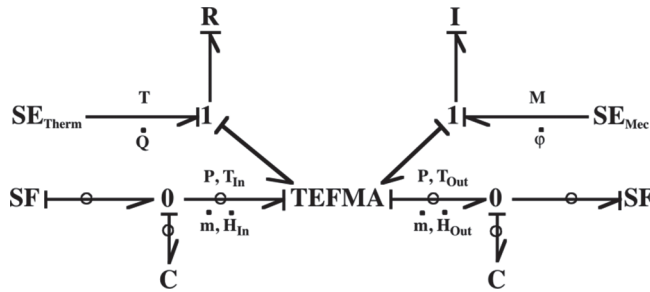Figure 22: Bond graph of a HEXA with a resistor added to adjust the causalities.

Figure 23: Bond graph of a thermofluid machine TEFMA in which a mechanical branch with inertia I is added at the top right to adapt the causalities. (i) At the top right, we have the mechanical bond that generates the rotation frequency of the shaft in response to the torque of the turbine and the torque of the flywheel which are both connected in series. This is a true bond graph. (ii) The thermal branch, at the top left, receives its temperature at the thermofluid machine TEFMA and goes into a thermal element R: this part of the bond graph is a pseudo-bond graph.

In the elements above, the causality lines are all important: we see that the horizontal vector bonds, representing flows of gas, have as efforts p and T and as flows $\dot{m}$ and $\dot{H}$. These are pseudo vector bonds.

3.4 Bond graph of a thermofluid machine

One can assemble all of the elements described above at will to represent any heat engine, a thermofluid one in particular.

Figure 24 brings together the three basic elements we have prepared to construct bond graphs of a hot gas heat engine. At the bottom, we have added two coupling capacitors C to ensure correct causalities. Apart from the coupling capacitors, the flywheel on the turbine shaft also provides consistent causalities. Two small precautions are to be taken when modelling any turbomachine:

- reticulate the pipes well by parallel junctions followed by a leak resistor to control the losses in the pipes, and

- ensure that the velocities at the coupling capacitors are quite low, of a magnitude such that their kinetic power is negligible compared with the static power of the enthalpy flow.

The difference between reversible and irreversible thermodynamics is not brought out clearly in many texts. Using the bond graph method it becomes very simple: if the R elements impose only negligible dissipation, the system is reversible. If not, we have an irreversible system which produces entropy in RS fields. All the frictions reside there, including entropy conduction under finite temperature drop, mixing and chemical effects.

4 Icons for thermofluid machines

Our bond graph representation of thermofluid machines is perfectly adequate, but some people need a schematic representation of the RECO, HEXA and TEFMA machine elements, if only for illustration purposes.
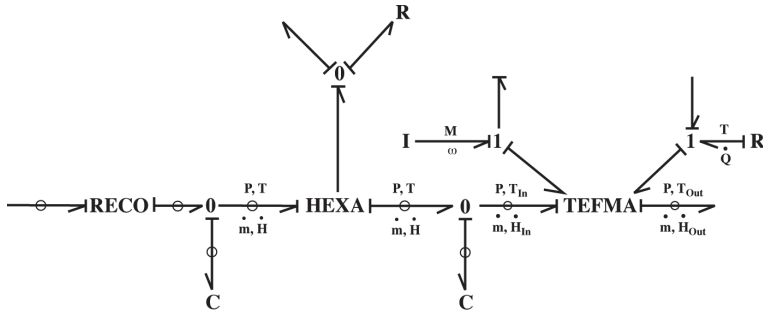
Figure 24: Bond graph containing the three elements RECO, HEXA and TEFMA, separated by coupling capacitors.
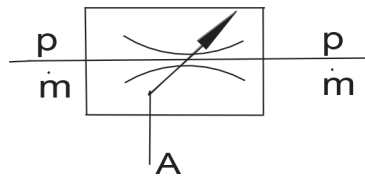

Figure 25: Icon to represent a RECO. The icon represents a small passage in a resistor, the letter A representing the smallest section of the passage.
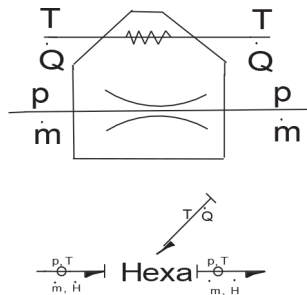

Figure 26: Icon for a HEXA. An additional thermal line in a HEXA (with bond graph below). This figure is an extension of the heat exchanger, in which the hot fluid is depicted by a zig-zag line within the casing.

For the RECO, we can make the proposition as in Fig. 25:

With regard to the heat exchanger HEXA, the icon is a little more complex because it has to contain two lines of power, Fig. 26:

The icon for the TEFMA, for its part, must contain a third line of power, Fig. 27:

Note that the coupling capacitors do not appear in the RECO icon. Therefore, Fig. 28 proposes an icon for a turbomachine with coupling capacitors and a flywheel on the shaft:
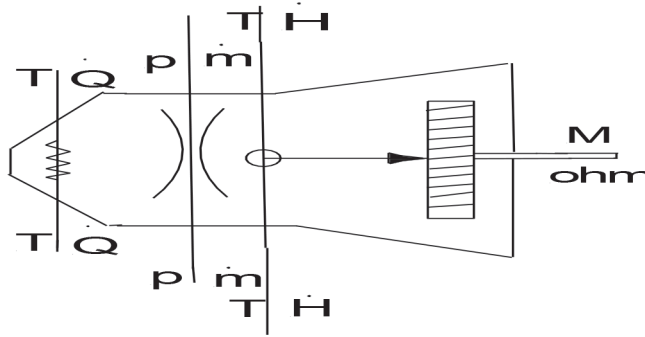
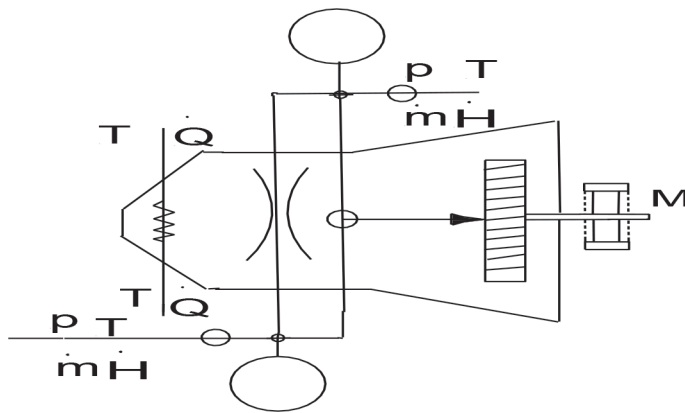Figure 27: Icon of a thermofluid machine TEFMA with a turbine.

Figure 28: Complete icon of a TEFMA, with icons for the coupling capacitors and for the inertia of the shaft.

5 The Carnot cycle

Carnot's original heat engine was a steam engine that consisted of a cylinder, a piston, a connecting rod and a transmission shaft, together with a heat source and a cold source, alternatively entering into thermal contact with the cylinder to exchange entropy.

Carnot's idea was to introduce two partial cycles, in which the working medium is completely isolated thermally, adiabatics in modern terminology. At the end of heating, for example, the medium is disconnected from the heat source, the piston continues its expansion and consequently the medium cools, until it reaches the low temperature at which it can be connected to the cold source without thermal shock (without thermal short circuit, or 'jolt' one might say).

Next, the steam used is expelled by movement of the piston. At the end of the expulsion, the medium is again disconnected from the cold source, but the piston continues to move. It compresses the medium and raises its temperature until it reaches the high temperature. Then, the cycle starts over again.

By this decomposition of the process, Carnot avoids the two thermal short circuits, or, as he put it, the production of 'calorique' (entropy) by conduction between a finite temperature difference. The instants at which switching occurs between the isotherms and the adiabatics must be well synchronised with the inversion of piston movement: errors in these instants diminish the power of the machine. Carnot, moreover, also understood that a heat engine can be used inversely, by passing the 'calorique' from the cold source to the heat source. The power then outputs the 'calorique' from the cold source, causing its temperature to fall further by injection of mechanical power.

5.1 Bond graph of the Carnot cycle

We can show an elementary Carnot heat engine schematically, Fig. 29:

Such a heat engine can be represented very easily using bond graphs (Fig. 30).

The working cycle of this machine is the following: to start with, the piston is at the extreme right and begins to move to the left. The switch connects the heat source and the hot vapour enters the cylinder. At a certain point, the switch goes off, the piston continues to travel and the medium to expand by cooling until it has reached the temperature of the cold source. Then, the piston reverses its movement and discharges the cold medium, cooled by expansion. Towards the end of expulsion, the switch disconnects the cold source, the piston reverses its travel and compresses the remaining medium until it again reaches the temperature of the heat source. And the cycle can begin again with new 'calorique' or steam. By this process, Carnot avoids thermal contact by the steam with different temperatures, that is, thermal short circuits.

Sometimes it is said that the Carnot cycle must take place infinitely slowly to avoid entropy being generated by the irreversibilities. This is not the case because, apart from the fact that such a machine would have no useful power output, thermal short circuits are avoided by the adiabatics whereby the temperatures adjust their values. On the contrary, it must be executed at a suitable intermediate speed at which the resistors in series and the resistor in parallel towards the environment produce only a tolerable amount of entropy.
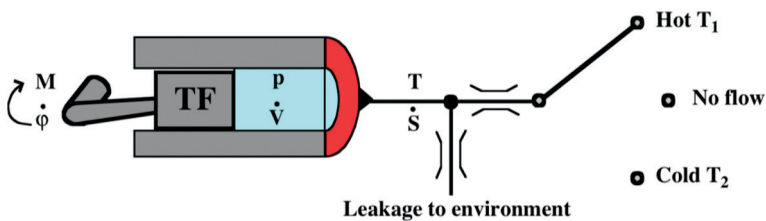

Figure 29: A thermal machine producing the Carnot cycle. It is made up of a horizontal cylinder, a piston, a link mechanism and thermal connections with the heat and cold sources and the environment. The three-way switch on the right is important in that it allows the machine to be disconnected from the sources to produce the adiabatic parts of Carnot's cycle.
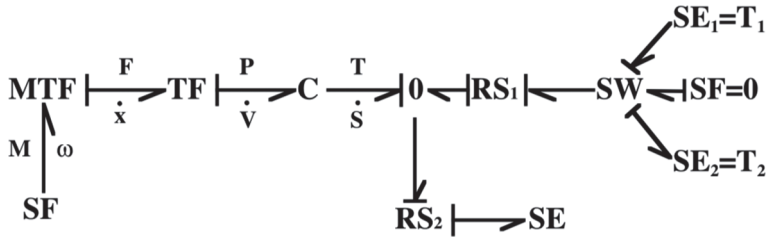
Figure 30: Bond graph of a Carnot heat engine. (i) At the bottom left, we have a mechanical load represented by the source of flow SF. The power conveyed by the mechanical energy bond that goes upwards from SF is characterised by moment M, which is its extensive flow variable, its angular momentum flow in other words, and by angular frequency ω which is the intensive variable, or effort. (ii) The mechanical energy rotating the shaft is transformed into mechanical energy for translatory motion of the piston by the crankshaft represented by a modulated transformer MTF. (iii) The linear mechanical energy obtained, whose extensive variable is displacement x and intensive variable force F (inverse mechanical analogy here), is transformed into hydraulic energy by the piston itself represented by transformer TF. At the output of this transformer, the intensive variable is therefore pressure P and the extensive variable is volume V. (iv) The medium contained in the cylinder is represented by the multiport C since it accumulates both hydraulic energy on the left, whose intensive variable is pressure P and extensive variable is volume V, and on the right thermal energy whose intensive variable is temperature T and extensive variable is entropy S. (v) The part of the machine that allows entropy (but not matter) to pass easily is represented by thermal resistors RS.

5.2 Properties of 'calorique' (entropy)

We come now to the properties of entropy that we can consider as a thermal charge, since it is to thermal energy transmission what electric charge is to electric energy transmission.

It is generated by two principal mechanisms:

- the transfer of thermal power by a finite temperature difference, such a process conserving thermal power.
- friction, whether produced in electric wires or mechanically, for example, such a process also conserving power.

Two additional processes produce entropy:

- chemical friction, in chemical reactions;
- the mixing of chemicals, which must be avoided in good practice in steam engines, and which Carnot calls the mixing of cold air and hot air (or steam), which produces useless calorique, that is to say entropy in modern terms. He also said in a roundabout way: temperature must change only by compression and expansion and not by conduction.

We shall not deal here with the latter two phenomena, that is, chemical phenomena, as they have been dealt with elsewhere [13, p. 74]. We shall note simply that a reversible mixture can be produced by using, in a cylinder, selective pistons which do not produce entropy, and that the combustion part of a heat engine can be represented by bond graphs in which the intensive variable is the chemical potential and the extensive variable the quantity of matter, as mentioned in Table 1.

5.3 Removing entropy

Entropy, this sort of thermal charge, is always necessary for transmitting energy in the form of thermal power, just as an electric charge is necessary for transmitting energy in the form of electric power. After its use, the thermal charge must be eliminated in a cold source, the inverse power necessary at the cold source thus limiting the power output of thermal engines.

When introducing bond graphs, we said that entropy is a simple concept resulting directly from the notion of heat as conceived by Carnot in his equation:

$$\dot{Q} = \dot{S} \cdot T$$

Although the concept of entropy is *a priori* simple, two subtleties regarding it are worth mentioning here:

- Zero entropy is a difficult notion because entropy is not well defined: in general, one considers entropy from a reference value, usually 0° or 273 K. It follows from this that the practice of postulating null entropy at absolute zero temperature must be relativised since entropy can be set in some bodies by very rapid cooling to extremely low temperatures.

- All irreversible processes occurring on earth are negligible due to the earth's ejection back into space of the immense flow of entropy that it receives from solar radiation. It follows that a calculation according to Boltzmann's formula, stipulating an equality of powers, gives a rather low temperature at the surface and a difference between surface and clouds, an essential cause of the greenhouse effect occurring in the atmospheric layer.

5.4 Carnot's efficiency formula

A reversible heat engine is an engine that does not produce new entropy in other words, that conserves entropy. No entropy leaves from the mechanical side and only the entropy that arrives must be eliminated in the cold source, which requires some power:

$$\dot{Q}_2 = T_2 \cdot \dot{S}$$

in such a way that only the difference with the power that arrives is available for conversion, which limits efficiency to

$$\eta_{Carnot} = \frac{(T_1 - T_2) \cdot S}{T_1 \cdot S} = \frac{T_1 - T_2}{T_1}$$

A point to note in passing is that all temperatures must be taken as absolute. If we take the temperatures as excesses of temperature, then we need to add a offset T_{Offset} = 273 K, which gives the following formula:

$$\eta_{Carnot} = \frac{(T_1 - T_2) \cdot S}{(T_1 + T_{offset}) \cdot S} = \frac{(T_1 - T_2)}{(T_1 + T_{offset})}$$

for the Carnot efficiency, expressed with temperatures in degrees centigrade (Celsius).

For example, for a heat source at 200°C and a cold source at 100°C, we can obtain a Carnot efficiency of 21%. These temperatures are typical of simple machines without a condenser in which the steam used is ejected to the environment, as in steam locomotives. This value is an upper limit, obtained in an ideal engine without losses. It shows, as Carnot pointed out, that the high temperature should be made the highest possible and the low temperature the lowest possible to obtain good levels of efficiency.

5.5 Conclusion on the Carnot cycle

Our incursion into the nascent practice of steam engines shows that Carnot's ideas were truly astounding for understanding these thermal engines. It is a theory of thermal engines for generating mechanical power that today still informs the design of modern internal combustion engines. It can be summed up by the following main points:

- Repeated or cyclic operation to obtain a machine with a useful life.
- Discharge of the used steam to the environment.
- The use of inverted sources for discharging entropy.

All theory on heat engines is deduced from the above ideas. As Carnot said: 'avoid increases in temperature by conduction and raise temperature by reducing volume.'

Appendix

2005 Edition of Carnot's *Reflections on the Motive Power of Fire*

In 2005, Dover Publications republished Carnot's work as edited by E. Mendoza [2].

We have reproduced, with the permission of the Publishers, the book's front cover. This shows Sadi Carnot at work with an oil lamp of the time.
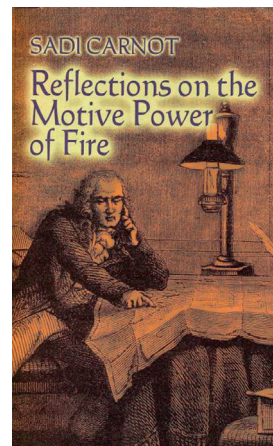
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