Chapter 3

Dam-break wave routing

D. De Wrachien¹, S. Mambretti²
¹State University of Milan, Milan, Italy
²Politecnico di Milano, Milan, Italy

Abstract

Studies of dam-break consider, mainly, situations of clear-water surges. However, under natural conditions a dam-break flow can generate extensive debris or encounter floating debris in the valley downstream of the dam. Debris flows differ from other natural unconfined flows in the nature of both the floating material and the flow itself, which is rapid, transient and may significantly influence surge height and speed.

Phenomenological analysis defines debris flows as mixtures of water and clastic material with high coarse particle content, in which collisions between particles and dispersive stresses are dominant mechanisms in energy dissipation. Therefore, their nature mainly changes according to the sediment concentration and characteristics of the sediment size.

The rheological property of a debris flow depends on a variety of factors, such as suspended solid concentration, cohesive property particle size distribution, particle shape, grain friction and pore pressure. So, modelling these flows requires both a rheological model and constitutive equations for sediment–water mixtures.

To be reliable, a rheological model should possess three major features. It should describe: the dilatancy of the sediment–water mixtures; the soil yield criterion, as proposed by Mohr–Coulomb; and the role of inter-granular fluid.

Achieving a set of debris-flow-constitutive equations is a task which has been given particular attention by the scientific community. To properly tackle this problem, relevant theoretical and experimental studies have been carried out during the second half of the last century.

Research work on theoretical studies has traditionally specialized in different mathematical models. They can be roughly categorized on the basis of three characteristics: the presence of bed evolution equation, the number of phases and the rheological model applied to the flowing mixture.

Because a debris flow essentially constitutes a multi-phase system, any attempt at modelling this phenomenon that assumes, as a simplified hypothesis, homogeneous mass and constant density conceals the interactions between the phases and prevents the possibility of investigating further mechanisms such as the effect of sediment...
separation (grading). Modelling the fluid as a two-phase mixture overcomes most of these limitations and allows for a wider choice of rheological models.

In this context, debris flows resulting from a sudden collapse of a dam are often characterized by the formation of shock waves.

It is commonly accepted that a mathematical description of these phenomena can be accomplished by means of 1D or 2D De Saint Venant (SV) or shallow water (SW) equations. These equations yield discontinuous solutions in the form of shocks or bores, which can be difficult to represent accurately without the use of appropriate shock-capturing schemes.

In recent years, great effort has been devoted to the construction of efficient and accurate procedures to solve these problems. Along this line of work, further efforts need to be made to analyse dam-break waves and show the distribution of different size of the solid phase within the mixture profile in a better way.

With reference to these issues, the chapter aims to provide the state-of-the-art of shock waves and debris-flow rheology, modelling and laboratory investigations, along with a glance to the direction that in-depth studies regarding these phenomena are likely to follow in future.

1 Introduction

Debris flows and surges, such as those triggered by the sudden collapse of dams, are unique unsteady-flow phenomena in which the flow changes rapidly and the properties of the moving fluid mixture of debris and water are very different from those of clear-water floods.

Compared with the open-channel flow of pure water, where the resistance behaviour is mainly attributed to the boundary turbulent shear stress, the resistance behaviour of the debris flow depends on the relative importance of the shear stresses arising from different sources. These sources include:

1. the turbulent shear stress due to the channel boundary roughness;
2. the solid–liquid mixture viscous stress and yield stress;
3. the mixture’s dispersive stress due to sustained frictional contacts;
4. the inelastic collisions of solid particles within the fluid mixture.

Consequently, the characteristics of debris flows may differ significantly between events. For these reasons, clear-water surges and debris flows triggered by dam-break phenomena will be studied on the basis of the most advanced numerical and experimental investigations nowadays available.

2 Clear–water wave routing

In practical applications, flood forecasting involves two steps. First, a flood routing model is used to obtain the flood wave by routing flood events between stream flow gauging stations. This flood wave must then be put into a hydraulic model based on detailed channel geometry to forecast flood levels at key sites. The first
step, flood routing, can be defined as the process of determining progressively the timing and shape of a flood wave at successive points along or throughout a reservoir. In other words, flood routing is a mathematical procedure for predicting the changes in magnitude, speed and shape of a flood wave as a function of time at one or more points along a watercourse, waterway or channel (Fread [1]).

Flood routing may be classified as either hydrologic (lumped) or hydraulic (distributed).

Hydrologic routing involves the balancing of inflow, outflow and volume of storage through the use of continuity equation. A second relationship, the storage–discharge relation, is also required between outflow rate and storage in the system. This approach has been widely used in the past by hydrologists, because of its simplicity in implementation, calibration and use, but it will not be covered in this book because it has been replaced by more physically based models.

2.1 Governing equations

Hydraulic routing is based on the solution of the continuity and momentum equations for unsteady flow in open channel. These two differential equations, known as the SV or SW equations, can be written under the following hypotheses:

1. The flow is 1D. This implies that both depth and velocity vary only in the longitudinal direction of the channel.
2. The flow is assumed to vary gradually along the longitudinal direction so that the hydrostatic pressure prevails and vertical accelerations can be neglected.
3. The longitudinal axis of the channel can be assumed as a straight line.
4. The bottom slope of the channel is small and the channel bed is fixed.
5. Resistance coefficients for steady uniform turbulent flow are applicable.
6. The fluid is incompressible and therefore of constant density.

These hypotheses do not hold in the case of dam-break phenomena, where the flow is multi-dimensional and the vertical accelerations cannot be neglected. To deal with these problems different approaches have been proposed, such as those based on the impulsive wave equations (Favre [2]; Arredi [3]). Anyway, the SV equations have proved to be effective even in cases when not all of the above-mentioned hypotheses hold, but some treatments are required to prevent instability in the integration phase.

The mass-conservation equation can be written as:

\[ \frac{\partial Q}{\partial t} + \frac{\partial A}{\partial x} = 0 \]  \hspace{1cm} (1)

and the momentum equation as:

\[ \frac{\partial Q}{\partial t} + \frac{\partial (Q^2 / A)}{\partial x} + gA \frac{\partial h}{\partial x} + gA(S_f - S_o) = 0 \]  \hspace{1cm} (2)
where \( A(x, t) \) is the wetted cross-sectional area; \( Q(x, t) \) the flow rate; \( x \) the spatial co-ordinate; \( t \) the temporal co-ordinate; \( g \) the acceleration due to gravity; \( S_0 \) the bed slope; \( S_f \) the bed resistance term or friction slope, that can be modelled using different rheological laws (Fread [4]) and \( h \) depth of flow.

In the past, owing to the mathematical complexity of the SV equations, simplifications were necessary to obtain feasible solutions for important characteristics of a flood wave and its movement. This resulted in the development of many simplified flow-routing methods such as the so-called diffusive wave, by neglecting acceleration terms, which can be written as:

\[
gA \frac{\partial h}{\partial x} + gA(S_f - S_0) = 0
\]

(3)

For negligible pressure terms, the equation may be further simplified to yield:

\[
gA(S_f - S_0) = 0
\]

(4)

known as kinematic wave.

While the diffusive wave model neglects the local and convective acceleration terms but incorporates the pressure term, the kinematic wave’s assumption is that the inertial and pressure effects are unimportant and that the gravity force of fluid is approximately balanced by the resistive force of bed friction. This means that flood waves can be observed as a uniform rise and fall in water surface over a relatively long period of time, and therefore represent the characteristic changes in discharge, velocity and water-surface elevation with time at any location on an overland flow plane or along the stream channel. Kinematic waves can be classified as uniform and unsteady flow.

More powerful expressions of the SV equations are their conservation form with additional terms to account for different characteristics such as lateral flows (Stoker [5]), off-channel storage areas (Dronkers [6]) and sinuosity effects (De Long [7]). The extended SV equations consist of the mass conservation equation:

\[
\frac{\partial Q}{\partial x} + \frac{\partial x_c(A + A_0)}{\partial t} - q = 0
\]

(5)

and the momentum equation:

\[
\frac{\partial (s_mQ)}{\partial t} + \frac{\partial (\beta Q^2 / A)}{\partial x} + gA \left( \frac{\partial h}{\partial x} + S_f - S_0 + S_{\infty} \right) + L + W_f B = 0
\]

(6)

where, \( q \) is the lateral inflow or outflow per linear distance along the watercourse (inflow is positive and outflow is negative); \( A_0 \) is the inactive cross-sectional area (off-channel storage), \( s_c \) and \( s_m \) are depth-weighted sinuosity coefficients that correct for the departure of a sinuous in-bank channel from the \( x \) axis of the flood-plain; \( \beta \) is the momentum coefficient for non-uniform velocity distribution within
the cross-section (often let equal to 1.0); \( S_{ec} \) is the expansion–contraction (large eddy loss) slope; \( L \) is the momentum effect of lateral flows; \( W_fB \) is the wind effect on the water surface and \( B \) is the wetted top width of the active flow portion of the cross-section.

The different terms may be evaluated with a number of formulas.

Friction slope \( S_f \) is usually evaluated by means of the Manning equation:

\[
S_f = \frac{n^2 |Q|Q}{A^2 R^{4/3}}
\]

where \( n \) is the Manning roughness coefficient and \( R \) the hydraulic radius.

The term \( S_{ec} \) can be computed as follows:

\[
S_{ec} = \frac{K_{ec} \Delta (Q/A)^2}{2g \Delta x}
\]

where \( K_{ec} \) is the expansion and contraction coefficient (negative for expansion, positive for contraction) which varies from \(-1.0\) to \(0.4\) for an abrupt change in section. It is obviously equal to \(0.0\) when the watercourse is straight (Skogerboe et al. [8]; Fouladi Nashta and Garde [9]). The term \( \Delta (Q/A)^2 \) represents the difference in velocity at two adjacent cross-sections separated by a distance \( \Delta x \).

The lateral flow momentum \( L \) can be evaluated as:

\[
L = -qv_x
\]

where \( v_x \) is the velocity of lateral inflow in the \( x \) direction of the main channel flow.

The wind effect \( W_f \), most often neglected, can be evaluated as:

\[
W_f = |V_w|V_w^2 c_w
\]

where \( V_w \) is the velocity of the wind relative to the water along the direction of the main channel, and \( c_w \) the wind friction coefficient.

### 2.2 Initial and boundary conditions

#### 2.2.1 Initial conditions

Values of water-surface elevation \( h \) and discharge \( Q \) for each cross-section must be specified initially at time \( t = 0 \) to obtain solutions of the SV equations. Initial conditions may be obtained from any of the following:

1. observations at gauging stations, or interpolated values between gauging stations for intermediate cross-sections in large rivers;
2. computed values from a previous unsteady-flow solution;
3. computed values from a steady-flow solution, which is the most commonly used method.

In the case of dam break, the initial conditions are sketched in fig. 1. The problem is due to the presence of the discontinuity in the section of the dam when it is completely and instantaneously removed at time $t = 0\,^+$.

The first solution was found by Ritter [10], who integrated the equations in the case of sudden collapse of a dam on a horizontal rectangular channel with infinite extension and no roughness. From the Ritter’s solution, the following value for the water depth in the section where the dam is positioned can be obtained:

$$h(t = 0^+) = \frac{4}{9} h_0$$

(11)

where $h_0$ is the initial water depth at the dam site, which can be used as internal condition for the solution of the numerical problem.

Most authors prefer not to have an internal condition and to adopt more elegant self-contained solutions (Aureli et al. [11]).

To this end, a number of shock-capturing schemes have been developed, as will be described in another paragraph. Moreover, to avoid numerical (non-physical) instabilities artificial dissipation terms are sometimes introduced in the equations.

Most codes solve the set of eqns. (1) and (2), neglecting all terms that have been introduced in eqn. (6). Nevertheless, a number of authors (Miller and Chaudry [12]) focused on the effects of curves in channels and proposed numerical models suitable to simulate converging channels, presence of a hydraulic jump, flow near a spur-dike and other cases (Dammuller et al. [13]; Molls and Chaudry [14]).

One key assumption of SV equations is that streamline curvature effects are small, so that the pressure distribution is hydrostatic. This feature, however, was taken into account by Basco [15] who extended the SV equations to the Boussinesq system.

![Figure 1: Initial and boundary conditions for dam-break wave.](image-url)
Initial stages of the waves have similar problems and should be simulated taking in due account the turbulence terms (Shigematsu et al. [16]). Other problems are mainly related to the scheme adopted for the numerical integration and to the sediment transport.

2.2.2 Boundary conditions

Boundary conditions can be external and internal.

Values for the unknowns at external boundaries (the upstream and downstream extremities of the routing reach) of the watercourse must be specified to obtain solutions to the SV equations. In fact, in most unsteady-flow applications the unsteady disturbance is introduced at one or both the external boundaries.

Either a specified discharge or water-surface elevation time series (hydrograph) can be used as the upstream boundary condition. The hydrograph should not be affected by downstream flow conditions. Boundary conditions have to be assigned upstream and downstream when the flow is subcritical, and both of them have to be assigned upstream when the flow is supercritical.

Specified water-surface elevation time series, or a tabular relation between discharge and water-surface elevation (single-valued rating curve) can be used as the downstream boundary condition. Another downstream boundary condition can be a loop-rating curve based on the Manning equation.

This condition allows the unsteady wave to pass the downstream boundary with minimal disturbance by the boundary itself, which is desirable when the routing is terminated at an arbitrary location along the watercourse and not at a location of actual flow control such as a dam. The downstream boundary condition can also be a critical flow section such as the entrance to a waterfall or a steep reach.

When the downstream boundary is a stage–discharge relation (rating curve), the flow at the boundary should not be otherwise affected by flow conditions farther downstream. Although there are often some minor effects due to the presence of cross-sectional irregularities downstream of the chosen boundary location, these usually can be neglected unless the irregularity is so pronounced as to cause significant backwater or drawdown effects. When these situations are unavoidable, the routing reach should be extended downstream to the dam in the case of the reservoir or to a location downstream of where the major tributary enters. Sometimes the routing reach may be shortened by moving the downstream boundary to a location farther upstream where backwater effects are negligible.

Along a watercourse, there are locations such as a dam, bridge or waterfall (short rapids) where the flow is rapidly varied rather than gradually varied in space. In these cases, empirical water elevation–discharge relations, as weir flow, are utilized for simulating rapidly varying flow.

Some authors (Miller [17]) considered the role of spatially varying boundary conditions and flow patterns as determinants of geomorphic effectiveness. Slope-area measurements are typically made along straight reaches of uniform or nearly uniform width, and therefore cannot account for significant
longitudinal variations in boundary conditions that may affect flood hydraulics elsewhere along the valley. Hydraulic reconstructions based on the standard step method can account for longitudinal variability, but cannot predict lateral variations within a cross-section or the spatial pattern of flow vectors. To explore these features of the flow field it is necessary to use two-dimensional (2D) flow models.

2.3 Cross-sections

Much of the characteristics of a specific flow-routing application is featured in particular cross-sections located at selected points along the watercourse. These sections may be classified as either active or inactive (dead) sections (fig. 2).

The portion of the channel cross-section in which flow occurs is called active. Cross-sections may be of regular or irregular geometrical shape. Usually, a cross-section is described by tabular values of channel top width and water-surface elevation which constitute a piecewise linear relationship. Areas or widths associated with a particular water-surface elevation are linearly interpolated from the tabular values. Cross-sections at gauging station locations are generally used as computational points. Such points are also specified at locations along the river where significant cross-sectional or flow-resistance changes occur or at locations where major tributaries enter.

There can be portions of a cross-section where the flow velocity is negligible relative to the velocity in the active portion. The inactive portion is called off-channel (dead) storage: it is represented by the term $A_0$ in eqn. (5). Off-channel storage areas can be used to effectively account for adjacent embankments, ravines or tributaries which connect at some elevation with the flow channel but do not convey flow in the main direction; they serve only to store some of the passing flow. Sometimes,
off-channel storage can be used to simulate a heavily wooded floodplain that primarily stores some of the floodwaters while conveying a very minimal portion of the flow. Generally, dead storage cross-sectional properties are described by width (dead storage) vs. elevation tables.

2.4 2D flow models

2D unsteady-flow models, which account for momentum conservation across the waterway perpendicular to the longitudinal direction, can be categorized as complete (all terms in the momentum equation appear) or simplified (inertial or acceleration terms in each momentum equation are neglected). Complete or simplified 2D flow-routing models have been used for unsteady flows in complex floodplains (Cunge [18]). Generally, the additional accuracy gained does not justify their use to predict water-surface elevations and average flows in usual unsteady-flow applications dealing with floodplains.

2.5 Hybrid models

Until recently, hydraulic models were not reckoned a practical alternative for flood routing because they were considered not economically viable to obtain cross-section data over the reaches involved in flood routing. Recent investigations (Hicks [19]) have revealed that hydraulic routing can be successfully used to determine discharge hydrographs in reaches where little channel geometry data are available, by approximating the model reach by a rectangular channel.

It was found that this ‘limited geometry’ modelling approach – based on 1D SV equations – could accurately determine discharge hydrographs, making it an effective and suitable alternative to hydrologic flood routing. It was also found that this hybrid model offers the advantage of operationally combining the flood routing and the determination of flood levels (Blackburn and Hicks [20]). In addition, the use of a hydraulic model opens up the potential for modelling more dynamic flood events such as ice jam release surges, which cannot be handled by traditional hydrological modelling approaches.

3 Debris-flow routing

Studies of dam-break flows consider, mainly, situations of clear-water surges. However, under natural conditions, a dam-break flow can generate extensive debris or encounter floating debris in the valley downstream of the dam. This chapter investigates the effects of floating debris on dam-break surges. Debris flows differ from other natural unconfined flows (Johnson and Rodine [21]; Meunier [22]) in two main respects:

- the nature of the flowing material, constituted by a mixture of water, clay and granular materials;
the nature of the flow itself, which is rapid, is transient and includes a steep front mainly constituted of boulder.

The rheological property of a debris flow depends on a variety of factors, such as suspended solid concentration, cohesive property, particle size distribution, particle shape, grain friction and pore pressure.

Various researchers have developed models of mud and debris-flow rheology. These models can be classified as: Newtonian models (Johnson [23]; Hunt [24]), linear and non-linear viscoplastic models (Johnson [23]; O’Brien and Julien [25]; Liu and Mei [26]; Huang and Garcia [27], [28]), dilatant fluid models (Bagnold [29]; Takahashi [30]; Mainali and Rajaratnam [31]), dispersive or turbulent stress models (Arai and Takahashi [32]; O’Brien and Julien [25]; Hunt [24]), biviscous modified Bingham model (Dent and Lang [33]) and frictional models (Norem et al. [34]; Iverson [35]). Among these, linear (Bingham) or non-linear (Herschel–Bulkey) viscoplastic models are most widely used to describe the rheology of laminar debris/mud flows (Liu and Mei [26]; Jang [36]; Huang and Garcia [28]; Jiang and LeBlond [37]).

Because a debris flow is, essentially, a multi-phase system, any attempt at modelling this phenomenon that assumes, as a simplified hypothesis, homogeneous mass and constant density, conceals the interactions between the phases and prevents the possibility of investigating further mechanisms such as the effect of sediment separation (grading).

Modelling the fluid as a two-phase mixture overcomes most of the limitations mentioned above and allows for a wider choice of rheological models such as Bagnold’s dilatant fluid hypothesis (Takahashi and Nakagawa [38]; Shieh et al. [39]), Chézy type equation with constant value of the friction coefficient (Hirano et al. [40]; Armanini and Fraccarollo [41]), models with cohesive yield stress (Honda and Egashira [42]) and the generalized viscoplastic fluid or Chen’s model (Chen and Ling [43]).

3.1 Flow profile

Debris-flow dynamics can be divided into three distinct time phases (Lorenzini and Mazza [44]). The first is a brief initial phase in which the density of the mixture differs very slightly from the water, the solid particles do not interact and rapid ‘saturation’ takes place. This is followed by a longer intermediate phase, which corresponds to the developed motion of the debris flow, in which density is particularly high and interactions become predominant. In the third and final phase, the process ends, following deceleration, thus resulting in the deposition of the debris and the formation of a rigid skeleton, corresponding to the consolidated solidification, through which the water filters out.

A typical debris-flow wave can be spatially divided into three main regions, as shown in fig. 3.

The anterior section of the flow (also known as the snout) forms the head; it has a forward protuberation composed mainly of rock fragments due to the
reverse grading phenomenon (Takahashi [45]). The second region (commonly
known as the body), characterized by an increasingly tapered flow, is composed
of a turbulent mixture of water, fine particles and sediment transported along to
course, whereas the last region, which is slender and much more diluted than the
previous two, is characterized by a high mud concentration and continues to flow
long after the wave has passed.

3.2 Rheological models

Modelling debris flow requires a rheological model (or constitutive equations) for
sediment–water mixtures. Various models relating stress, strain and time, among
other variables, have been developed.

Advanced theoretical models, despite their general validity and applicability, are
too complicated to be useful in practice, whereas the use of simpler semi-empirical
models is limited to a narrow range of applications for lack of adaptability. There-
fore, the selection of a rheological model has constituted one of the major issues in
debris-flow modelling.

A general model which can realistically describe the rheological properties of
debris flow should possess three main features. The model should:

- describe the dilatancy of sediment–water mixtures;
- take into account the soil yield criterion, as proposed by Mohr–Coulomb;
- assess the role of inter-granular or interstitial fluid.

On the whole, a rheological model of hyper-concentrated flow should involve
the interactions of several physical processes.

3.2.1 Non-Newtonian fluid models

The non-Newtonian behaviour of the fluid matrix is ruled, in part, by the cohe-
sion between fine sediment particles. This cohesion contributes to the yield
stress, which must be exceeded by an applied stress to trigger fluid motion. For
large rates of fluid matrix shear (as might occur on steep alluvial fans), turbulent
stresses may be generated. In these cases, an additional shear stress component

Figure 3: The various regions within a debris flow wave.
arises in turbulent flow from the collision of solid particles under large rates of deformation.

In general terms, the total shear stress $\tau$ in debris and hyper-concentrated flows can be assessed from the summation of five shear stress components (O’Brien et al. [46]):

$$\tau = \tau_c + \tau_{mc} + \tau_v + \tau_t + \tau_d$$

(12)

where $\tau_c$ is cohesive yield stress; $\tau_{mc}$ Mohr–Coulomb shear stress; $\tau_v$ viscous shear stress; $\tau_t$ turbulent shear stress and $\tau_d$ dispersive shear stress.

When written in terms of shear rates $du/dy$, the total shear stress can be expressed according to the following quadratic model (O’Brien and Julien [47]):

$$\tau = \tau_y + \mu \left( \frac{du}{dy} \right) + c \left( \frac{du}{dy} \right)^2$$

(13)

where:

$$\tau_y = \tau_c + \tau_{mc}$$

(14)

and:

$$c = \rho_m l^2 + f(\rho_m, c_v)d_s^2$$

(15)

with $\mu$: dynamic viscosity; $u$: local velocity; $y$: co-ordinate axis; $c$: inertial shear stress coefficient; $\rho_m$: mass density of the mixture; $l$: Prandtl mixing length; $d_s$: sediment size; $c_v$: volumetric sediment concentration.

The Mohr–Coulomb stress can be assessed as:

$$\tau_{mc} = p \tan \Phi$$

(16)

$p$ being the inter-granular pressure and $\Phi$ the angle of repose of the material.

Bagnold [29] defined the function $f(\rho_m, c_v)$ as:

$$f(\rho_m, c_v) = a_i \rho_m \left( \frac{c}{c_v} \right)^{1/3} - 1$$

(17)

where $a_i$ is an empirical coefficient equal to 0.01; $c^*$ the maximum static volume concentration for the sediment particles.

The first two terms in eqn. (13) are referred to as the Bingham shear stresses, due to the fact that they represent the internal resistance stresses of a Bingham fluid. The sum of the yield stress and viscous stress defines the shear stress of a cohesive, hyper-concentrated sediment fluid in a viscous flow regime, while the last term represents the sum of the dispersive and turbulent shear stresses, which depend on the square of the vertical velocity gradient (Julien and O’Brien [48]).
In view of theoretical soundness behind the development of the non-Newtonian rheological models, Bailard [49], Jenkins and Cowin [50], Bailard and Imman [51] and Hanes [52] have questioned the validity of Bagnold’s empirical relations. Moreover, when the flow changes from a quasi-static state to a fully dynamic or macro-viscous state, the relation (16) between the total (grain) shear stress and the inter-granular pressure does not hold. To overcome these problems, Chen [53] developed a generalized viscoplastic fluid (GVF) model, based on two major rheological properties (i.e. the normal stress effect and soil yield criterion) for general use in debris-flow modelling.

3.2.2 GVF model
The analysis that Chen [54] conducted on the various flow regimes of a granular mixture identified three regimes: a quasi-static one, which is a condition of incipient movement with plastic behaviour; a macro-viscous one at low shear rates, in which viscosity determines the mixture behaviour and finally a granular inertial state, typical of rapid flowing granular mixtures, dominated by inter-granular interactions. Chen developed his model starting from the assumption that a general solution should be applicable through all three regimes.

Chen observed that an applicable model, able to realistically describe debris flows, should consider both an independent term and one dependent on the shear rate in a perpendicular direction to the main flow (fig. 4) as well as the main rheological properties: the effect of normal stress and the Mohr–Coulomb yield criterion.

By introducing the hypothesis according to which the viscous effect should not coexist with turbulent and dispersive effects, Chen developed the following set of theoretical relations:

$$\tau_{xz} (= \tau_{zx}) = c \cos \phi + p \sin \phi + \mu \left( \frac{du}{dz} \right)$$

(18)

Figure 4: Definition diagram for a uniform and steady debris flow under a bidimensional state of stress in Chen’s GVF model.
with \( x, z \) the co-ordinates in the longitudinal and normal (positive, upward) direction of flow; \( \tau_{zx}, \tau_{zz} \) the total shear and normal stresses, respectively, at a point \((x,z)\) as shown in fig. 4; \( c \) the cohesion; \( \mu_1 \) and \( \mu_2 \) the consistency and cross-consistency indices, respectively and \( \eta \) the flow-behaviour index.

The summation of the first two terms on the right-hand side of eqn. (18) is normally referred to as the yield stress:

\[
S = c \cos \phi + p \sin \phi
\]  
(20)

and represents an extended form of the Mohr–Coulomb yield criterion. The value of \( \eta \) may vary from 1 to 2 (or possibly higher) as the flow changes from a macro-viscous to a grain-inertial state, covering the entire spectrum of Newtonian models.

If \( c \) and \( \phi \) do not vary with \( z \), eqns. (18) and (19) can be expressed in the following differential form (Chen [53], [55]):

\[
\frac{d\tau_{zx}}{dz} = \left( \frac{dp}{dz} \right) \sin \phi + \frac{d}{dz} \left[ \mu_1 \left( \frac{du}{dz} \right)^\eta \right]
\]  
(21)

\[
\frac{d\tau_{zz}}{dz} = - \frac{dp}{dz} + \frac{d}{dz} \left[ \mu_2 \left( \frac{du}{dz} \right)^\eta \right]
\]  
(22)

Further generalization of eqns. (18) and (19) or eqns. (21) and (22) have been proposed by Tsubaki et al. [56], by extending the Bagnold's model of a binary collision of grains to a more general case of the multiple collision of grains.

### 3.2.3 Multi-layer models

All the models previously reviewed feature monotonic velocity profile that, generally, do not agree with experimental and field data. In many experimental tests (Takahashi [57]; Shen and Ackermann [58]) ‘S’ reversed shaped trends have been observed, where the maximum shear rate is not achieved near the bed, but rather between the bed and the free surface. The main discrepancy is derived from the assumption of a debris flow as a uniform mixture. In fact, the solid concentration distribution is usually non-uniform due to the action of gravity, so that the lower layer could, consequently, have a higher concentration than the upper layer. This is specifically the case of ‘immature’ debris flow, while, in mature flows, due to the inverse grading effect, larger debris are on the surface (and front), and finer ones towards the bottom.

Higher concentration means higher cohesion, friction and viscosity in the flow. For this reason, the velocity profile for non-uniform concentration is altered. To clear this hurdle, Wan and Wang [59] proposed a multi-layer model, known as the laminated layer model, that features a stratified debris flow into three regions.
from the bed to the surface: a bed layer, in which an additional shear stress is
dominant in momentum exchange; an inertial layer, where the dispersive stress of
the grains is dominant and an upper viscoplastic layer, which can be represented
by the Bingham’s model.

Later on, Takahashi [60] proposed a so-called unified model of inertial debris
flow, by altering the constitutive equations from the theory of granular mixtures
and suspended-load transport. In this model, the author hypothesized the flow
as a two-layer system: a lower layer dominated by collisions, and an upper one
composed of the turbulent suspension (fig. 5).

The relative extension of the two layers depends on the concentration and
diameter of the particles: the range goes from the stony debris flows in which only
collision layer exists, to muddy debris flows in which the entire flow is composed
of a turbulent layer.

As previously described, the rheological behaviour of a mixture depends on
the hydrodynamic characteristics of the flow and the properties of the sediment,
which can vary from one flow to another.

The one-layer models are unable to adequately feature the entire thickness
of the flow and, therefore, it has recently become common to use multi-layers
models that combine two or more constitutive relationships to analyse adequately
these phenomena. The coefficients of the rheological models have wide ranges of
variation and, therefore, in evaluating them considerable errors are committed.
On the other hand, some empirical equations of velocity are necessary in any
debris-flow disaster-forecasting measure, although the hydraulics of debris flow
is not theoretically comparable to that of a traditional water flow, due to the high
sediment condition and the interaction between the solid particles. Being more
diluted than the head, the main body of the flow could be described by a tradi-
tional law of resistance such as that of Manning or Chézy, and simulated by a set
of mass and momentum equations averaged on the flow depth.

3.3 Governing equations

Shock waves with floating debris are complex two-phase systems, and so model-
ing the flow as a two-phase mixture is the best way to predict these phenomena.
The change in debris-flow density can be modelled through mass and momentum balance of both phases (solid and liquid) and interactions between the two phases could be assessed by means of appropriate additional terms (Wallis [61]; Wang and Hutter [62]), while the erosion/deposition rate can be controlled by the excess of the local instantaneous concentration over the equilibrium concentration (Egashira and Ashida [63]; Honda and Egashira [42]). Many other models, the so-called quasi-two-phase models, use different forms of motion equations – SW or SV equations – together with erosion/deposition and mass conservation equations for the solid phase and take into account mixtures of varying concentrations.

Recently, Mambretti et al. [64], [65] and De Wrachien and Mambretti [66] proposed a general model suitable to analyse both non-stratified (mature) and stratified (immature) flows, i.e. when the solid/liquid mixture is present in the lower layer, while only water is present in the upper one (fig. 6). The model, based on 1D SV equations, takes into account mass and momentum conservation balance for each phase and layer and energy exchange between layers. The level of maturity of the flow is assessed by an empirical, yet experimental based, criterion. The model could also be improved to predict and assess the propagation and stoppage processes of debris and hyper-concentrated flows, triggered by extreme hydrological events, once validated on the basis of laboratory or field data.

### 3.3.1 One-phase model

The 1D approach for unsteady debris flow triggered by dam-break is governed by the SV equations. This set of partial differential equations describes a system of hyperbolic conservation laws with source term (S) and can be written in compact vector form as:

\[
\frac{\partial \mathbf{V}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} = \mathbf{S}
\]  

(23)
where:

\[
V = \begin{bmatrix} A \\ Q \end{bmatrix}, \quad F = \begin{pmatrix} Q \\ (Q^2/A) + gl_1 \end{pmatrix}, \quad S = \begin{pmatrix} 0 \\ gA(i - S_i) + gl_2 \end{pmatrix}
\]

where \( A(x, t) \) is the wetted cross-sectional area; \( Q(x, t) \) the flow rate; \( x \) the spatial co-ordinate; \( t \) the temporal co-ordinate; \( g \) the acceleration due to gravity; \( i \) the bed slope; \( S_i \) the bed resistance term or friction slope, that can be modelled using different rheological laws (Rodriguez et al. [67]).

The pressure force integrals \( I_1 \) and \( I_2 \) are calculated in accordance with the geometrical properties of the channel. \( I_1 \) represents a hydrostatic pressure form term and \( I_2 \) represents the pressure forces due to the longitudinal width variation, expressed as:

\[
I_1 = \int_0^h (H - \eta) \sigma(x, \eta) d\eta, \quad I_2 = \int_0^h (H - \eta) \frac{\partial \sigma}{\partial x} d\eta
\]

(24)

where \( H \) is the water depth; \( \eta \) the integration variable indicating distance from the channel bottom; \( \sigma(x, \eta) \) the channel width at distance \( \eta \) from the channel bed, expressed as:

\[
\sigma(x, \eta) = \frac{\partial A(x, \eta)}{\partial \eta}
\]

(25)

To take into account erosion/deposition processes along the debris flow propagation path, which are directly related to both the variation of the mixture density and the temporal evolution of the channel bed, a mass-conservation equation for the solid phase and an erosion/deposition model have been introduced in the SV approach. Defining the sediment discharge as:

\[
q(x, t) = EB
\]

(26)

where \( E \) is the erosion/deposition rate; \( B \) the wetted bed width, the modified vector form of the SV equations can be expressed as follows:

\[
\frac{\partial V}{\partial t} + \frac{\partial F}{\partial x} = S
\]

(27)

where:

\[
V = \begin{bmatrix} A \\ Q \end{bmatrix}, \quad F = \begin{pmatrix} Q \\ (Q^2/A) + gl_1 \end{pmatrix}, \quad S = \begin{pmatrix} q(x, t) \\ gA(i - S_i) + gl_2 \end{pmatrix}
\]

www.witpress.com, ISSN 1755-8336 (on-line)
where $c_s$ is the volumetric solid concentration in the mixture and $c^*$ the bed volumetric solid concentration.

### 3.3.2 Two-phase mathematical model

#### 3.3.2.1 Non-stratified (mature) flows

The model includes two mass and momentum balance equations for both the liquid and solid phases, respectively. The interaction between phases is simulated according to Wan and Wang hypothesis [59]. The system is completed with equations to estimate erosion/deposition rate derived from the Egashira and Ashida [63] relationship and by the assumption of the Mohr–Coulomb failure criterion for non-cohesive materials.

*Mass and momentum equations for water* can be expressed as:

\[
\frac{\partial Q_1(x, t)}{\partial x} + \frac{\partial (c_l A(x, t))}{\partial t} = 0
\]

\[
\frac{\partial Q_1}{\partial t} + \frac{\partial}{\partial x} \left( \beta \frac{Q_1^2}{c_l A} \right) = g c_l A \left( i - j \frac{\partial H}{\partial x} \right) - F
\]

where $Q_1(x, t)$ is the flow discharge; $c_l$ the volumetric concentration of water in the mixture; $\beta$ the momentum correction coefficient (assumed $\beta = 1$); $J$ the slope of the energy line according to Chézy’s formula; $i$ the bed slope and $F$ the friction force between the two phases.

According to Wan and Wang [59], the interaction of the phases at single granule level $f$ is given by:

\[
f = c_D \frac{\pi d_{50}^2}{4} \frac{\rho_l (u_l - u_s)}{2} |u_l - u_s|
\]

where $c_D$ is the drag coefficient; $u_l$ the velocity of water; $u_s$ the velocity of the solid phase; $d_{50}$ the mean diameter of the coarse particle and $\rho_l$ the liquid density.

Assuming grains of spherical shape and defining the control volume of the mixture as:

\[
V_c = BH \cos \vartheta \, dx \approx BHdx
\]

where $\vartheta$ is the channel slope angle, which holds for low channel slopes, the whole friction force $F$ between the two phases for the control volume can be written as:

\[
F = \frac{3}{4} c_D \rho_l (u_l - u_s) |u_l - u_s| \frac{c_s}{d_{50}} HBdx
\]
Mass and momentum conservation equations for the solid phase of the mixture can be expressed as:

\[
\frac{\partial(c_s A)}{\partial t} + \frac{\partial Q_s}{\partial x} = E \ast B
\]

(33)

\[
\frac{\partial Q_s}{\partial t} + \frac{\partial}{\partial x}\left(\beta \frac{Q_s^2}{c_s A}\right) = -g \frac{\rho_s - \rho_l}{\rho_s} c_s (1 + \Gamma^2) \frac{\partial H}{\partial x} A + F
\]

+ \frac{g \rho_s - \rho_l}{\rho_s} c_s (\Gamma^2 - 1) g \delta A + g \frac{\rho_s - \rho_l}{\rho_s} c_s A\]

(34)

where \(Q_s(x, t)\) is the discharge of the solid rate; \(\rho_s\) the solid phase density.

According to Ghilardi et al. [68] and Egashira and Ashida [63], the bed volumetric solid concentration \(c^*\) is assumed to be constant and the erosion velocity rate \(E\) a function of the mixture velocity \(u\):

\[
E = u k_E \tan(\varphi_f - \varphi_e)
\]

(35)

with \(k_E\): coefficient equal to 0.1 according to experimental data (Egashira and Ashida [63]; Gregoretti [69], [70]; Ghilardi et al. [68]).

Positive or negative values of \(E\) correspond to granular material erosion or deposition, respectively.

\(\varphi_f\) and \(\varphi_e\) represent the energy line and the bed equilibrium angles, respectively, expressed as (Brufau et al. [71]):

\[
\varphi_f = \arctan \left( \frac{J}{\cos \varphi} \right)
\]

(36)

\[
\varphi_e = \arctan \left( \frac{c_s (\rho_s - \rho)}{c_s (\rho_s - \rho) + \rho \phi} \right)
\]

(37)

where the debris-flow density is defined as:

\[
\rho = (\rho_s - \rho_l) c_s + \rho_l
\]

(38)

and \(\phi\) is the static internal friction angle. \(u\) is defined as follows:

\[
u = c_s u_s + c_l u_l
\]

(39)

For \(J\) the Takahashi [45] equation can be assumed, according to the dilatant fluid hypothesis developed by Bagnold [29]:

\[
J = S_l = \frac{u^2}{(2/5 d_{s0})(H/\lambda)^2 (1/a_s \sin \delta)[c_s + (1 - c_s)(\rho_l/\rho_s)]} g R
\]

(40)
where $S_i$ is the friction term and $R$ the hydraulic radius given by:

$$R = \frac{A}{P}$$  \hspace{1cm} (41)

where $P$ is the wetted perimeter.

The quantity $\lambda$ (linear concentration) depends on the granulometry of the solids in the form:

$$\lambda = \frac{c_s^{1/3}}{c_m^{1/3} - c_s^{1/3}}$$  \hspace{1cm} (42)

where $c_m$ is the maximum packing volume fraction (for perfect spheres $c_m = 0.74$); $a_b$ the empirical constant.

For high values of sediment concentration, the resistance is mainly caused by dispersive stress and the roughness of the bed does not influence the resistance (Scotton and Armanini [72]). For low values of the same characteristic the energy dissipation is mainly due to turbulence in the interstitial fluid and the influence of the wall roughness becomes important. In such a case, Takahashi [45] suggests to use the Manning’s equation or similar resistance laws.

### 3.3.2.2 Stratified (immature) flows

As shown in fig. 6, debris flows are categorized as stratified or immature whenever the solid/liquid mixture is present in the lower layer, while only water is present in the upper one.

Assuming $h_{mx}$ and $h_{cw}$ as the depths of the mixture and of the clear water respectively, the total depth of the debris flow $h_{df}$ is equal to:

$$h_{df} = h_{mx} + h_{cw}$$  \hspace{1cm} (43)

while the maturity degree $d_m$ is assessed as the ratio:

$$d_m = \frac{h_{mx}}{h_{df}}$$  \hspace{1cm} (44)

Larcan et al. [73] has suggested – on the basis of laboratory experiments – to distinguish mature and immature debris flow by means of a criterion based on mixture velocity and concentration (fig. 7).
The figure underlines the effectiveness of the above-mentioned criterion and depicts a boundary between mature and immature debris flow. The boundary $C_s$ boundary can be expressed by:

$$C_s = \begin{cases} \frac{0.5 - 0.08 \cdot Fr}{Fr} & \text{when} \quad Fr < 5 \\ 0.1 & \text{when} \quad Fr \geq 5 \end{cases}$$

(45)

where $Fr$ is the Froude number, while the maturity degree $d_m$ can be assessed as:

$$d_m = \frac{C_s \text{ effective}}{C_s \text{ boundary}}$$

(46)

The experimental tests showed that in the first phase the flow is stratified; then, usually, it becomes mature, because the velocities and the concentrations are quite high. Finally, the tail of the wave is characterized by low velocities, due to the fact that the solid phase tends to deposit, and thus the flow becomes again stratified.

**Mass and momentum equations for clear water in the higher layer (cw)** can be expressed in conservative form as:

$$\frac{\partial Q_{cw} (x,t)}{\partial x} + \frac{\partial \bar{A}_{cw} (x,t)}{\partial t} = 0$$

(47)

$$\frac{\partial Q_{cw}}{\partial H} + \frac{\partial}{\partial x} \left( \beta \frac{Q_{cw}^2}{A_{cw}} \right) = g A_{cw} \left( i - \bar{J}_{cw} - \frac{\partial \bar{H}_{cw}}{\partial x} - J_{two \ layers} \right)$$

(48)
The resistance term \( J_{cw} \) can be assessed on the basis of bank shear stress, while the slope of the energy line, \( J_{\text{two layers}} \), due to the lower layer, according to Chézy’s formula, is expressed as:

\[
J_{\text{two layers}} = \frac{n^2 (u_{cw} - u_{mx})^2}{R^{1/3}}
\]

\( n \) being the Manning’s number and \( u_{mx} \) the velocity of the lower layer. The drag force \( T_{\text{two layers}} \) between the higher layer and the lower one, can be expressed as:

\[
T_{\text{two layers}} = g A_{cw} J_{\text{two layers}}
\]

In the same ways as eqns. (28) and (29), mass and momentum equations for the liquid phase in the lower layer (mx) can be expressed as:

\[
\frac{\partial Q_{l,mx}(x,t)}{\partial x} + \frac{\partial (c_{l,mx} A_{mx}(x,t))}{\partial t} = 0
\]

\[
\frac{\partial Q_{l,mx}}{\partial t} + \frac{\partial}{\partial x} \left( \beta \frac{Q_{l,mx}^2}{c_{l,mx} A_{mx}} \right) = g c_{l,mx} A_{mx} \left( i - J_{mx} \frac{\partial H_{mx}}{\partial x} \right)
\]

\[-F + c_{l,mx} T_{\text{two layers}}
\]

\( T_{\text{two layers}} \) is opposite in sign with respect to eqn. (50) due to the fact that the higher layer, with greater velocities, exerts a drag force to the mixture.

Likewise as in eqns. (33) and (34), mass and momentum equations for the solid phase in lower layer can be expressed as:

\[
\frac{\partial (c_s,mx A_{mx})}{\partial t} + \frac{\partial Q_{s,mx}}{\partial x} = E c_s B
\]

\[
\frac{\partial Q_{s,mx}}{\partial t} + \frac{\partial}{\partial x} \left( \beta \frac{Q_{s,mx}^2}{c_{s,mx} A_{mx}} \right) = -g \frac{\rho_s - \rho_l}{\rho_s} c_{s,mx} (i^2 + 1)
\]

\[
\frac{\partial H_{mx}}{\partial x} A_{mx} + F + g \frac{\rho_s - \rho_l}{\rho_s} c_{s,mx} (i^2 - 1) g b A_{mx}
\]

\[
+ g \frac{\rho_s - \rho_l}{\rho_s} c_{s,mx} A_{mx} l + c_{s,mx} T_{\text{two layers}}
\]

3.3.3 Numerical model
The SV equations for 1D two-phase unsteady debris flow can be expressed in compact vector form as follows:

\[
\frac{\partial \mathbf{V}}{\partial t} + \frac{\partial \mathbf{F}'}{\partial x} + C \frac{\partial \mathbf{F}^*}{\partial x} = \mathbf{S}
\]
where, for a rectangular section channel and for a completely mixed fluid:

\[
V = \begin{pmatrix}
  c_A & A \\
  c_s & A \\
  Q & 1 \\
  Q_s & 1
\end{pmatrix}
\quad F' = \begin{pmatrix}
  Q_1 \\
  Q_s \\
  Q_1^2 / (c_A A) \\
  Q_s^2 / (c_s A)
\end{pmatrix}
\]

\[
F^* = \begin{pmatrix}
  0 \\
  0 \\
  (1/2)g(A^2 / B) \\
  (1/2)g((\rho_s - \rho_h) / \rho_h)(1 + i^2)(A^2 / B)
\end{pmatrix}
\]

\[
S = \begin{pmatrix}
  0 \\
  E_c B \\
  gc_A(i - J) - (3/4)c_D(u_1 - u_2)(c_A / d_{50}) \\
  gc_s P_s - P_l [(i^2 - 1)tg + i] + \\
  (3/4)c_D(\rho_1 / \rho_h)(u_1 - u_2)(c_s A / d_{50})
\end{pmatrix}
\]

\[
C = \begin{pmatrix}
  0 & 0 & c_1 & c_s
\end{pmatrix}
\]

and for a stratified (immature) flow:

\[
V = \begin{pmatrix}
  A_{cw} & c_{l,mx} A_{mx} \\
  c_{s,mx} A_{mx} & A_{cw} \\
  Q_{cw} & Q_{l,mx} \\
  Q_{s,mx} & Q_{l,mx}
\end{pmatrix}
\quad F' = \begin{pmatrix}
  Q_{cw} \\
  Q_{l,mx} \\
  Q_{cw}^2 / A_{cw} \\
  Q_{s,mx}^2 / (c_{l,mx} A_{mx}) \\
  Q_{l,mx}^2 / (c_{s,mx} A_{mx})
\end{pmatrix}
\]

\[
F^* = \begin{pmatrix}
  0 \\
  0 \\
  (1/2)g(A_{cw}^2 / B) \\
  (1/2)g(A_{mx}^2 / B) \\
  (1/2)g((\rho_s - \rho_h) / \rho_h)(1 + i^2)(A_{mx}^2 / B)
\end{pmatrix}
\]
This model can be easily extended to 2D as follows

\[
\frac{\partial \mathbf{V}}{\partial t} + \frac{\partial \mathbf{E}}{\partial x} + \frac{\partial \mathbf{F}}{\partial y} = \mathbf{S}
\]  

(56)

where

\[
\mathbf{E} = \mathbf{E} + c_1 \mathbf{E}'
\]

\[
\mathbf{F} = \mathbf{F} + c_2 \mathbf{F}'
\]
\[
E' = \begin{bmatrix}
0 & 0 & 0 & c_i & 0 & 0 & c_s \\
0 & 0 & (1/2)gh_{cw}^2 & 0 & 0 & (1/2)gh_{mx}^2 & 0 \\
0 & 0 & 0 & (1/2)gh_{mx}^2 & 0 & 0 & (1/2)g((\rho_s - \rho_i) / \rho_i)(1 + i_y^2)h_{mx}^2 \\
\end{bmatrix}
\]

\[
E'' = \begin{bmatrix}
0 & 0 & 0 & c_i & 0 & 0 & c_s \\
0 & 0 & (1/2)gh_{cw}^2 & 0 & 0 & (1/2)gh_{mx}^2 & 0 \\
0 & 0 & 0 & (1/2)gh_{mx}^2 & 0 & 0 & (1/2)g((\rho_s - \rho_i) / \rho_i)(1 + i_y^2)h_{mx}^2 \\
\end{bmatrix}
\]

\[
S = E_x + E_y
\]

\[
\begin{align*}
&-\frac{(3/4)c_D (c_s h_{mx}) / D)(u_{mx} - u_{smx})^2 + gh_{mx}[i_x - J_{mx}] \\
&-\frac{(3/4)c_D (c_s h_{mx}) / D)(v_{mx} - v_{smx})^2 + gh_{mx}[i_y - J_{mxy}] \\
&\frac{(3/4)c_D (\rho_s - \rho_i)(c_s h_{mx}) / D)(u_{mx} - u_{smx})^2} \\
&\frac{g(c_{l1} h_{mx})((\rho_s - \rho_i) / \rho_i)[i_x - (1 - i_y^2) \tan \phi] + g(c_{l2} h_{mx})((\rho_s - \rho_i) / \rho_i)[i_y - (1 - i_y^2) \tan \phi]}
\end{align*}
\]

\[\vec{u} \text{ and } \vec{v} \text{ being the velocities along the } x \text{ and } y \text{ axes, respectively.} \]

Numerical treatments of such equations, generally, require schemes capable of preserving discontinuities, possibly without any special shift (shock-capturing schemes). Most numerical approaches have been developed in the last two or three
decades, which include the use of finite differences, finite elements or discrete/distinct element methods (Asmar et al. [74]; Rodriguez et al. [67]).

Whatsoever be the solver adopted, at each time step the degree of maturity has to be assessed, to choose the appropriate terms to incorporate in the SV equations.

4 Numerical models and solvers for dam-break shock waves

Debris flow resulting from a sudden collapse of a dam (dam-break) are often characterized by the formation of shock waves caused by many factors such as valley contractions, irregular bed slope and non-zero tail water depth. It is commonly accepted that a mathematical description of these phenomena can be accomplished by means of 1D SV equations (Bellos and Sakkas [75]; Bechteler et al. [76]; Aureli et al. [11]).

During the last century, much effort has been devoted to the numerical solution of the SV equations, mainly driven by the need for accurate and efficient solvers for the discontinuities in dam-break problems.

A rather simple form of the dam-failure problem in a dry channel was first solved by Ritter [10] who used the SV equations in the characteristic form, under the hypothesis of instantaneous failure in a horizontal rectangular channel without bed resistance.

Later on, Stoker [77] on the basis of the work of Courant and Friedrichs [78] extended the Ritter solution to the case of wet downstream channel. Dressler [79], [80] used a perturbation procedure to obtain a first-order correction for resistance effects to represent submerging waves in a roughing bed.

Lax and Wendroff [81] pioneered the use of numerical methods to calculate the hyperbolic conservation laws. McCormack [82] introduced a simpler version of the Lax–Wendroff scheme, which has been widely used in aerodynamics problems. Van Leer [83] extended the Godunov scheme to second-order accuracy by following the Monotonic Upstream Schemes for Conservation Laws (MUSCL) approach. Chen [84] and Chen and Ambruster [85] applied the method of characteristics, including bed resistance effects, to solve dam-break problems for reservoir of finite length.

Sakkas and Strelkoff [86], [87] provided the extension of the method of the characteristics to a power-law cross-section and applied this method to a dam-break steady right channel, in the case of rectangular and parabolic cross-section shapes. Strelkoff and Falvey [88] presented a critical review of the method of characteristics of power-law cross-sections. Hunt [89] proposed a kinematic wave approximation for dam failure in a dry sloping channel.

Flux-splitting-based schemes, like that of the implicit Beam-Warming [90], were applied to solve open-channel flow problems without source terms and, in general, reported good results. However, these schemes are only first-order accurate in space and employ the flux splitting in a non-conservative way. When applied to some cases of dam-break problems, these tools gave much slower front celerity and higher front height when compared to experimental tests. Later, Jha et al. [91] proposed a modification for achieving full conservative form of both
the continuity and momentum equations, employing the use of the Roe average approximate Jacobian (Roe [92]). This produced significant improvement in the accuracy of the results.

Total variation diminishing (TVD) and essentially non-oscillation (ENO) schemes were introduced by Harten [93] and Harten and Osher [94] for efficiently solving 1D gas dynamic problems. Their main property is that they are second-order accurate and oscillation free across discontinuities.

Recently, several 1D and 2D models using approximate Riemann solvers have been reported in the literature. Such models have been found very successful in solving open-channel flow and dam-break problems. Zhao et al. [95] reported implementation of an approximate Riemann solvers with Osher scheme in finite volume and later extended that work by including flux-vectors splitting and flux-difference splitting (Zhao et al. [96]).

In the past ten years, further numerical methods to solve flood routing and dam-break problems have been developed, which include the use of finite elements or discrete/distinct element methods (Asmar et al. [74]; Rodriguez et al. [67]).

Finite element methods (FEMs) have certain advantages over finite difference methods, mainly in relation to the flexibility of the grid network that can be employed, especially in 2D flow problems.

In this context, Hicks and Steffer [97] used the characteristic dissipative Galerking (CDG) FEM to solve 1D dam-break problems for variable width channels. The McCormack predictor–corrector explicit scheme is widely used for solving dam-break problems, due to the fact that it is a shock-capturing technique, with second-order accuracy both in time and in space, and that the artificial dissipation terms, such as TVD correction, can be easily introduced (Garcia and Kahawita [98]).

Garcia-Navarro and Saviron, [99] used the TVD-McCormack scheme to compute open-channel flows, particularly those involving hydraulic jumps and bores. Yang et al. [100] solved numerically 1D and 2D free surface flows by using second-order TVD and ENO schemes, while Delis and Skeels [101], [102] made a comparison with several different TVD solvers to predict 1D dam-break flows.

The main disadvantage of this solver regards the restriction to the time step size to satisfy Courant–Friedrichs–Lewy (CFL) stability condition. However, this is not a real problem for dam-break debris-flow phenomena that require short time step to describe the evolution of the discharge.

To ease the time step restriction, implicit methods could be considered. In this case, the variables are calculated simultaneously at a new time level, through the resolution of a system with as many unknowns as grid points. For non-linear problems, such as the SV equations, the resulting system of equations is also non-linear and either a linearization or an iterative procedure is required. This extra computation time is, usually, compensated by the possibility of achieving unconditional or near unconditional stability for the scheme or allowing the use of very high CFL numbers. To this end, implicit TVD schemes have been proved to be unconditionally stable, even when a linearization technique is applied to solve a non-linear hyperbolic equation (Yee [103], [104]). Attempts along this line of work were presented by Alcrudo et al. [105] introducing in the
McCormack scheme TVD corrections to reduce spurious oscillations around discontinuities, for both 1D and 2D flow problems, and by Delis et al. [106] developing new implicit TVD methods to solve SV equations.

Mambretti et al. [64], [65] and De Wrachien and Mambretti [66] used an improved TVD–McCormack–Jameson scheme to predict the dynamics of both mature (non-stratified) and immature (stratified) debris flows in different dam-break conditions.

4.1 Numerical solutions

The SV set of partial differential eqns. (23) describe a system of hyperbolic conservation laws with source term (S). The equations, as previously described, are based on the assumption of hydrostatic pressure distribution and incompressibility of water.

The homogeneous part of the system is responsible for most of the difficulties when it is numerically integrated (Delis et al. [106]).

The flux vector \( \mathbf{F} \) is related to the flow variable \( \mathbf{V} \) through the Jacobian \( \mathbf{J} \) of \( \mathbf{F} \) with respect to \( \mathbf{V} \) as:

\[
\frac{\partial \mathbf{V}}{\partial t} + \mathbf{J} \frac{\partial \mathbf{V}}{\partial x} = 0 \tag{57}
\]

with:

\[
\mathbf{J} = \frac{d \mathbf{F}}{d \mathbf{V}} = \begin{bmatrix} 0 & 1 \\ g(A/\sigma) & -u^2 2u \end{bmatrix} \tag{58}
\]

The hyperbolic nature of eqn. (57) ensures that matrix \( \mathbf{J} \) has a complete set of independent and real eigenvectors expressed as:

\[
\mathbf{e}^{1,2} = (1, u \pm c)^T \tag{59}
\]

with \( u = Q/A \) flow velocity and \( c = \sqrt{g(A/\sigma)} \) wave celerity. The eigenvalues of \( \mathbf{J} \) are expressed as:

\[
\lambda^{1,2} = u \pm c \tag{60}
\]

and correspond to the two characteristic wave speeds with their signs providing information about the direction of the wave.

4.2 Homogeneous conservation laws

High-order shock-capturing schemes have been recently proposed from the nonlinear scalar case to 1D non-linear systems, based on the use of Riemann solvers.
Consider the initial value problem for a 1D system of hyperbolic conservation laws represented by:

\[
\frac{\partial \mathbf{V}}{\partial t} + \frac{\partial \mathbf{F}(\mathbf{V})}{\partial x} = 0
\]  

and

\[
\mathbf{V}(x,0) = \mathbf{V}_0(x)
\]

where \(\mathbf{V}_0(x)\) is the initial datum with a single discontinuity:

\[
\mathbf{V}_0(x) = \begin{cases} 
\mathbf{V}_L & x < 0 \\
\mathbf{V}_R & x > 0 
\end{cases}
\]

with \(\mathbf{V}_L\) and \(\mathbf{V}_R\), respectively, the left and right constant states of the initial discontinuity \(x = 0\).

### 4.3 Riemann solvers

According to Riemann, for a given set of initial data, a conservation law may have different weak solutions one of which has physical meaning, known as the entropy-satisfying solution or solution of the Riemann problem. This solution is composed of two types of elementary waves, called shock (bores) and rarefactions (depressions) (Delis et al. [106]).

Assuming there are \(n\) independent conservation laws, then the solution of the Riemann problem will, in general, consist on \(n\) centred waves. A convergent explicit different scheme, in conservation form for eqn. (60), may be written as:

\[
\mathbf{V}_i^{n+1} = \mathbf{V}_i^n + \frac{\Delta t}{\Delta x} \left( \tilde{\mathbf{F}}_i^n - \tilde{\mathbf{F}}_{i-1/2}^n \right)
\]

where \(i\) and \(n\) are the spatial and temporal grid levels, \(\Delta x\) and \(\Delta t\) the spatial and temporal increments and \(\tilde{\mathbf{F}}_{i+1/2}\) the numerical flux function that approximates the time average flux across the cell interfaces.

#### 4.3.1 Local Lax–Friedrichs (LLF) Riemann solver

The simplest Riemann solver is the LLF solver (Lax [107], Davis [108]). The form of the numerical flux function is given by:

\[
\tilde{\mathbf{F}}_{i+1/2} = \frac{1}{2} [\mathbf{F}_{i+1} + \mathbf{F}_i - a_{\text{max}} (\mathbf{V}_{i+1} - \mathbf{V}_i)]
\]

where:

\[
a_{\text{max}} = \max \left( |a_{i1}|, |a_{i1}^2|, |a_{i1+1}^2|, |a_{i1+1}^2| \right)
\]

with \(a^k\) the characteristics’ wavespeeds.
4.3.2 Roe’s approximate Riemann solver

In the approximate Riemann solver developed by Roe [92], [109], a solution is sought for the linearized form of eqn. (57):

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{J} \frac{\partial \mathbf{V}}{\partial x} = 0$$  \hspace{1cm} (67)$$

where $\mathbf{J}$ is an approximation of the Jacobian $\mathbf{J}$.

For each computational call, $\mathbf{J}$ is chosen such that it satisfies the following conditions:

1. for any $\mathbf{V}_i$ and $\mathbf{V}_{i+1}$ the following must hold:

$$\Delta_{i+(1/2)} \mathbf{F} = \mathbf{J}(\mathbf{V}_i, \mathbf{V}_{i+1}) \Delta_{i+(1/2)} \mathbf{V} = \mathbf{J}_{i+(1/2)} \Delta_{i+(1/2)} \mathbf{V} \hspace{1cm} \forall i$$  \hspace{1cm} (68)$$

where the operator $\Delta_{i+(1/2)}(\bullet) = (\bullet)_{i+1} - (\bullet)_i$ and $\mathbf{F}_i = \mathbf{F}(\mathbf{V}_i)$;

2. for $\mathbf{V}_i = \mathbf{V}_{i+1} = \mathbf{V}$ the matrix $\mathbf{J}(\mathbf{V}, \mathbf{V}) = \mathbf{J}(\mathbf{V}) = \frac{\partial \mathbf{F}}{\partial \mathbf{V}}$;

3. $\mathbf{J}$ has real eigenvalues with linearly independent eigenvectors.

A matrix satisfying the above conditions is called Roe matrix.

From condition (3) it follows that the eigenvalues $\tilde{a}^{1,2}$ and the eigenvectors $\tilde{c}^{1,2}$ of $\mathbf{J}$ are given by:

$$\tilde{a}_{i+(1/2)}^{1,2} = \tilde{a}_{i+(1/2)} \pm \tilde{c}_{i+(1/2)}$$  \hspace{1cm} (69)$$

and the problem of finding $\mathbf{J}$ is now transferred to that of finding the average values of $\tilde{a}$ and $\tilde{c}$ that meet the conditions (1)–(3).

The following equations for $\tilde{a}$ and $\tilde{c}$ can be obtained (Dalis and Skeels [102]):

$$\tilde{a}_{i+(1/2)} = \left(\frac{Q_{i+1}}{\sqrt{A_{i+1}}} + \frac{Q_i}{\sqrt{A_i}}\right) \sqrt{A_{i+1}} + \sqrt{A_i}$$  \hspace{1cm} (70)$$

$$\tilde{c}_{i+(1/2)}^2 = \begin{cases} \frac{I_{i,i+1} - I_{i,i}}{A_{i+1} - A_i} & \text{when } A_{i+1} - A_i \neq 0 \\ c_i^2 & \text{when } A_{i+1} - A_i = 0 \text{ or } \frac{I_{i,i+1} - I_{i,i}}{A_{i+1} - A_i} < 0 \end{cases}$$  \hspace{1cm} (71)$$
The diagonalized form of \( \mathbf{J} \), \( \text{diag} \left[ \tilde{a}^k_{i+1/2} \right] \), can be expressed as:

\[
\mathbf{J} = \mathbf{R}_{i+1/2} \text{diag} \left[ \tilde{a}^k_{i+1/2} \right] \mathbf{R}^{-1}_{i+1/2}, \quad k = 1, 2
\]  

(72)

where \( \mathbf{R}_{i+1/2} \) represents the right eigenvectors matrix.

Condition (3) implies that:

\[
\Delta_{i+1/2} \mathbf{V} = \sum_{k=1}^{2} a^k_{i+1/2} \bar{a}^k_{i+1/2}
\]  

(73)

where:

\[
da^{k}_{i+1/2} = \frac{[\Delta_{i+1/2} \mathbf{Q} + (-\tilde{u}_{i+1/2} \pm \tilde{c}_{i+1/2}) \Delta_{i+1/2} \mathbf{A}]}{\pm 2\tilde{c}_{i+1/2}}
\]  

(74)

The numerical flux for a Roe’s Riemann solver can be expressed as:

\[
\bar{F}^k_{i+1/2} = \frac{1}{2} \left[ \mathbf{F}_{i+1} + \mathbf{F}_i - \sum_{k=1}^{2} a^k_{i+1/2} \bar{a}^k_{i+1/2} \right]
\]  

(75)

This form of the interface numerical flux can be interpreted as the average flux plus a ‘dissipation’ term.

An important drawback of this procedure is that Roe’s scheme admits unphysical stationary jumps, because it cannot properly distinguish a shock from a rarefaction wave. Therefore an entropy correction is needed to ensure entropy-satisfying solution (Delis et al. [106]).

### 4.3.3 Harten–Lax–van Lier–Einfeldt (HLLE) Riemann solver

The HLLE solver approximates the solution of any Riemann problem (Einfeldt [110]) with the following three constant states:

\[
\mathbf{V}(x_{i+1/2,t}) = \begin{cases} 
\mathbf{V}^n_{i} & \text{when } \tilde{x} < b^r_{i+1/2} \\
\mathbf{V}^n_{i+1/2} & \text{when } b^r_{i+1/2} < \tilde{x} < b^l_{i+1/2} \\
\mathbf{V}^n_{i+1} & \text{when } b^l_{i+1/2} < \tilde{x}
\end{cases}
\]  

(76)

where \( \tilde{x} = x - x_{i+1/2} \) and \( b \) coefficients are defined as:

\[
b^r_{i+1/2} = \max \left( u_{i+1} + c_{i+1} \tilde{a}^1_{i+1/2} \right)
\]

\[
b^l_{i+1/2} = \min \left( u_{i+1} - c_{i+1} \tilde{a}^2_{i+1/2} \right)
\]  

(77)

with \( \tilde{a}^{1,2}_{i+1/2} \) : eigenvalues of the Roe’s solver; \( \tilde{b}^{r,l}_{i+1/2} \) : numerical approximations for the maximum and the minimum characteristic speeds at the respective locations.
The average flow vector $V_{i+1/2}^n$ is defined as:

$$V_{i+1/2}^n = \frac{b_{i+1/2}^+ - b_{i+1/2}^-}{b_{i+1/2}^+ - b_{i+1/2}^-} \frac{F_{i+1} - F_i}{b_{i+1/2}^+ - b_{i+1/2}^-}$$  \tag{78}$$

The numerical flux at the cell interface can be written as:

$$\tilde{F}_{i+1/2}^{\text{HLLE}} = \frac{1}{2} \left[ F_{i+1} + F_i - \sum_{k=1}^2 q_{i+1/2}^k a_{i+1/2}^k \tilde{F}_{i+1/2}^k \right]$$  \tag{79}$$

where:

$$q_{i+1/2}^k = \frac{b_{i+1/2}^+ + b_{i+1/2}^-}{b_{i+1/2}^+ - b_{i+1/2}^-} q_{i+1/2}^{1,2} - 2 \frac{b_{i+1/2}^+ - b_{i+1/2}^-}{b_{i+1/2}^+ - b_{i+1/2}^-}$$  \tag{80}$$

$$b_{i+1/2}^+ = \max\left(b_{i+1/2}^+, 0\right)$$  \tag{81}$$

$$b_{i+1/2}^- = \min\left(b_{i+1/2}^-, 0\right)$$

The HLLE solver does not require an artificial entropy correction.

### 4.4 Inhomogeneous conservation laws

The overwhelming majority of the real problems in dam-break shock waves include source terms quantity that should be considered when using a numerical solver.

The general form for a 1D implicit numerical scheme of the SV equations can be written as:

$$V_{i+1}^{n+1} + \frac{\Delta t}{\Delta x} \vartheta \left( \tilde{F}_{i+1/2}^{n+1} - \tilde{F}_{i-1/2}^{n+1} \right) - \vartheta \Delta S_i^{n+1} = V_i^{n+1} + \frac{\Delta t}{\Delta x} \left( 1 - \vartheta \right) \left( \tilde{F}_{i+1/2}^n - \tilde{F}_{i-1/2}^n \right) + \left( 1 - \vartheta \right) \Delta S_i^n$$  \tag{82}$$

with $0 \leq \vartheta \leq 1$. For $\vartheta = 0$ the scheme is explicit.

Two other important cases for $\vartheta$ can be identified. For the case where $\vartheta = 0.5$ (Crank–Nicholson scheme) second-order accuracy in time can be achieved but severe stability restrictions apply (Delis et al. [106]). The case $\vartheta = 1$ (fully implicit scheme) appears to give the preferable scheme for solving open-channel flow equations.

#### 4.4.1 Linearized conservative implicit (LCI) solver

Numerical schemes for $V_{i+1}^{n+1}$ in eqn. (82) generally involve systems of non-linear algebraic equations that have to be solved iteratively. To avoid this...
procedure, the implicit operator is linearized and solved (Yee [103]; Alcrudo et al. [105]). In this approach, a family of schemes with the numerical flux vector in (82) written as:

$$\mathbf{F}_{i+\frac{1}{2}} = \frac{1}{2} \left[ \mathbf{F}_i + \mathbf{F}_{i+1} + \mathbf{R}_{i+\frac{1}{2}} \Phi_{i+\frac{1}{2}} \right]$$

(83)

can be derived for different vector functions $\Phi_{i+\frac{1}{2}}$. $\mathbf{R}$ is the matrix of the eigenvectors of $\mathbf{J}$ (eqn. (72)) and the elements of $\Phi$ (dissipater vector) can be calculated according to:

$$\mathbf{B}_{i+\frac{1}{2}} = \mathbf{R}_{i+\frac{1}{2}} \text{diag} \left[ -\Phi_{i+\frac{1}{2}} \right] \mathbf{R}^{-1}_{i+\frac{1}{2}}$$

(84)

where $\Phi_{i+\frac{1}{2}}^k$ are the elements of $\Phi_{i+\frac{1}{2}}$.

Dropping the subscript indexes, the matrix $\mathbf{B}$ can be defined as:

$$\mathbf{B} = \frac{1}{\tilde{a}^2 - \tilde{a}_i^2} \begin{bmatrix}
\Phi^1_{\text{a}^2} - \Phi^2_{\text{a}_i^2} & \Phi^2_{\text{a}} - \Phi^1_{\text{a}_i}
\tilde{a}_i^2 \left( \Phi^2_{\text{a}} - \Phi^1_{\text{a}_i} \right) & \Phi^2_{\text{a}^2} - \Phi^1_{\text{a}_i}^3
\end{bmatrix}$$

(85)

The numerical flux can, then, be expressed as:

$$\mathbf{F}_{i+\frac{1}{2}} = \frac{1}{2} \left[ \mathbf{F}_i + \mathbf{F}_{i+1} + \mathbf{B}_{i+\frac{1}{2}} \Delta_{i+\frac{1}{2}} \mathbf{V} \right]$$

(86)

The implicit terms $\mathbf{F}^{n+1}$ and $\mathbf{S}^{n+1}$, using a Taylor expansion, can be rewritten as:

$$\mathbf{F}_i^{n+1} = \mathbf{F}_i^n + \mathbf{J}_i^n \mathbf{\delta V}_i + O \left( \Delta t^2 \right)$$

$$\mathbf{S}_i^{n+1} = \mathbf{S}_i^n + \mathbf{J}_i^n \mathbf{\delta V}_i + O \left( \Delta t^2 \right)$$

(87)

where:

$$\mathbf{\delta V}_i = \mathbf{V}_i^{n+1} - \mathbf{V}_i^n$$

(88)

and $\mathbf{J}_i$ is the Jacobian of the source term $\mathbf{S}$ with respect to $\mathbf{V}$:

$$\mathbf{J}_i = \frac{\partial \mathbf{S}}{\partial \mathbf{V}}$$

(89)

The resulting implicit operator is strictly diagonally dominant. The $LCI$ scheme preserves the conservative form of the differencing scheme and it is applicable to both steady- and unsteady-flow conditions without stability restrictions.
### 4.4.2 Symmetric LCI solver

The elements of $\Phi_{i+1/2}$ in eqn. (84) are defined as (Yee [104]):

$$
\Phi^k_{i+1/2} = -\Psi\left(\frac{\tilde{a}_i^k - L^k_{i+1/2}}{a_i^k} \right) \quad k = 1, 2
$$

(90)

where the function $\Psi_{i+1/2}$ is the entropy correction to the eigenvalues $\tilde{a}_i^{1/2}$ that guarantees the physically valid discontinuities in the solution. The entropy correction can either take the form:

$$
\Psi(\tilde{a}) = \max(\tilde{a}, |\tilde{a}|)
$$

(91)

or the smooth one (Harten and Hyman [111]):

$$
\Psi(\tilde{a}) = \begin{cases} |	ilde{a}| & \text{when } |\tilde{a}| \geq \delta \\ \frac{\tilde{a}^2 + \delta^2}{2\delta} & \text{when } |\tilde{a}| < \delta \\ \end{cases}
$$

(92)

where $\delta$ is a small positive number, between 0.1 and 1.0.

The limit function $L$ can assume one of the following values:

$$
L^k_{i+1/2} = m\left(a_{i-1/2}^k, a_{i+1/2}^k, a_{i+1/2}^k \right)
$$

(93)

$$
L^k_{i+1/2} = m\left[2a_{i-1/2}^k, 2a_{i+1/2}^k, 2a_{i+1/2}^k + \frac{1}{2}\left(a_{i-1/2}^k + a_{i+3/2}^k \right) \right]
$$

(94)

$$
L^k_{i+1/2} = \left[a_{i-1/2}^k, a_{i+1/2}^k + a_{i+1/2}^k, a_{i-1/2}^k, a_{i+1/2}^k + a_{i+1/2}^k \right]
$$

(95)

$$
L^k_{i+1/2} = \max\left[0, \min\left(2a_{i+1/2}^k, a_{i-1/2}^k, a_{i+1/2}^k \right), \min\left(a_{i+1/2}^k, 2a_{i-1/2}^k, a_{i+1/2}^k \right) \right]
$$

(96)

where $m(a,b)$ is the minmod limiter function defined as:

$$
m(a,b) = \sgn(a) \max\left[0, \min\left(|a|, \sgn(a)b \right) \right]
$$

(97)

### 4.4.3 Monotonic upstream scheme for conservation laws (MUSCL) LCI solver

The MUSCL LCI solver, introduced by VanLeer [83], provides a one-parameter set of second-order schemes and one third-order scheme.
This procedure replaces the arguments $V_i$ and $V_{i+1}$ of the numerical fluxes by $V_{i+1/2}^L$ and $V_{i+1/2}^R$, calculated as follows:

$$V_{i+1/2}^L = V_i + \frac{1}{4} \left( (1-m)\Delta_{i+1/2}^\pm + (1-m)\Delta_{i+1/2}^\pm \right)$$

(98)

$$V_{i+1/2}^R = V_{i+1} - \frac{1}{4} \left( (1-m)\Delta_{i+1/2}^\pm + (1-m)\Delta_{i+1/2}^\pm \right)$$

(99)

where:

$$\Delta_{i+1/2}^\pm = m(\Delta_{i+1/2}V, \beta \Delta_{i-1/2}V)$$

(100)

$$\Delta_{i+1/2}^\pm = m(\Delta_{i+1/2}V, \beta \Delta_{i+1/2}V)$$

(101)

where $\beta$ is the compression parameter whose value is in the range:

$$1 \leq \beta \leq \frac{3-m}{1-m} \quad m \neq 1$$

(102)

Using MUSCL procedure linked with the LCI scheme it is possible to achieve third-order accuracy in space.

### 4.4.4 McCormack–Jameson solver

The McCormack–Jameson scheme incorporates forward (predictor) and backward (corrector) steps to solve the SV equations (McCormack [82]; Jameson [112]) that can be written as:

$$V_i^p = V_i^n - \Delta t \left[ (1-\vartheta) F_{i+1}^n - (1-2\vartheta) F_i^n - \vartheta F_{i-1}^n \right] + \Delta t S_i^n$$

$$V_i^c = V_i^n - \Delta t \left[ \vartheta F_{i+1}^p + (1-2\vartheta) F_i^p + (\vartheta-1) F_{i-1}^p \right] + \Delta t S_i^p$$

(103)

$$V_{i+1}^{n+1} = \frac{1}{2} \left( V_i^p + V_i^c \right) + \frac{\Delta t}{\Delta x} \left( d_{i+1/2}^n - d_{i-1/2}^n \right) \vartheta = 0,1$$

in which the superscripts ‘$p$’ and ‘$c$’ indicate the variables at predictor and corrector steps, respectively. The order of backward and forward differentiation in the scheme is governed by $\vartheta$ which can also be cyclically changed during the computations (Chaudry [113]).

In eqns. (103) $d$ is an artificial dissipation term that must be added to the original McCormack scheme, to avoid spurious oscillations and discontinuities without any physical significance.
The artificial dissipation terms introduced by Jameson [112], according to the classical theory developed in the field of aerodynamics, can be written as:

\[
\mathbf{d}_{i+1/2} = \epsilon^{(2)}_{i+1/2} (\mathbf{V}_{i+1} - \mathbf{V}_i) - \epsilon^{(4)}_{i+1/2} (\mathbf{V}_{i+2} - 3\mathbf{V}_{i+1} + 3\mathbf{V}_i - \mathbf{V}_{i-1})
\] (104)

where:

\[
\begin{align*}
\epsilon^{(2)}_{i+1/2} &= \max\left(\epsilon^{(2)}_i, \epsilon^{(2)}_{i+1}\right) \\
\epsilon^{(2)}_i &= \alpha^{(2)} \frac{h_{i+1} - 2h_i + h_{i-1}}{h_{i+1} + 2h_i + h_{i-1}} \\
\epsilon^{(4)}_{i+1} &= \max\left(0, \alpha^{(4)} - \epsilon^{(2)}_{i+1/2}\right)
\end{align*}
\] (105)

where \(h\) is the local free surface elevation.

The McCormack–Jameson predictor–corrector scheme is widely used for solving dam-break problems, due to the fact that it is a shock-capturing scheme, with second-order accuracy both in time and in space and that the artificial dissipation terms can be introduced, to avoid non-physical shocks and oscillations around discontinuities (García Navarro and Savirón [99]).

Recently, the scheme was applied by Mambretti \textit{et al.} [64, 65] and by De Wrachien and Mambretti [66] to predict the dynamics of both mature (non-stratified) and immature (stratified) debris and hyper-concentrated flows in different dam-break conditions.

5 Sediment concentration of the mixtures

Dam-break hydraulics involves not only water flow, but also the flow-induced sediment transport and morphological changes of the channel, which in turn modify the flow. In this context it is recognized that flow, sediment transport and morphological evolution are strongly linked to each other. So, the prediction of the grain-size-specific sediment transport of sand mixtures plays a role of paramount importance in studies of sorting processes in debris flow and surges triggered by the sudden collapse of dams. As a matter of fact, debris flows can rapidly transport large volumes of sediment, have a strong erosive force and, during movement, they collect debris or rocks, eroded from the beds or banks of the channel. They continue flowing downstream, growing in volume and sediment concentration with the addition of water, sand, mud, boulders, trees and other materials (Hui-Pang and Fang-Wu [114]).

A typical debris flow is a dense, poorly sorted, solid–fluid mixture that commonly contains more than 50% sediment by volume, and consists of particles that range widely in size from clay to boulders of several meter in diameter (Major and Pierson [115]).

Therefore, the nature of debris flow mainly changes according to the sediment concentration and characteristic of the sediment size (Egashira \textit{et al.} [116]).
debris flow of very high concentration, it is necessary to assess the equilibrium sediment concentration, and its vertical distribution to properly analyze the triggering, movement and deposition processes of dam-break shock waves with floating debris.

This paragraph describes current knowledge on sediment properties, transport, velocity distribution and suspended sediment concentration, within debris flow’s bodies, and the most advanced analytical and mathematical models, nowadays available, for sediment-laden flows.

5.1 Fundamental parameters of sediment particles

The velocity and concentration profiles in sediment-laden flows has been the subject of many theoretical, experimental, and numerical studies over the past fifty years. The main reason for such interest is that accurate prediction of the profiles allows for accurate prediction of the flow depth and suspended sediment discharge, key parameters for morphological prediction models.

Early on, Vanoni [117] and Einstein and Chen [118] experimentally observed an increase in the velocity gradient in the presence of suspended sediment. It was hypothesized that the effect was due to a ‘damping’ of the turbulence, thus decreasing turbulent mixing (Vanoni [119]). One mechanism identified as the cause for this ‘damping’ is the vertical density stratification induced by the concentration gradient. It was also observed that the density gradient creates a buoyancy force that makes it more difficult to flux heavier fluid/sediment upward into lighter fluid/sediment, and similarly restricts the downward flux of lighter fluid/sediment into heavier fluid/sediment.

The main characteristics that affect the above-mentioned processes are the size, shape, concentration and fall velocity of sediment particles (Shen and Julien [120]).

5.1.1 Size

The size of sediment particle is defined on three mutually perpendicular axes labelled a, b, c. The a axis is the direction of the longest dimension of a sediment particle. The other two mutually perpendicular axes are chosen with the c axis close to the shortest dimension and b being the intermediate perpendicular axis.

The most reliable method to measure sediment size distribution is by laboratory sieve analysis. Other methods feasible to measure the sediment size distribution are (US Inter-Agency Committee on Water Resources [121]):

- visual accumulation tube method, for sediment size between 0.062 and 2 mm;
- pipette method, for sediment particles between 0.002 and 0.062 mm;
- hydrometer method, which relies on the relationship between the progressive decrease in density that occurs at a given elevation in a well-dispersed sediment suspension and the sediment diameter of the particles.
5.1.2 Shape of sediment particles

The shape of a sediment particle has a strong influence on its fall velocity. The shape factor can be defined as:

\[ k = \frac{c}{\sqrt{ab}} \]  

(106)

where \(a\), \(b\), and \(c\) are the particle's length, width, and height, respectively (Garde and Ranga Raju [122]).

5.1.3 Sediment concentration

Sediment concentration measures the amount of sediment carried by the flow. Commonly, three parameters are used to express sediment concentration:

- Sediment concentration by volume:

\[ S_v = \frac{S_w}{S_G - S_w(S_G - 1)} \]  

(107)

where \(S_G\) is the specific gravity of the sediment;

- \(S_w\) is the sediment concentration by weight (mass):

\[ S_w = \frac{S_m}{\rho_w + (1 - (1/S_G))S_m} \]  

(108)

where \(\rho_w\) is the density of water and \(S_m\) is the sediment concentration by a mixture of weight (mass) and volume

\[ S_m = \frac{S_w\rho_wS_G}{S_G - (S_G - 1)S_w} \]  

(109)

The sediment concentration is usually expressed as parts per million (ppm) by weight. Another sediment concentration measure is by Newton (weight) per cubic meter (volume).

5.1.4 Fall velocity

The fall velocity is defined as the terminal fall velocity of a sediment particle. It depends on the size, shape and density of the particle and the effects of fluid density and turbulence. According to Schultz et al. [123], the fall velocity of a particle varies greatly with the Reynolds number based on particle size.

5.2 Sediment concentration distribution

Sediment concentration, defined as the amount of sediment carried by the flow, represents a parameter of paramount importance in debris and hyper-concentrated flow movement and deposition.
Einstein [124] was one of the first to propose a method for the calculation of sediment transport on a grain-size-specific basis, proposing separate relations for bed and suspended load. Later on, the log-law velocity profile and Rouse concentration profile (Vanoni [119]) were used, along with the Einstein bed-load equation, for the near-bed concentration. McLean [125] proposed a similar approach with several improvements that account for density stratification effects in both the velocity and concentration profiles.

Another class of relations take a more empirical approach for partitioning the total sediment transport on a grain-size-specific basis. A review of these methods can be found in Molinas and Wu [126].

While these methods predict the total sediment transport by grain size, there are other procedures that predict the grain-size-specific bed load and/or suspended load separately (Samaga et al. [127]). The suspended load is assumed to be related to the bed shear stress.

More advanced analytical and mathematical models for sediment-laden flows are based on the governing equations for single-phase flows, and are valid for very-low-sediment situation. Usually a model of this type is referred to as a decoupled one. Examples of this kind are the widely accepted Rouse formula for suspended sediment concentration distribution and the well-known logarithmic velocity profile developed initially for a single-phase flow. Recently, more and more attention has been paid to the influences of the interphase (water–sediment) and sediment particle–particle interactions, which become predominant factors and cannot be neglected for dam-break shock waves with floating debris, with heavy sediment concentrations.

A two-phase flow approach appears to be more appropriate for this kind of phenomena. Up to now, much effort has been performed in this direction. Kobayashi and Seo [128] proposed a model for water–sediment interaction in the bed-load layer on the basis of two-phase flow equations. Later on, Cao et al. [129] provided a two-phase hydraulic model, based on mass and momentum conservation equations, feasible to predict the distribution of velocity and suspended sediment concentration. The flow is divided vertically into a suspension flow, a near-bed layer and a viscous sublayer.

To determine the velocity and sediment concentration at the interfaces of the three layers, the shear and dispersive stresses, stemming from the interactions between sediment particles, are taken into account.

5.3 Hydraulic-based routing modelling

The model is based on the continuum assumption for both the fluid phase (water) and the dispersed solid one (sediment).

5.3.1 Governing equations

In the following, the Cartesian co-ordinate system and time are denoted by \( x_i \) \( (i = 1, 2, 3) \) and \( t \), respectively.
The equations of mass conservation for the fluid and solid phases are:

\[
\frac{\partial \rho_f (1-c)}{\partial t} + \frac{\partial \rho_f (1-c) c u_j}{\partial x_j} = 0 \quad (110)
\]

\[
\frac{\partial \rho_s c u_j}{\partial t} + \frac{\partial \rho_s c u_j}{\partial x_j} = 0 \quad (111)
\]

The equations of motion for the fluid and solid phases are:

\[
\frac{\partial \rho_f (1-c) u_{i,j}}{\partial t} + \frac{\partial \rho_f (1-c) u_j u_i}{\partial x_j} = \rho_f (1-c) g_i - (1-c) \frac{\partial p}{\partial x_j} + \frac{\partial T_{ij}}{\partial x_j} - f_i \quad (112)
\]

\[
\frac{\partial \rho_s c u_i u_j}{\partial t} + \frac{\partial \rho_s c u_i u_j}{\partial x_j} = \rho_s c g_i - c \frac{\partial p}{\partial x_j} + \frac{\partial T_{ij}}{\partial x_j} + f_i \quad (113)
\]

where \( \rho_f, \rho_s \) denote the constant densities of the fluid and solid phases, respectively; \( c \) the volumetric concentration of the solid phase; \( u_{i,j}, u_j \): \( x_j \)-components of instantaneous velocities of the fluid and solid phases, respectively; \( g_i \) the full pressure of the fluid–solid system; \( g_i \) the \( x_i \)-component of gravitational acceleration; \( T_{ij} \), the viscous stress tensor of incompressible fluid phase; \( T_{ij} \), the stress tensor stemming from interaction between dispersed solid particles; \( f_i \) the \( x_i \)-component of interphase interaction force per unit volume; and the subscripts ‘f’ and ‘s’ denote, respectively, the fluid and solid phases. The fractional pressures of the fluid and solid phases are assumed to be proportional to their volumetric concentrations, respectively (Pai [130]).

The density of the fluid–solid mixture is defined as:

\[
\rho_m = \rho_f (1-c) + \rho_s c \quad (114)
\]

where (and hereafter) the subscript \( m \) denotes the mixture.

The continuity and motion equation of the mixture can be written as:

\[
\frac{\partial \rho_m}{\partial t} + \nabla \left( \rho_m \textbf{u}_{m, MF} \right) = 0 \quad (115)
\]

\[
\frac{\partial \rho_m u_i}{\partial t} + \frac{\partial \rho_m u_i u_j}{\partial x_j} = \rho_m g_i - \frac{\partial p}{\partial x_j} + \frac{\partial T_{ij}}{\partial x_j} - \left[ \rho_s c u_j (u_i - u_j) \right] \quad (116)
\]
where $u_i, u_j$ denote the $x_i, x_j$-components of the mixture's velocity vector $u_{m, MF}$ defined as:

$$\rho_m u_{m, MF} = \rho_f (1-c)u_f + \rho_s C u_s$$  \hspace{1cm} (117)

with $u_f, u_s$: velocity vectors of the fluid and solid phases, respectively, and $i_f, i_s$ the $x_i$-components of the dispersive velocity vectors of the same phases:

$$i_f = u_f + u_{m, MF}$$  \hspace{1cm} (118)

$$i_s = u_s + u_{m, MF}$$  \hspace{1cm} (119)

Under gravitational action, a solid particle is considered to have an effective settling velocity $\omega$ (time independent) relative to the mixture’s velocity in the vertical direction, that is:

$$i_s = -\omega k$$  \hspace{1cm} (120)

where $k$ represents a unit vector in the vertical direction, positive upward.

### 5.3.2 Sediment concentration profile

On the basis of the governing equations, the flow, as previously mentioned, is divided into three parts vertically (fig. 8):

- suspension flow: $y \in [a, h]$;
- near-bed layer: $y \in [y_b, a]$;
- viscous sublayer: $y \in [0, y_b]$.

![Figure 8: Two-dimensional uniform flow.](image-url)
For the suspension flow, it is assumed that the inertia of the sediment particles is negligible and that the sediment velocity differs from the fluid–sediment mixture’s velocity only in the vertical direction by an effective settling velocity.

This allows to establish the following diffusion equation for suspended sediment concentration:

\[
\varepsilon_{\text{sy}} \frac{dc}{dy} = -\omega C - \frac{\rho_s - \rho_f}{\rho_f} \omega C^2
\]  

(121)

where \( \varepsilon_{\text{sy}} \) denotes the \( y \)-coefficient of the turbulent diffusion of sediments, feasible to provide sediment concentration profiles according to different models for turbulent diffusion.

Solution of eqn. (121) requires appropriate initial and boundary conditions. Usually the mean sediment concentration is set equal to a reference concentration at the interface between the suspension flow and the near-bed layer. Moreover, a closure method is required to determine \( \varepsilon_{\text{sy}} \). Conventionally, the following relationship is assumed:

\[
\varepsilon_s = \beta \nu_1
\]  

(122)

where \( \nu_1 \) is the coefficient of turbulent viscosity; \( \beta \) the coefficient usually supposed equal to 1 (Van Rijn [131]).

The two-phase model was tested extensively using measured data of Coleman [132] and Einstein and Chen [118]. The agreement is satisfactory for low-sediment concentration flows, and fairly good for heavy sediment concentration cases.

5.4 Stochastic approach

Much research on sediment concentration of debris flow triggered by dam-break events has been conducted using hydraulic-based routing models. However, debris flow always has uncertainties in variables and model parameters. The properties of the moving fluid mixtures of debris and water are very different from those of purely water floods (Jin and Fread [133]).

Unlike sediment-laden flow, debris flow is composed of a dense mixture of water and solid particles rather than of a continuous medium so that the sediment concentration is not continuous as its profile. Therefore, the traditional approach to debris-flow studies using a physically hydraulic-based is limited. Despite advances made with this approach, major problems and barriers still exist in debris-flow analysis. In this context, Hui-Pang and Fang-Wu [114] proposed a model, based upon probability and the concept of entropy, feasible to solve sediment concentration problems of debris flow. The entropy concept provided a new approach for investigating the stochastic behaviour of certain debris-flow
phenomena, including vertical distribution of suspended sediment concentration in both open-channel and debris flows.

5.4.1 Maximum entropy principle
Assuming the entropy as a measure of uncertainty, the probability distribution function of a hydraulic variable, like the sediment concentration of debris flow, can be assessed by maximizing the entropy of the variable in an appropriate way (Chiu [134]).

From this assumption, the maximum value of the sediment concentration function $H(c)$ can be written as:

$$\text{maximize } H(c) = -\int p(c) \ln p(c) dc$$

(123)

where $p(c)$ is the probability density function and $p(c)dc$ the probability of the sediment concentration being between $c$ and $c+dc$.

From experimental studies, different authors (Tsubaki et al. [56]; Takahashi [45], Egashira et al. [135]) proposed monotonic variations of the sediment concentration along the debris-flow profile, under steady and uniform flow (fig. 9).

The figure shows that the sediment concentration, in a steady uniform debris flow with depth $h_0$, decreases monotonously in the vertical direction from the maximum value of $c_m$ at the channel bed, to any arbitrary value $c_h$ at the water surface. In such flows, $c$ represents the sediment concentration at the vertical distance $y(0) \leq y \leq h_0$ from the channel bed. Then, at any distance greater than $y$, the sediment concentration is less than $c$. Under this hypothesis, the density function $p(c)$ can be written as:

$$p(c) = \left(\frac{h_0}{\frac{dc}{dy}}\right)^{-1}$$

(124)

Figure 9: Sketch of uniform debris flow.
The density function $p(c)$ must satisfy the following two conditions:

\[ \int_{c_i}^{c_i} p(c) dc = 1 \]  
\[ \int_{c_i}^{c_e} cp(c) dc = c_D \]

The first condition (125) is the definition of general probability, the second one (126) defines the equilibrium (or mean) sediment concentration by volume, $c_D$.

5.4.2 Equilibrium sediment concentration

Solution of the differential eqn. (124), using the boundary condition $c = c_m$ at $y = 0$, provides sediment concentration profiles. This solution can be written as:

\[ \frac{c}{c_m} = \frac{1}{E} \ln \left[ e^E + \left( e^{E(c/c_m)} - e^E \right) \frac{y}{h_b} \right] \]

where the dimensionless entropy parameter $E$ is defined as:

\[ E = \lambda c_m \]

where $\lambda$ is the Lagrangian multiplier.

To validate the model, comparisons have been made between its prediction and experimental results carried out by different authors (Takahashi [30]). The comparisons have shown that the procedure based on the maximum entropy principle is feasible to predict the sediment concentration distribution profile of debris flow once its channel gradient and bed materials are known or given.

5.5 Gradual earth-dam failure

Gradual earth-dam failure (dam breach) involves not only water flow, but also the flow-induced sediment transport and morphological changes, which in turn modify the flow. Therefore, the strong interaction between flow, sediment and morphological evolution has to be taken into account in any experimental and/or computational study of dam-breath hydraulics (Cao et al. [136]).

In the context of computational studies of dam-breath hydraulics, two basic observations are warranted. First, it has to be recognized that the flow, sediment transport and morphological evolution are strongly coupled to each other, the rate of bed deformation being considerable compared to that of the flow evolution.

Second, it has to be assumed that the most important mechanism of sediment transport due to dam-breath flows is the local entrainment of bed sediment, in response to excessively energetic turbulent motions.

In these processes, erosion and sediment transport are triggered by some low or weak points in the crest or in the downstream face of the dam, by piping or by overtopping. The water overtopping the crest shapes a little breach.
Progressive erosion, then, widens and deepens the breach, increasing outflow and erosion rate.

With reference to these issues, dam-breach models over mobile beds provide a unique opportunity to gain enhanced insight into the interaction between flow, sediment transport and morphological evolution. In the context of fluvial mobile bed hydraulics, it has long been known that the flow tends to adapt itself (velocity and depth) to reach a potential dynamic equilibrium. Under this equilibrium state, sediment entrainment from, and deposition to, the channel bed is well balanced and the flow bears minimum energy dissipation rate (Yan [137]). Bed erodibility can modify the free surface profile and hydrographs greatly, and has considerable implication for flood prediction. The velocity is strongly reduced over a mobile bed, which is accompanied by an increase in flow depth. This behaviour can provoke very active sediment exchange between the water column and the bed, and also produce a sharp spatial gradient concentration. These features are sensitive to bed sediment particle size. The finer the sediment, the greater the effects bed mobility will have. These processes significantly affect the shape of the mobile bed, which can be scoured. Therefore the rate of bed deformation is not negligible compared to that of flow change, characterizing the need for coupled modelling of the interacting flow–sediment–morphology system.

An attempt to simulate this interacting system, was made using geotechnical principles and simplifying hypotheses to obtain empirical equations relating the amount of eroded material to the flow through the breach (Cristofano [138]). Later on, Brown and Roger [139] developed a model using the Schoklitsh relationship to compute suspended sediment. Ponce and Tsivoglou [140] presented a mathematical model of the gradual failure of an earth dam, which can be considered as the first effort to tackle this problem, within the framework of an implicit numerical solution scheme. Afterwards, many other authors have proposed their own models. These models can be grouped into the following categories (Leopardi et al. [141]):

- **comparative models**: they rely on data of similar, previously failed dams, which represent input data to predict the breach evolution and discharge values of the simulated structures;
- **statistical models**: correlations are established to link unknown variables and typical dam characteristics (McDonald and Langridge-Monopolis [142]);
- **hydraulic models**: they require as input the size of the breach, the time expected for the failure to develop and the height of the surface water at which breaching begins (Fread [4], US ARMY [143]);
- **physically based model**: the breach development depends on the erosive strength of the stream flowing through it. Erosion laws are assumed as linear or quadratic functions of the average flow velocity (Singh and Scarlatos [144]). The evolution of the dam-breach process can be based on hydrologic, geometric and geotechnical considerations, or on the use of erosion formulas, like those proposed by Einstein [124]. The physically based models can be coupled with sediment routing schemes to simulate the triggering of a debris flow due to the water overtopping the crest of a dam (Takahashi and Nakagawa [38]).
6 General remarks

Studies of dam-break flows consider, mainly, situation of clear-water surges. However, under natural conditions a dam-break flow can generate extensive debris or encounter floating debris in the valley downstream of the dam. Debris flows differ from other natural unconfined flows in the nature of both the floating material and the flow itself, which is rapid, transient and may significantly influence surge height and speed.

Because a debris flow is, essentially, a multi-phase system, any attempt at modelling this phenomenon that assumes, as a simplified hypothesis, homogeneous mass and constant density, conceals the interactions between the phases and prevents the possibility of investigating further mechanisms such as the effect of sediment separation (grading). Therefore, modelling the fluid as a two-phase mixture overcomes most of the limitations mentioned above and allows for a wider choice of rheological models.

The rheological behaviour of a mixture depends on the hydrodynamic characteristics of the flow and the characteristics of the sediment. To be reliable, a rheological model should possess three major features (Chen [54]). It should describe: the dilatancy of the sediment–water mixtures; the soil yield criterion, as proposed by Mohr--Coulomb; and the role of inter-granular fluid.

On the whole, the one-layer models are unable to adequately describe the entire thickness of the flow, so, recently, multi-layer models suitable to combine two or more constitutive relationships to fit experimental velocity and concentration profiles have been proposed (Wan and Wang [59]).

The coefficients of these models depend, strongly, on factors such as the dimension, concentration and distribution of the sediments, as well as the flow regime. The assessment of this dependency constitutes one of the major challenges of debris-flow studies.

Different methods have been proposed for the prediction of the flow depth, grain-size specific near-bed concentration and bed-material suspended sediment transport rate in sand-bed rivers. The salient improvements, related to the need to enhance the existing procedures to encompass the full range of sand-bed streams, and in particular large, low-slope sand-bed watercourses, can be summarized as follows (Wright and Parker [145]):

- the inclusion of density stratification effects, which are particularly relevant for large, low-slope, sand-bed rivers;
- a new predictor for near-bed entrainment rate into suspension;
- a new predictor for form drag based on the shear stress component.

However, debris flow always has uncertainties in variables and model parameters. The properties of the moving fluid mixture of debris and water are very different from those of purely water floods (Jin and Fread [133]).

The traditional approach to debris-flow studies using a physically hydraulic-based model is limited. Despite advances made with this approach major problems
and barriers exist in debris-flow studies. Therefore, efforts to develop new tools for debris-flow forecasting are needed. Some investigations have, recently, explored the feasible application of the theory and concept of entropy, based upon probability, to solve the sediment concentration problems of debris flow (Hui-Pang and Fang-Wu [114]). The entropy concept provides a new approach suitable to analyse the stochastic nature of certain hydraulic and morphologic mechanisms, including vertical distribution of the velocity, shear stress and the distribution of suspended sediment concentration in both open-channel and dam-break flows.

The stochastic approach proved feasible to predict sediment concentration distribution profiles, within debris-flow phenomena, once the channel gradient and bed materials are known or given.

In modern hydraulic engineering practice, there is a need for efficient and accurate mathematical models and solvers that should be able to numerically represent all the physically significant phenomena in a given flow scenario.

In this context, debris flows resulting from a sudden collapse of a dam (dam-break) are often characterized by the formation of shock waves. It is commonly accepted that a mathematical description of these phenomena can be accomplished by means of 1D SV equations (Bellos and Sakkas [75]). These equations yield discontinuous solutions in the form of shocks or bores, which can be difficult to represent accurately without the use of appropriate shock-capturing methods.

Several classical finite difference or finite element schemes are commonly used in practical applications, but they are highly inaccurate in modelling discontinuous flow and provide unreliable approximations in subcritical and supercritical flow conditions.

In recent years, a great effort has been devoted to the construction of efficient and accurate procedures for systems of conservation laws. This resulted in schemes known as high-resolution shock-capturing schemes. Much of the development of recent high-resolution numerical methods written in conservative form is applied to the flow equations via the use of Riemann solvers. Some of these solvers have been analysed in this chapter.

TVD schemes were introduced by Harten [93] for efficiently solving the Euler equations in gasdynamics. Their main property is that they are second-order accurate (except at extremes) and oscillation free across discontinuities. Their main disadvantage lies in the restriction on the time step which is imposed by the CFL condition. However, this is not a real problem for dam-break debris-flow phenomena that require short time steps to describe the evolution of the discharge. Attempts along this line of work were presented by Alcrudo et al. [105] introducing in the McCormack scheme TVD corrections to reduce spurious oscillations around discontinuities, for both 1D and 2D flow problems.

Recently, Mambretti et al. [64], [65] and De Wrachien and Mambretti [66] used an improved TVD McCormack–Jameson scheme to predict the dynamics of both stratified and non-stratified debris flow in different dam-break conditions that overcome some of the above-mentioned constraints. Along this line of work further efforts need to be made to analyse reverse grading (sorting) and to better feature the distribution of the material of different size of the solid phase within the mixture profile.
References


138 DAM-BREAK PROBLEMS, SOLUTIONS AND CASE STUDIES


