CHAPTER 6

Transmission line models for high-speed conventional interconnects and metallic carbon nanotube interconnects

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Abstract

The transmission line is a powerful model to describe in a simple and accurate way the propagation of electric signals along interconnects of different kind. The ‘standard’ transmission line (STL) model is derived under a series of assumptions involving both the physical structures and the carried signals, which are satisfied for a large amount of cases of practical interest. Nowadays the signal speed is growing rapidly due to market requirements and progress in technology. As the velocity of the electrical signals increases, high-frequency effects due to dispersion and radiation losses, which the STL model is unable to describe, are no more negligible.

In the future large scale integration electronics the interconnect cross-sections will become smaller and smaller down to nanometric dimensions. As interconnect sizes shrink copper resistivity increases due to grain and surface scattering effects and wires become more and more vulnerable to electro-migration due to the higher current densities that must be carried. In order to overcome these limitations the use of metallic carbon nanotubes (CNTs) as interconnects has been proposed and discussed recently.

Here both an ‘enhanced’ transmission line model able to describe the high-frequency effects due to dispersion and radiation losses in conventional high-speed interconnects and a new transmission line model for metallic CNT interconnects are reviewed. Some applications to interconnects of particular interest in present high-speed electronics and in future nanoelectronics are presented.

1 Introduction and historical background

The transmission of electric signals through metallic wires is one of the most important contributions to the development of modern technology. S.F.B. Morse
invented the electric telegraph in 1838 and the first commercial telegraph line was erected in 1844, between New York, Baltimore and Washington. Nevertheless, at that time the theory of electric circuits was still at its dawn and hardly anything was known about the transmission of electric signals along conducting wires. The paper in which G. Kirchhoff formulated his well-known laws has been published in 1845.

The rapid development of telegraphic signal transmission by means of overland lines and undersea cables (the first undersea cable was laid between France and England in 1851 and in 1853 the first transatlantic cable was installed) gave rise to a long series of theoretical investigations on the transmission of electrical signals through conducting wires.

Lord Kelvin (1855) studied the effects of transients in telegraphic signal transmission through long cables and formulated the first distributed parameter model for an electric cable. He assumed that the effects of magnetic field were negligible, and modeled the effects of electric induction by means of the per-unit-length (p.u.l.) capacitance of the cable and the lossy effects by means of the p.u.l. resistance, so deriving the well-known voltage diffusion equation (Lord Kelvin, 1855).

Shortly after Kirchhoff (1857), using Weber’s electromagnetic theory [1], analyzed the transmissions of electric signals through two wires with finite conductivity, including the effects of the magnetic field, and obtained what we can define as the first transmission line model [2]. He deduced that the electric signals propagate along the conductors with the same velocity as that which light propagates in the vacuum, several years before Maxwell published his fundamental paper demonstrating the electromagnetic nature of light [3]. Unfortunately, for reasons that are still not fully clear, Kirchhoff’s work has never been widely acknowledged and is even today largely unknown. There is an interesting work by Ferraris in which Kirchhoff’s model is reviewed and studied in depth [4].

O. Heaviside (1881–87) was the first to study the ‘guided’ propagation of electric signals along couples of rectilinear and parallel conducting wires, with finite conductivity, immersed in a lossy homogeneous dielectric, using Maxwell’s electromagnetic theory. He developed the transmission line theory as it is still known today [5]. Hereafter, the Heaviside transmission line model is called the ‘standard’ transmission line (STL) model.

Kirchhoff obtained his transmission line model starting with an integral formulation of the problem based on Weber’s theory of electromagnetism. This theory is based on interaction at distance, described by two variables that can be considered as a forerunner of the electric scalar potential and the magnetic vector potential. Heaviside, instead, obtained his transmission line model starting from a formulation based directly on Maxwell’s field theory under the assumption that the configuration of the electromagnetic field is quasi-transverse electromagnetic (TEM).

The STL model has since been extended to interconnects, even non-uniform ones, with many wires, in the presence of conducting planes and non-homogeneous dielectrics. The reader is referred to many excellent books and reviews existing in the literature for a complete and comprehensive treatment of the subject [5–8].
The STL model for conventional interconnects is based on the assumptions that:

- The interconnect quasi-parallel wires are metals, whose electrical behavior is governed by Ohm’s law;
- The structure of the electromagnetic field surrounding the wires is of quasi-TEM type with respect to the wire axis;
- The total current flowing through each transverse section is equal to zero.

A TEM field structure is one in which the electric and magnetic fields in the space surrounding the conductors are transverse to the wire axis. The TEM fields are the fundamental modes of propagation of ideal multiconnected guiding structures, i.e. guiding structures with transverse section uniform along the wire axis, made by perfect conductors and embedded in a homogeneous medium [6, 8]. In actual interconnects the electromagnetic field is never exactly of the TEM type. In ideal shielded guiding structures, high-order non-TEM modes with discrete spectra can propagate as well as the TEM fundamental modes. In unshielded guiding structures there are also non-TEM propagating modes with continuous spectra. Actual guiding structures are most frequently embedded in a transversally non-homogeneous medium, and thus TEM modes cannot exist. However, even if the medium were homogeneous, due to the losses, the guiding structure could not support purely TEM modes. Furthermore, the field structure is complicated by the influence of non-uniformities present along the axis of the guiding structures (bends, crossovers, etc.). However, when the cross-sectional dimensions of the guiding structure are smaller than the smallest characteristic wavelength of the electromagnetic field propagating along it, the transverse components of the electromagnetic field give the ‘most significant’ contribution to the overall field and to the resulting terminal voltages and currents [9]. In other words, we have that the structure of the electromagnetic field is said to be of quasi-TEM type.

Nowadays, the speed of electronic signals is growing rapidly due to market requirements and to progress in technology, e.g. allowing switching times below 1 ns. Because of such high-speed signals the distance between the wires of interconnects existing at various levels in an electronic circuit may become comparable with the smallest characteristic wavelength of the signal themselves. As a consequence high-frequency effects such as dispersion and radiation losses are no more negligible and there is the need of a new model to describe the propagation of the signals along the interconnects.

Several efforts have been made to obtain generalized transmission line models from a full-wave analysis based on integral formulations to overcome the restrictions of the STL model [10–17]. Recently, the authors [18–20] have proposed an ‘enhanced’ transmission line (ETL) model derived from a full-wave analysis based on an integral formulation of the electromagnetic field equations, which has the same simplicity and structure as the STL model. The ETL model describes the propagation along interconnects in frequency ranges where the STL model fails, taking into account the shape effects of the transverse cross-section of the interconnect wires. It reduces to the STL model in the frequency ranges, where the distance between wires is electrically short. Specifically, the ETL model allows to
forecast phenomena that the STL model cannot foresee, such as the distortion introduced by the non-local nature of the electromagnetic interaction along the conductors, and the attenuation due to radiation losses in the transverse direction. Furthermore, the ETL model describes adequately both the propagation of the differential mode and the propagation of the common mode and the mode conversion. The ETL model considers thick quasi-perfect conducting wires and evaluate correctly the kernel that shows the logarithmic singularity that is typical of the surface distributions. Such a singularity plays a very important role in the radiation problems, e.g. it may regularize the numerical models [21, 22]. The approach on which the ETL model is based bears a resemblance to the Kirchhoff approach [2].

The STL model can be easily enhanced so to describe non-perfect conductors, provided that they satisfy Ohm’s law, as for instance copper does. Unfortunately, in future ultra-large-scale integrated circuits some problems will arise from the behavior of the copper interconnects. As the cross-section shrinks to nanometric dimensions, due to surface scattering, grain boundary scattering and electromigration, the copper resistivity rises to values higher than its bulk value. Because of heating, these high values will limit the maximum allowed current density. Nanometric copper conductors also suffer from the additional problem of mechanical stability. Carbon nanotubes (CNTs) are allotropes of carbon that have been discovered fairly recently [23] and are considered as an alternative to conventional technology for future nanoelectronic applications such as transistors, antennas, filters and interconnects [24, 25]. Metallic CNTs have been suggested to replace copper in nano-interconnects [26–28], due to their unique electrical, mechanical and thermal properties, such as the high-current density allowed (up to $10^{10}$ A/cm$^2$) which is three order of magnitude higher than the one of copper, the thermal conductivity as high as that of diamond, and the long mean free path (ballistic transport along the tube axis). Recently, the authors [29–31] have proposed a transmission line model to describe the propagation of electrical signals along metallic single wall CNT interconnects.

In this chapter, both the ETL model and the transmission line model for metallic CNT interconnects are reviewed. In Section 2, the derivation of transmission line models from a general integral formulation of the electromagnetic problem is presented. Section 3 is devoted to the transmission line model representation of conventional interconnects, like wire pairs or microstrips. In Section 4, the transmission line model for the propagation along metallic CNTs is presented. Finally, in Section 5 some case-studies are carried out showing either qualitative or quantitative analysis of the behavior of conventional and CNT interconnects.

## 2 General integral formulation and derivation of transmission line models

### 2.1 Integral formulation

Let us consider an interconnect made of $N$ conductors of generic cross-sections, with parallel axis $\hat{x}$ and total length $l$, as depicted in Fig. 1, where the $y$–$z$ plane
is shown. A perfect conductor ground is located at \( x = 0 \) and a stratified inhomogeneous dielectric is considered, made by several dielectric layers with relative permittivity \( \varepsilon_k \). Let \( S_k \) be the boundary surface of the \( k \)th conductor, and \( l_k \) be its contour at any given cross-section \( x = \text{const.} \) (\( s_k \) is the curvilinear abscissa along \( l_k \)). We assume that a sinusoidal steady state is reached, and that the operating frequencies and the geometrical dimensions are such that the current density is mainly located on the conductor surfaces \( S_j \). In the frequency domain, the Faraday–Neumann law relates the electric to the magnetic field as:

\[
\nabla \times \mathbf{E} = -i\omega \mathbf{B},
\]

where \( \omega \) is the radian frequency. In order to automatically solve (1) and to impose the solenoidality of \( \mathbf{B} \), implied by eqn (1) itself, we can introduce the magnetic vector potential \( \mathbf{A} \) and the electric scalar potential \( \varphi \) such that:

\[
\nabla \times \mathbf{A} = -i\omega \varphi, \quad \nabla \cdot \mathbf{B} = \nabla \times \mathbf{A}.
\]

The potentials \( \mathbf{A} \) and \( \varphi \) are not uniquely defined, unless a suitable gauge condition is imposed. In the present derivation we will use the so-called Lorenz gauge

\[
\nabla \cdot \mathbf{A} + i\omega \varepsilon \mu \varphi = 0,
\]

which is imposed in homogeneous regions, i.e. in regions where the dielectric permittivity \( \varepsilon \) and the magnetic permeability \( \mu \) are constant. Note that at the interfaces between homogeneous regions we have to impose the continuity of the tangential components of the fields.

The sources of the electromagnetic field are the (superficial) current and charge densities \( \mathbf{J}_s \) and \( \sigma_s \), which must satisfy the charge conservation law:

\[
\nabla_s \mathbf{J}_s + i\omega \sigma_s = 0,
\]

where \( (\nabla_s) \) is the surface divergence operator. These sources may be related to the potentials through the Green functions defined for the domain of interest:

\[
\mathbf{A}(\mathbf{r}) = \frac{1}{\mu_0} \int \int_{S} G_A(\mathbf{r}, \mathbf{r}') \mathbf{J}_s(\mathbf{r}') \ dS',
\]

\[
\varphi(\mathbf{r}) = \frac{1}{\varepsilon_0} \int \int_{S} G_\varphi(\mathbf{r}, \mathbf{r}') \sigma_s(\mathbf{r}') \ dS',
\]

\[\text{Figure 1: Generic cross-section of a multilayered interconnect.}\]
where \(\varepsilon_0\) and \(\mu_0\) are the dielectric constant and the magnetic permeability in the vacuum space, \(S\) represents the union of all the \(N\) conductor surfaces \(S_j\). Note that the Green function \(G_\lambda\) is in general dyadic.

In order to derive a multiport representation of the interconnect, we assume that it would be possible to characterize it regardless of the actual devices on which it is terminated. In other words, the terminal elements are taken into account only through the relations that they impose on the terminal currents and voltages, but the sources located on their surfaces are neglected in computing the potentials (5) and (6). This is a crucial point in the field/circuit coupling problem. This condition is approximately satisfied if the characteristic dimensions of the terminal devices are small compared to the interconnect length. Anyway, as a consequence of this approximation, the potentials in eqns (5) and (6) do not wholly satisfy the Lorenz gauge condition. Conversely, when the assumption does not hold, there is no way to separate the behavior of the interconnect from that of terminal devices and the electromagnetic system has to be analyzed as a whole.

### 2.2 Transmission line equations

The first fundamental assumption is that the surface current density is mainly directed along \(\hat{x}\): \(\mathbf{J}_s = J_s(x)\hat{x}\). In other words, we neglect any transverse component of the current density, taking into account only the longitudinal one. This assumption is well-founded when the interconnect length is infinite and only the fundamental mode is excited. Even with an infinite length, high-order propagation modes may exhibit non-longitudinal current density components; hence this assumption defines an upper limit in the frequency range.

The first consequence of this assumption is a drastic simplification of eqns (5) and (6). Indeed, the magnetic vector potential (eqn (5)) is directed only along \(\hat{x}\), and so the magnetic field is of Transverse Magnetic (TM) type. In this condition, it is uniquely defined the voltage between any couple of points lying on a plane \(x = \text{constant}\).

A second assumption is that the current and charge densities have a spatial dependence of separable type:

\[
\mathbf{J}_s(x) = J_k(x)F'_k(s_k), \quad \sigma(x)|_{\text{res}} = Q_k(x)F''_k(s_k),
\]

where \(I_k(x)\) and \(Q_k(x)\) are the total current and p.u.l. charge associated with the conductor and \(F'\) and \(F''\) are shape functions dimensionally homogeneous with \(m^{-1}\), describing the distribution of currents and charges along the contour \(l_k\). In other words, we are assuming that only the total current \(I_k(x)\) and p.u.l. charge \(Q_k(x)\) vary along \(x\), whereas the spatial distributions of current and charge densities are independent on \(x\).

Imposing the charge conservation law (4) on the \(k\)th conductor and using eqn (7), we obtain

\[
\frac{dI_k(x)}{dx}F'_k(s_k) + i\omega Q_k(x)F''_k(s_k) = 0,
\]
which yields:

\[ F'(s_k) = F''(s_k) = F_0(s_k), \quad (9) \]

\[ \frac{dI_k(x)}{dx} + ioQ_k(x) = 0. \quad (10) \]

The shape functions for the charge and current distributions must be the same. If we impose the following normalization condition:

\[ \oint_{l_k} F_k(s_k) ds_k = 1 \quad (11) \]

then the current and the p.u.l. charge are obtained by integrating eqn (7) along \( l_k \).

With the position (7), the problem may be solved by separating the transverse and longitudinal behavior of the current and charge distributions. When the characteristic transverse dimensions of the conductors are electrically short and the interconnect is geometrically long, the transverse behavior is obtained by solving once for all a quasi-static 2D problem in the transverse plane. This assumption imposes the high-frequency validity limit for the ETL model.

Equation (10) may be written for every conductor, introducing the numerical vectors

\[ I(x) = \{I_k(x)\}_{k=1,\ldots,N} \quad \text{and} \quad Q(x) = \{Q_k(x)\}_{k=1,\ldots,N} \]

\[ \frac{dI_k(x)}{dx} + ioQ_k(x) = 0. \quad (12) \]

This is the first of the two governing equations for any transmission line model. In order to derive the second one, we must impose the boundary conditions. Assuming an ohmic behavior, on the surface of the \( k \)th conductor the boundary condition may be written as

\[ E(r) \times n \big|_{r \in S_k} = \tau_k \mathbf{J}_0(r) \times n \big|_{r \in S_k}, \quad (13) \]

This assumption will be removed in Section 4, when dealing with CNTs. In eqn (13) the surface impedance \( \tau_k \) takes into account the ohmic losses inside the conductor. For high-frequency operating conditions, for instance, it reduces to the well-known Leontovich expression

\[ \tau_k = \eta_k \frac{1 + i}{\delta_k}, \quad (14) \]

where \( \eta_k \) and \( \delta_k \) are, respectively, the conductivity and the penetration depth of the \( k \)th conductor.

Let us now focus on the relation between the voltage and p.u.l. magnetic flux. Let \( a_k \) indicates a characteristic dimension of the cross-section of the \( k \)th conductor and let \( a = \max_k(a_k) \): assuming operating conditions such that \( a \) is electrically small it is possible to approximate at any abscissa \( x \) the values of \( A(x,y,z) \) and
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\( \phi(x,y,z) \) on the surfaces \( S_1 \) and \( S_2 \) with their average values along the conductor cross-sections contours, say \( \hat{A}_k(x) \) and \( \hat{\phi}_k(x) \). As already pointed out, it is possible to define uniquely the voltage between any two pair of points lying on a plane \( x = \text{const} \). We may then introduce the *grounded mode* voltage of the \( k \)th conductor as follows:

\[
V_k(x) = \hat{\phi}_k(x) - \hat{\phi}_{N+1}(x). \tag{15}
\]

The p.u.l. magnetic flux linked to a closed loop connecting the \( k \)th conductor and the ground one in the plane \( x-z \) may be expressed as

\[
\Phi_k(x) = \hat{A}_k(x) - \hat{A}_{N+1}(x). \tag{16}
\]

Let us introduce the vectors \( \mathbf{V}(x) = \{V_k(x)\}_{k=1...N} \) and \( \mathbf{\Phi}(x) = \{\Phi_k(x)\}_{k=1...N} \); by using eqns (15) and (16) in eqn (13) it is easy to obtain

\[
-\frac{d\mathbf{V}(x)}{dx} = i\omega \mathbf{\Phi}(x) + Z_s(i\omega) \mathbf{I}(x), \tag{17}
\]

where \( Z_s(i\omega) \) is a diagonal matrix with \( Z_s^{kk}(i\omega) = \varsigma/\pi a_k \).

Equations (12) and (17) must be now augmented with the relation between the p.u.l. flux and the current and that between the voltage and the p.u.l. charge. In the above assumptions these relations may be obtained from eqns (5) and (6):

\[
\mathbf{\Phi}(x) = \mu_0 \int_0^l H_I(x-x') \mathbf{I}(x') dx', \tag{18}
\]

\[
\mathbf{V}(x) = \frac{1}{\epsilon_0} \int_0^l H_Q(x-x') \mathbf{Q}(x') dx'. \tag{19}
\]

These *constitutive* relations are spatial convolutions, hence their meaning is straightforward: in the general case the value of the p.u.l. magnetic flux (the voltage) at a given abscissa \( x \) depends on the whole distribution of the current intensity (p.u.l. electric charge) along the line. The kernels in eqns (18) and (19) are \( N \times N \) matrices whose entries are:

\[
H_{I}^{ik}(\zeta) = \frac{1}{c_i} \int ds_i \int ds'_k G_A(s_i,s'_k;\zeta) F_I(s'_k) ds'_k \tag{20}
\]

\[
H_{Q}^{ik}(\zeta) = \frac{1}{c_i} \int ds_i \int ds'_k G_p(s_i,s'_k;\zeta) F_Q(s'_k) ds'_k \tag{21}
\]

The system of equations (12) and (17)–(19) represents a *generalized transmission line model*: in the following we will refer to it as the ETL model. The 3D
full-wave problem has been recast in a transverse quasi-static 2D problem and a 1D propagation problem. The first problem is solved once for all and provides the source distributions \( F_k(s) \) along the conductor contours. The 1D propagation problem provides, instead, the distributions of voltages, currents, p.u.l. charge and magnetic flux along the line axis.

Letting the frequency go to zero and the interconnect length go to infinity, it is possible to prove that the kernels in eqns (18) and (19) tend to spatial Dirac pulses [21]:

\[
H_f(x-x') \rightarrow H_f \delta(x-x'), \quad H_Q(x-x') \rightarrow H_Q \delta(x-x').
\]

Hence eqns (18) and (19) reduce to local relations:

\[
\Phi(x) = \mu_0 H_f I(x), \quad V(x) = \frac{1}{i_0} H_Q Q(x),
\]

which along with eqns (12) and (17) provide the classical expression of the telegraphers’ equations in frequency domain

\[
-\frac{dV(x)}{dx} = Z(\omega)I(x), \quad -\frac{dI(x)}{dx} = Y(\omega)V(x),
\]

where the p.u.l. impedance and admittance matrices are given by:

\[
Z(\omega) = \mu_0 n H_f(\omega) + Z_n(\omega), \quad Y(\omega) = \mu_0 n H_Q^{-1}(\omega).
\]

For the ideal case of a lossless transmission line \( Z(\omega) = i\omega L, \ Y(\omega) = i\omega C \), where \( L \) and \( C \) are, respectively, the p.u.l. inductance and capacitance matrices.

This means that the ETL model (eqns (12) and (17)–(19)) contains the STL model (eqn (24)) as a particular case, obtained when the interconnect is enough long to neglect the effect of the finite length and the frequency is enough low to make the transverse dimensions electrically small.

It is worth noting that, as all the transmission line models, the STL model is based on the separation between a transverse quasi-static 2D problem and a 1D propagation problem. The difference with respect to the ETL model is in the fact that the transverse 2D problem, solved once for all, provides the p.u.l. parameters (eqn (25)), whereas, as for all the transmission line models, the distributions of voltages and currents are the solutions of a 1D propagation problem (eqn (24)).

3 Transmission line model for conventional conductors

3.1 A cylindrical pair

Let us study the simple case of a straight pair in the vacuum space, made by two cylindrical perfect conductors of radius \( a \). Let \( h_c \) be the center to center distance in the transverse plane (see Fig. 2a) and the total length. The example can be also
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used to analyze the case of a cylindrical conductor above a perfect ground plane. In vacuum the Green functions in eqns (4) and (5) reduce to the function

\[ G(r) = \frac{\exp(-ikr)}{4\pi r}, \]  

where \( r \) is the distance between the source and field points and \( k = \sqrt{\mu/\epsilon} \) is the propagation constant.

The static distribution of the sources along the conductor contours may be expressed in closed form as a function of the angle \( \theta \) (see Fig. 2a) [19]:

\[ F(\theta) = \frac{1}{2\pi a}\left(1 - \frac{a}{h_c} \sin \theta \right), \quad \theta \in [0, 2\pi]. \]

Figure 2b shows the behavior of \( F(\theta) \) for \( a = 1 \) mm and for different values of the ratio \( h_c/a \): for small values of \( h_c/a \) (say <10) this distribution differs significantly from the uniform case because of the proximity effect.

When considering widely separated conductors it results \( F(\theta) = 1/2\pi a \) and it is possible to give a closed-form expression to the kernel (eqns (20) and (21)), which may be split as the sum of a static and a dynamic term, \( H = H_{\text{stat}} + H_{\text{dyn}} \):

\[ H_{\text{stat}}(\zeta) = \frac{\kappa[m(\zeta)]}{\pi^2} \frac{1}{R_s(\zeta)} - \frac{1}{2\pi} \frac{1}{R_m(\zeta)}, \]

\[ H_{\text{dyn}}(\zeta) = -\frac{ik}{\pi} \exp\left(-\frac{ikR_m(\zeta)}{2}\right) \sin c\left[\frac{kR_m(\zeta)}{2}\right]. \]
Here $k(m)$ is the complete elliptic integral of the first type, and

$$m(\zeta) = \frac{\zeta^2}{(4a^2 + \zeta^2)}, \quad R_m(\zeta) = \sqrt{h^2 + \zeta^2}, \quad R_4(\zeta) = \sqrt{4a^2 + \zeta^2}. \quad (30)$$

The dynamic term depends on the frequency and vanishes as $\omega \to 0$. The static term is independent on frequency but shows a singularity of logarithmic type:

$$H_s(\zeta) = -\frac{1}{2\omega\pi^2} \ln(\zeta) \quad \text{for} \quad \zeta \to 0. \quad (31)$$

As already pointed out, if we consider infinitely long lines and assume frequency operating conditions such that $h_c/\lambda \ll 1$, $\lambda$ being the characteristic signal wavelength, $H(\zeta)$ reduces to a spatial Dirac pulse $H(\zeta) \to H_0\delta(\zeta)$, where

$$H_0 = \int_{-\infty}^{\infty} H(x) \, dx = \frac{1}{\pi} \ln \left( \frac{h}{a} \right). \quad (32)$$

In this case the cylindrical pair is described by the classical telegrapher’s equations for ideal two-conductor lines, namely by eqn (24) with $Z(\omega) = \omega\mu_0 H_0 = \omega L$ and $Y(\omega) = \omega e_0 / H_0 = \omega C$.

### 3.2 A coupled microstrip

A structure of great interest for high-speed electronic applications is the microstrip line: Fig. 3 shows a simple example of a three conductor microstrip, made by two signal conductors on a dielectric layer and a ground plane. Figure 3a shows the references for the voltages and currents (note that the grounded modes are considered).

From a qualitative point of view, the results highlighted in Section 3.1 still hold: the kernels (20) and (21) show a singularity of logarithmic type and the STL model may be obtained as a limit case of the generalized one.
The first difference is in the fact that the shape functions are no longer known in analytical form. However, they may be easily numerically computed by solving the electrostatic problem in the cross-section: for instance Fig. 4 shows the computed behavior of the shape function for the signal conductor of a single microstrip with $w_1 = 5$ mm, $t = 1.25$ mm, $h = 8.7$ mm and $\varepsilon_r = 4$. It is here evident the effect due to the sharp edges of the rectangular section.

A second difference is due to the influence of the dielectric. In this case the kernels (20) and (21) are different, since we have to consider two different Green functions in eqns (5) and (6). As already pointed out, the Green function involved in eqn (5) is in general dyadic. Since the layers properties are assumed to change only along $\hat{z}$ (see Fig. 1), $G_A$ has the structure

$$
G_A = \begin{bmatrix} G_{xx} & 0 & G_{zx} \\ 0 & G_{yy} & G_{zy} \\ G_{zx} & G_{zy} & G_{zz} \end{bmatrix}.
$$

(33)

In many practical applications the thickness of conductors $t$ is small compared to their width $w$. If we assume zero-thickness for the signal conductors, since the current density $\mathbf{J}$ directed along $\hat{x}$ we have the simple expression $G_A = G_{xx}$.

For the considered structure the Green functions may be evaluated in closed form in the spectral domain: let $\tilde{G}_{xx}(k_{\rho})$ and $\tilde{G}_{yy}(k_{\rho})$ be their transforms in such a
domain, where $k_r$ is the spectral domain variable. The spatial domain functions are obtained by evaluating the Sommerfeld integrals [32]:

$$G_{xx}(r) = \frac{1}{4\pi} \int_{-\infty}^{+\infty} \tilde{G}_{xx}(k_r) H_0^{(2)}(k_r r) k_r \, dk_r,$$

(34)

$$G_{y}(r) = \frac{1}{4\pi} \int_{-\infty}^{+\infty} \tilde{G}_{y}(k_r) H_0^{(2)}(k_r r) k_r \, dk_r,$$

(35)

where $H_0^{(2)}$ is the Hankel function. Such integrals are hard to compute practically, due to the slowly decaying and oscillating nature of the kernels. The cost for computing such integrals is extremely high because of the slow decay of the integrands.

A way to overcome this problem is to extract analytically the terms which are dominant in the low-frequency range, referred to as the quasi-static terms. For the single-layer microstrip structure of Fig. 3b they may be expressed as follows [33]:

$$G_{xx}^0(r) = \frac{e^{-ik_0\sqrt{x^2+y^2}} - e^{-ik_0\sqrt{x^2+y^2+(2h)^2}}}{4\pi\sqrt{x^2+y^2}},$$

(36)

$$G_{y}^0(r) = (1+K) \frac{e^{-ik_0\sqrt{x^2+y^2}} + (K^2 - 1) \sum_{n=1}^{\infty} K^{n-1} \frac{e^{-ik_0\sqrt{x^2+y^2+(2nh)^2}}}{4\pi\sqrt{x^2+y^2+(2nh)^2}},$$

(37)

where $K = (1 - \varepsilon)/(1 + \varepsilon)$ and $k_0$ is the vacuum space wavenumber.

Once these terms have been extracted, the remainders (dynamic terms) may be evaluated in an efficient way by approximating the corresponding expressions in the spectral domain [34]. The quasi-static terms are associated to the fundamental mode, are the only terms left when $f \to 0$ and dominate the local range interactions. The dynamic terms are associated to parasitic waves (surface waves, leaky waves), vanish as $f \to 0$ and dominate the long-range interactions.

Figure 5 gives an example of scalar potential Green function $G_{y}$ computed at 2.1 GHz for a single microstrip with $\varepsilon = 4.9$, $h = 0.7$ mm.

The quasi-static term dominates the near-field region, whereas for increasing distances the dynamic terms become the principal ones.

Unless very high frequencies are considered, in practical interconnects the quasi-static terms are dominant, hence the approximation of the remainder is usually satisfactorily pursued by a low-order model. A reliable criterion [35] states that the Green functions are accurately represented by the quasi-static terms when $k_0 h_{c}/\varepsilon_r - 1 < 0.1$.

4 Transmission line model for CNT interconnects

CNTs are allotropes of carbon that have been discovered fairly recently [23] and are considered as an alternative to conventional technology for future nanoelectronic...
applications such as transistors, antennas, filters and interconnects [24, 25]. Metallic CNTs have been suggested to replace copper in nano-interconnects, due to their unique electrical, mechanical and thermal properties [26–28]. Table 1 shows typical values for current density allowed, thermal conductivity and mean free path [28].

A single wall carbon nanotube (SWCNT) is a single sheet of a mono-atomic layer of graphite rolled-up (Fig. 6a). It possesses four valence electrons for each carbon atom: three of these form tight bonds with the neighboring atoms in the plane, whereas the fourth electron is free to move across the positive ion lattice. When the sheet is rolled up it may become either metallic or semiconducting, depending on the way it is rolled up.

To describe the electrodynamics of CNTs we need to model the interaction of the free electrons with the fixed positive ions and the electromagnetic field produced by the electrons themselves and the external sources. This requires, in principle, a quantum mechanical approach, because the electrical behavior of the electrons depends strongly on the interaction with the positive ion lattice. However, under suitable assumption the problem may be modeled by using a linearized fluid model to describe the dynamics of the effective conduction electrons, and by coupling the fluid equations to the Maxwell equations through the Lorentz force.
4.1 A fluid model for CNTs

We model a SWCNT as an infinitesimally thin cylinder shell with radius \( r_c \) and length \( l \). The graphene has valence electrons (\( \pi \)-electrons) whose dynamics depends on the electric field due to interactions with ions and other electrons (atomic field), with the other \( \pi \)-electrons (collective field) and with external fields. If the atomic field is much stronger than the collective and the external fields (the sum of these two is denoted with \( e(r, t) \)) and if \( e(r, t) \) varies slowly compared to the atomic time-space scale, the \( \pi \)-electron may be described as a quasi-classical particle: the dynamics is the same as for a classical particle with the same charge and an effective mass (which takes into account quantum effects) moving under the action of \( e(r, t) \).

In these conditions the conduction electrons (distributed on the cylinder surface \( S \)) may be described as an electron fluid with surface number density \( n(r, t) \), velocity \( V(r, t) = u(r, t) \hat{x} \) and 2D hydro-dynamical pressure \( p = p(r, t) \), of quantum nature [29]. We have assumed the velocity to be directed along the CNT axis \( \hat{x} \). Assuming small perturbations around equilibrium condition \( (n_0, p_0) \), i.e. expressing the conduction electron density and the pressure as \( n = n_0 + \delta n \) and \( p = p_0 + \delta p \), the interaction between \( e(r, t) \) and the electron fluid is assumed to be governed by the linearized Euler’s equation

\[
m_{\text{eff}} n_0 \frac{\partial u}{\partial t} = -\frac{\partial \delta p}{\partial x} + e n_0 e_x - m_{\text{eff}} n_0 \delta p_x u
\]  

(38)

Table 1: Properties of CNTs compared to copper.

<table>
<thead>
<tr>
<th>Property</th>
<th>CNT</th>
<th>Cu</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum current density [A/cm²]</td>
<td>~10¹⁰</td>
<td>~10⁶</td>
</tr>
<tr>
<td>Thermal conductivity [W/mK]</td>
<td>~6000</td>
<td>~400</td>
</tr>
<tr>
<td>Mean free path [nm]</td>
<td>~1000</td>
<td>~40</td>
</tr>
</tbody>
</table>

(a) (b) Figure 6: Schematic representation of a CNT (a); picture of a CNT bundle (b) [28].
where $m_{\text{eff}}$ and $e$ denote, respectively, the effective mass and the charge of the electron and $\nu$ is a parameter which accounts for the collisions. Equation (38) is augmented with a ‘state equation’ relating $\Delta p$ to $\Delta n$

$$\Delta p = m_{\text{eff}} c_s^2 \Delta n$$  \hspace{1cm} (39)

where $c_s$ is the thermodynamic speed of sound. The continuity condition imposes the following relation:

$$\frac{\partial (\Delta n)}{\partial t} = - \frac{\partial (n_0 \nu)}{\partial x}$$  \hspace{1cm} (40)

Introducing in eqns (38) and (40) the charge density $\sigma = -e \Delta n$ and the current density $j = -en_0 \nu$ on the surface $S$, we obtain the following system:

$$\frac{\partial j}{\partial t} + c_s^2 \frac{\partial \sigma}{\partial x} + \nu j = \frac{e^2 n_0}{m_{\text{eff}}} \varepsilon_s$$  \hspace{1cm} (41)

$$\frac{\partial \sigma}{\partial t} = - \frac{\partial j}{\partial x}$$  \hspace{1cm} (42)

To complete the fluid model, we have to fix the values of the parameters $n_0/m_{\text{eff}}$, $c_s$ and $\nu$. First of all, the equilibrium number density $n_0$ is determined by requiring that the longitudinal electric conductivity obtained from this model agreed with the expression obtained from a semi-classical transport theory for a sufficient small CNT radius [36]:

$$n_0 \equiv \frac{4 \nu_F}{\pi \hbar r_c}$$  \hspace{1cm} (43)

where $\hbar$ is the Planck constant and $\nu_F$ is the Fermi velocity. Next, $c_s$ is assumed to be equal to $\nu_F$ and finally for the collision frequency $\nu$ we use the expression

$$\nu = a \frac{\nu_F}{l_{\text{mfp}}}$$  \hspace{1cm} (44)

where $l_{\text{mfp}}$ is the mean free path and $a$ is a correction factor, which can be used as a ‘tuning’ factor able to take into account, for instance, the slight dependence of $l_{\text{mfp}}$ from the CNT radius [37].

### 4.2 A transmission line model for a SWCNT above a ground plane

Let us consider a SWCNT above a perfect conducting plane, as schematically represented in Fig. 7; $h_c$ is the distance between the axis nanotube and the plane. We assume the same operating conditions used for the general formulation introduced
in Section 3, hence the governing equations in the frequency domain are still given by eqns (12) and (17), which read for this case:

$$\frac{dI(x)}{dx} + i\omega Q(x) = 0, \quad -\frac{dV(x)}{dx} = i\omega \Phi(x) + E(x).$$

(45)

Note that in this case the longitudinal component of the electric field \(E(x)\) appearing in the RHS of the second of eqn (45) is not expressed through the simple ohmic relation as in eqn (17), but should be derived from eqn (41) assuming all the above mentioned conditions on the sources:

$$E(x) = vL_k I(x) + i\omega L_k I(x) + \frac{1}{C_q} \frac{dQ(x)}{dx}.$$  \hspace{1cm} (46)

where and \(L_k\) and \(C_q\) are, respectively, the kinetic inductance and the quantum capacitance, given by

$$L_k = \frac{h}{8e^2\nu_F}, \quad C_q = \frac{1}{L_k} e^2 = \frac{8e^2}{\hbar\nu_F}.$$  \hspace{1cm} (47)

The parameters and \(L_k\) and \(C_q\) derived here agree with those obtained in literature starting from different models (e.g. in [27], using a phenomenological approach based on Luttinger liquid theory).

Equations (45) and (46) must be augmented with the constitutive relations (18) and (19). Assuming a quasi-TEM approximation, in this case they reduce to the simple relations (23):

$$\Phi(x) = L_m I(x), \quad V(x) = \frac{Q(x)}{C_e}.$$  \hspace{1cm} (48)

where \(L_m\) and \(C_e\) are the classical p.u.l. magnetic inductance and electrical capacitance for a single wire above a ground plane:

$$L_m = \frac{\mu_0}{2\pi} \ln \left( \frac{2h_c}{r_c} \right), \quad C_e = \frac{2\pi\varepsilon_0}{\ln(2h_c/r_c)}.$$  \hspace{1cm} (49)
By using eqns (46) and (48) in eqn (45), we obtain that the interconnect is described by a simple lossy RLC transmission line model:

\[
- \frac{dV(x)}{dx} = (R + i\omega L)I(x), \quad - \frac{dI(x)}{dx} = i\omega CV(x),
\]

where the p.u.l. parameters are given by:

\[
L = \frac{L_m + L_k}{1 + C_e/C_q}, \quad C = C_e, \quad R = \frac{L_k}{1 + C_e/C_q}.
\]

As will be shown in the case-studies analyzed in Section 5, the behavior of this particular transmission line is strongly affected by the influence of the kinetic inductance and quantum capacitance. For instance, assuming \(v_p = 8.8 \times 10^5 \text{ m/s}, \ l_{mfp} = 1 \mu\text{m}, \) and \(h/r_c = 5\) we have \(L_k/L_m = 8 \times 10^3\) and \(C_e/C_q = 7 \times 10^{-2}\). The result on the inductances is quite insensitive to variation of the geometry of the line: the kinetic inductance always dominates over the magnetic one. As for the capacitance, if different dielectrics are considered the quantum capacitance may be comparable to the electrostatic one. As a consequence, the propagation speed and the lossless characteristic impedance

\[
c_{\text{CNT}} = \frac{1}{\sqrt{LC}}, \quad Z_{0\text{CNT}} = \sqrt{\frac{L}{C}}.
\]

may be well different from those theoretically obtained using the same geometry for the transmission line and replacing the CNT with a perfect conductor, say \(c_0\) and \(Z_0\). Typical values are \(c_{\text{CNT}}/c_0 = 10^{-2}\) and \(Z_{0\text{CNT}}/Z_0 = 10^2\).

As for the resistance, by using eqns (51) and (44) with the same parameters as above and with \(a = 1\), we obtain \(R = 3 \text{ k}\Omega/\text{km}\). The high values of this p.u.l. resistance and of the characteristic impedance in eqn (52) suggest using as interconnect stacks or bundles of CNTs rather than single CNT [31, 37–39].

In order to analyze multiconductor structures such as bundles, it is useful to extend the model to interconnects made by \(n\) CNTs over a ground plane. Following the same steps described above, the relation between voltage \(v(x,t) = [v_1(x,t), ..., v_n(x,t)]^T\) and current \(i(x,t) = [i_1(x,t), ..., i_n(x,t)]^T\) is given by the multiconductor transmission line equations

\[
- \frac{\partial v}{\partial x} = L \frac{\partial i}{\partial t} + Rk, \quad - \frac{\partial i}{\partial x} = C \frac{\partial v}{\partial t},
\]

where the p.u.l. parameter matrices are given by

\[
L = (I + C/C_q)^{-1}(L_m + L_k I), \quad C = C_q, \quad R = (I + C/C_q)^{-1}R_c,
\]

\(I\) being the identity matrix.
Finally, we have to remark that given the assumptions at its basis, the transmission line model introduced here describes the propagation in the low-bias voltage condition (corresponding to a longitudinal field less than 0.1 V/µm) and assuming \( l \geq l_{mfp} \). In high-bias condition this model should be modified with the insertion of a non-linear resistance [30].

## 5 Examples and applications

### 5.1 Finite length and proximity effect

A first simple application (Case 1) of the ETL model is the high-frequency analysis of a simple cylindrical pair as in Fig. 2, with \( a = 1 \) mm, \( h_c = 1 \) cm and total length \( l = 0.1 \) m. The conductors are ideal and the pair is in the vacuum space. Although simple, this example exhibits a lot of phenomena, which can be found also in more complex applications.

Figure 8 shows the spatial current distributions when the line is fed at the near-end and is left open at the far-end: \( I(x = 0) = 1 \) a.u. and \( I(x = l) = 0 \). The prediction of the ETL model is compared to those provided by the STL model and by a full-wave numerical solution obtained by means of Numerical Electromagnetics Code (NEC), a full-wave commercial simulator based on the method of moment technique [40]. The agreement between the ETL solution and the full-wave one is very satisfactorily. As expected, for \( kh_c > 0.1 \) the full-wave solution starts to deviate from the STL one: Fig. 8a refers to an operating frequency of \( f = 1 \) GHZ, which means \( kh_c \approx 0.21 \). For higher frequencies the STL solution is completely inadequate to describe the real full-wave solution, whereas the ETL model is still accurate. Figure 8b refers to \( f = 5 \) GHZ, which means \( kh_c \approx 1.05 \).

![Figure 8: Case 1, amplitude (in arbitrary units) of the current distribution for the mismatched case, computed at 1 GHz (a) and 5 GHz (b).](image)
To investigate the phenomena which are at the basis of such a behavior, it is useful to exploit the possibility given in eqns (28) and (29) to split the static and dynamic terms in the kernels. Let us consider the same conditions as above, except for the far end, which is now assumed to be matched (it is loaded by the characteristic impedance $Z_0 = \sqrt{LC} = 276.2\ \Omega$ of the STL case).

Figure 9 shows the STL solution, the ETL complete solution and the ETL solution due only to the static kernel. The main contribution to the difference is given, at low frequencies, by the static part $H_s$, while for high frequencies also the dynamic part $H_d$ provides a significant contribution. This means that, when entering the high-frequency range $kh_c > 0.1$, the first effect experienced by the solution is due to the finite length of the structure, whereas the effect due to unwanted radiation in the transverse plane starts acting for higher frequencies.

Finally, Fig. 10 shows the frequency behavior of the input impedance of the line (normalized to $Z_0 = 276\ \Omega$), when the far-end is left open.

The ETL model is able to predict the shift of the resonance frequencies toward lower values. Note that the shift to lower frequencies with respect to those of STL model means that the interconnect is electrically shorter than it actually is. Besides, the ETL model well predicts the amplitudes at the resonance frequencies that are finite and decreasing with increasing frequency, which is typical of a lossy line with frequency-dependent losses.

In very large-scale integration (VLSI) applications it is of great interest the study of the proximity effect, because of the short distances between the signal traces. Case 1 referred to a condition of widely separated wires, with $h_c/a = 10$. For such a condition the distribution of the sources along the wire contours may assumed to be uniform (see Fig. 2b). For a cylindrical pair, we may assume as a rule of thumb that the proximity effect should be considered for $h_j/a = 2.5$. Let us study again a wire pair, with $a = 2.5\ mm$, $h_c = 5.7\ mm$ and total length $l = 1\ m$

![Figure 9: Case 1, amplitude (in arbitrary units) of the current distribution for the matched case, computed at 1 GHz (a) and 5 GHz (b).](image-url)
(Case 2). The line is fed at one end by a voltage source of 1 V and is terminated on a short circuit at the other end. This case has been analyzed in [14], where a full-wave solution is provided by using the wire antenna theory. The proximity effect is there taken into account by introducing a set of ‘equivalent’ wires, whose artificial electrical axes are positioned so to satisfy the static problem in the transverse plane.

Figure 11 shows the current distribution at 1.2 GHz for this case, computed by means of ETL and STL models and compared to the quoted full-wave solution. An approximated ETL solution is also plotted, obtained by disregarding the proximity effect and hence assuming uniform distributions.

5.2 High-frequency losses

In high-speed integrated circuit technology losses play a crucial role in the overall system performance. With respect to a full-wave solution provided by brute-force numerical simulators, one of the most important advantages in using the ETL solution is the possibility to have a qualitative insight on the lossy phenomena affecting the high-frequency solution. We can distinguish at least three different lossy
mechanisms: (i) conductor losses; (ii) dielectric losses; (iii) excitation of parasitic modes (leaky waves, surface waves); (iv) radiation.

Let us consider the same pair of Case 1, assuming the conductors to be real, with a conductor resistivity \( \eta = 1.7 \times 10^{-8} \, \Omega \, m \) (Case 3). These losses are very sensitive to the frequency because of the skin-effect and this may be taken easily into account by using a suitable definition of surface impedance as in eqn (14). The line is fed by a unitary current source (arbitrary units) and is opened at the other end. We consider the frequencies \( 0.1 f_0 - 2.5 f_0 \) (\( f_0 = 1.5 \, \text{GHz} \)), corresponding to a range where the STL model fails. We have evaluated the difference between the values of the mean power absorbed at \( x = 0 \)

\[
P_{in}(\omega) = \frac{1}{2} \text{real} \{ V(\omega) I^* (\omega) \},
\]  

evaluated with ideal and real conductors. In the first case the ohmic losses are not considered, whereas in the second case they add to the radiation losses. Figure 12a shows the radiated mean power computed in these two conditions. In the low-frequency range the absorbed power is dominated by the ohmic losses whereas the radiation losses are more relevant in the high-frequency range. The ratio between ohmic and radiated mean power is plotted in Fig. 12b. The effect of a finite resistivity is relevant for frequency ranges where the STL model may be still used. For frequencies where the ETL model should be used, the losses are mainly due to radiation.

Figure 11: Case 2, amplitude of the current distribution computed at 1.2 GHz.
Let us now consider a printed circuit board microstrip, with the geometry of Fig. 3, assuming a single signal conductor above a ground plane and a length of 36 mm (Case 4). The signal conductor has zero thickness, width \( w_1 = 1.8 \text{ mm} \), and lies on a FR-4 dielectric layer of thickness \( h = 1.016 \text{ mm} \), dielectric constant \( \varepsilon_r = 4.9 \) and magnetic permeability \( \mu = \mu_0 \). The conductors and dielectric are assumed ideal.

The ETL model solution is compared to the STL one and to two 3D full-wave solutions, one provided by the commercial finite element method code HFSS [41] and the other by the tool SURFCODE, which is based on the electric field integral equation formulation [42]. Assuming for this case \( h_c = h \), since \( \varepsilon_r,_{\text{eff}} = 3.65 \) we have \( kh_c = 0.1 \) at 1.4 GHz, which is in agreement with the results shown in Fig. 13, where it is plotted the absolute value of the input impedance of the line with the far-end left open. Indeed, the results of all the models agree satisfactorily in the low-frequency range (Fig. 13a), whereas in the high-frequency range the full-wave solutions start to deviate significantly from the ideal STL solution.

As for the previous case-studies, since the conductors and the dielectric are assumed to be ideal, the finite amplitude of the peak is only due to the lossy effects related to the presence of unwanted parasitic modes (surface waves, leaky waves). In this condition a small but not negligible amount of power is associated with radiation in the transverse plane. Using eqn (55), the real power absorbed by the interconnect fed at one end by a sinusoidal current of r.m.s. value \( I_0 \) and left open at the other end is given by \( P_\text{in}(\omega) = \text{real}[Z_{\text{in}}(\omega)]I_0^2/2 \). Figure 14a shows the absorbed real power computed with \( I_0 = 1 \text{ mA} \). The ETL solution is in good agreement with the full-wave one around the peak, whereas there is a deviation in the other ranges (where, however the values of power are very low). Note that, since we are in the ideal case, the STL input impedance is strictly imaginary, hence the absorbed real power predicted by the STL model is always zero.

![Figure 12: Case 3, dissipated mean power in arbitrary units (a); ratio between ohmic and radiated mean power (b).](image-url)
Next, let us assume the dielectric to be real, i.e. let us introduce frequency-dependent dielectric losses by using, for instance, a simple Debye model [43]

\[ \varepsilon_r(\omega) = \varepsilon_r + \frac{\varepsilon_r - \varepsilon_{\infty}}{1 + i\omega\tau}, \]  

(56)

where \( \varepsilon_r \) and \( \varepsilon_{\infty} \) are, respectively, the low- and high-frequency limit, whereas \( \tau \) is a relaxation time constant. For the considered case, we assume \( \varepsilon_r = 4.178 \), \( \varepsilon_{\infty} = 4.07 \) and \( \tau = 1.15 \text{ ps} \).

Figure 14b shows the power dissipated in the high-frequency range assuming again \( I_0 = 1 \text{ mA} \) and comparing the real dielectric described by (56) to the ideal one with \( \varepsilon_r = 4.178 \). It is clear that in this case the dielectric losses are negligible with respect to the losses associated to the other high-frequency phenomena.
5.3 High-frequency crosstalk and mode-conversion

In VLSI applications the crosstalk noise and the differential to common mode conversion are unwanted phenomena, which may lower dramatically the performances. The crosstalk between adjacent traces may cause false signaling and is a serious bottleneck in the miniaturizing process for incoming scaled technologies. A correct evaluation of the common-mode currents is a crucial point in the analysis of systems like printed circuit boards, because of their remarkable effect on the overall electromagnetic interference performance. Although they may be even some order of magnitude lower than the differential mode currents, their effects may be comparable, for instance in terms of radiated emissions. Both phenomena may be analyzed by studying a simple three-conductor structure, like the one depicted in Fig. 3b.

Case 5 refers to a coupled microstrip in air (Fig. 3), with \( w_1 = 5 \text{ mm} \), \( w_2 = 10 \text{ mm} \), \( w = 2.5 \text{ mm} \), \( h = 8.7 \text{ mm} \), \( t = 1.25 \text{ mm} \), and a total length of 50 mm. For such a structure, we assume \( h_c = 9.35 \text{ mm} \), and investigate the frequency range 0.1–3 GHz, corresponding to \( kh_c \in (0.02 − 0.59) \).

The line is assumed to be in the free space and to be open at the far end: \( I_{12} = I_{22} = 0 \) (see Fig. 3a for references). Figure 15 shows the frequency behavior of the self and mutual terms of the input impedance, computed, respectively, as \( Z_{11} = V_{11}/I_{11} \) and \( Z_{21} = V_{21}/I_{11} \) when \( I_{21} = 0 \). The three models agree in the low-frequency range, up to 0.5 GHz, corresponding to \( kh_c = 0.1 \). For higher frequencies the full-wave and ETL solutions deviate from the STL one, capturing not only the frequency shift that has been already observed in the previous cases, but also the additional small peaks due to the resonance in the transverse plane. The impedance \( Z_{12} \) is an index for the crosstalk noise: it would be the near-end crosstalk voltage assuming \( I_{11} = 1 \text{ A} \) and all the other currents equal to zero. Figure 15b clearly shows that above 1.5 GHz the crosstalk noise level predicted by the STL model is well below the full-wave solution.

![Figure 15: Case 5, amplitude of self (a) and mutual (b) impedance at the near end.](image-url)
Let us analyze, for the same structure, the problem of the common mode excitation. Usually the common-mode currents are due to unwanted effects, as the presence of external fields, asymmetric conductor cross-sections and non-ideal behavior of the ground. In the differential signaling technique, however, a signal is defined as the difference between the signals of two conductors with respect to a third reference one and hence, due to the presence of the ground, a common-mode solution propagate. In order to study this 'mixed-mode' propagation it is convenient to introduce the common-mode variables: assuming the references as in Fig. 3a, the differential and common-mode variables are

\[ I_d(z) = \frac{I_1(z) - I_2(z)}{2}, \quad V_d(z) = V_1(z) - V_2(z), \]  

(57)

\[ I_c(z) = I_1(z) + I_2(z), \quad V_c(z) = \frac{V_1(z) + V_2(z)}{2}. \]  

(58)

In order to study the mode conversion, let us assume the line to be excited by a pure differential mode current at the near end, with the far end left open: \( I_{d1} = 1 \) (arbitrary units) and \( I_{c1} = 0 \). Figure 16 shows the distribution of the excited common mode currents computed at 1.7 and 2.5 GHz. For low frequencies the mode conversion due to asymmetric signal conductors may be neglected, whereas for frequencies above 1 GHz the excited common mode current starts to be relevant.

![Figure 16: Case 5, common-mode current distribution for a pure differential excitation.](image-url)
5.4 A comparison between CNT and copper interconnects for nanoelectronic applications

In future ultra-large-scale integrated circuits the use of copper nano-interconnects will be seriously limited by the strong degradation of its performances. The main challenge for Cu nano-interconnects is the trade-off between the request for increasing current density and the steep increase of the resistivity which, at nanometric dimensions, rises to values higher than its bulk value of $\rho = 1.7 \, \mu\Omega/\text{cm}$. Because of heating, this high value will limit the maximum allowed current density. For 45 nm node technology, the International Technology Roadmap for Semiconductors (ITRS) [44] foresees, for instance, a request of a current density in local vias of $8 \times 10^6 \, \text{A/cm}^2$, whereas the maximum allowed current density for copper will be about $4.5 \times 10^6 \, \text{A/cm}^2$. This limitation suggests considering the use of metallic CNTs, given their excellent electrical and thermal properties (see Table 1 in Section 4).

As first case-study, we consider the simple interconnect structure of Fig. 7, made by a single CNT of radius $r_c = 2.712 \, \text{nm}$ at a distance $h = 20 \, \text{nm}$ from a perfect ground, and compare its performances to those which would be in principle obtained by scaling the copper technology to the same dimensions (Case 6). For this case, we assume $l = 1 \, \mu\text{m}$ and $\nu = 3.33 \times 10^{11} \, \text{s}^{-1}$. To investigate the validity limits of the transmission line model equation (50), its predictions have been compared to those provided by a full-wave three-dimensional electromagnetic numerical model based on the same fluid description of the conduction [29]. Figure 17 shows the absolute value of the input admittance of the interconnect terminated on a short circuit: the transmission line model provides accurate results up to 1–2 THz.

![Figure 17: Case 6, absolute value of the input admittance.](image-url)
Let us first consider lossless interconnects. For the considered case using the definition in eqn (52), we have \( c_{\text{CNT}} \approx 0.0120 c_0 \), hence the electrical length of CNT interconnects is completely different from that of the copper one interconnect. Figure 18a shows the frequency behavior of the absolute value of the input impedance compared to that of the equivalent copper interconnect, assuming an interconnect length of \( l = 10 \mu m \). The CNT interconnect shows resonances at much lower frequencies. Resonances are extremely undesirable for interconnects, hence the above result seems to limit to short lengths (<1 \( \mu m \)) the possibility to use CNT interconnects in high-speed circuits. However, if we take into account the damping effect due to the huge p.u.l. resistance \( R \) predicted by eqn (51) this conclusion may change. For the considered case it is \( R \approx 1.16 \, \text{k}\Omega/\mu m \): it introduces a strong damping effect able to cancel out the resonance peaks, as shown in Fig. 18b, where the absolute value of the input impedance for CNT interconnect is computed both considering (real CNT) and disregarding (ideal CNT) the effect of \( R \). This result agrees with experimental evidence [16].

In order to compare the performances between CNT and Cu interconnects, it is useful to investigate the behavior of the scattering parameters. Figure 19a and b shows the absolute value of \( S_{11} \) and \( S_{12} \) computed for line lengths of 1 and 10 \( \mu m \), respectively. For Cu interconnect, we disregard the increase of copper resistivity, assuming a constant value of \( \rho_{\text{Cu}} = 1.7 \times 10^{-8} \Omega m \). The reference impedance for the definition of all the \( S \)-parameters is chosen equal to the lossless characteristic impedance of the CNT interconnect, \( Z_{0\text{CNT}} \approx 13 \, \text{k}\Omega \) for this case (this is the reason for the particular behavior of such parameters for the copper case).

As a conclusion, provided that it would be possible to load the line with such an impedance, it is clear that CNT interconnect are suitable for short and intermediate lengths, while they introduce a strong attenuation for longer lines. In addition, for high frequencies they seem to outperform the ideally scaled conventional technology.

![Figure 18: Case 6, absolute value of the input impedance for the ideal (a) and real (b) cases.](image-url)
The high values of characteristic impedance, the p.u.l. resistance and the presence of huge parasitic resistance due to imperfect metal-CNT contacts make impossible the use of a single CNT as an interconnect. A more realistic condition should consider bundles of CNTs and compare their performance with that of copper, taking into account the increase of copper resistivity too at nanometric scale. Case 7 will refer to a microstrip, where the signal trace is made by a bundle of CNTs (Fig. 20), compared to a Cu conductor with the same cross-section $tw$. For the dimensions and the values of the parameters, we refer to the indications proposed by the ITRS for the 45 nm technology (year 2010) [44]. Let us consider a 200 ($10 \times 20$) CNT bundle, assuming $r_c = 1.35 \text{ nm}$, $d = 2r_c$ (hence $w = 27 \text{ nm}$, $t = 2w$), $h = 2t$ and $\varepsilon_{r,\text{eff}} = 2.2$.

The propagation speed along CNTs is $3.2 \times 10^7 \text{ m/s}$, whereas for the Cu interconnect it is $2 \times 10^8 \text{ m/s}$. Note that at 30 GHz the wavelength is 1.6 mm for CNT and 10 mm for Cu interconnects: therefore up to lengths of 100 $\mu\text{m}$ (local and intermediate level) the interconnects are electrically short. The effects of propagation should be taken into account only for global level (order of mm).

![Figure 19: Case 6, absolute value of $S_{11}$ (a) and $S_{12}$ (b).](image)

![Figure 20: Case 7, the considered microstrip structure (a); realization of the signal trace with a CNT bundle (b).](image)
Let us refer to the simple signaling system depicted in Fig. 21, where $R_p$ is a generic parasitic resistance. As a consequence of the above considerations, the interconnect delay in this system is strongly dominated by the resistance. Let us compare the delay introduced by the CNT bundle to that produced by an equivalent Cu line, with resistivity $\rho = 4.08 \, \mu\Omega/cm$ [44]. Figure 22 shows the results obtained for an ideal case (ideal drivers and contacts, ideal load: $C_{load} = 0$) and for a real case (ideal drivers, $R_p = 100 \, k\Omega$ and $C_{load} = 0.01 \, pF$). The performances of the two interconnects are very close and an accurate control of the parasitic contact resistance for CNT bundles would lead to CNT delays comparable to the Cu ones. The result suggests considering CNT interconnects as possible alternative to Cu ones at least for local and intermediate level, since they provide similar delays but much better performances in terms of current density allowed, heating dissipation and mechanical properties.
6 Conclusions

In this chapter, the extension of the popular transmission line model to high-speed interconnects and to CNT nano-interconnects is discussed. Starting from a full-wave integral formulation, an ETL model is derived, able to describe interconnects with transverse dimensions comparable to the characteristic wavelength of the propagating signals. The model allows us to describe, with a computational cost typical of a transmission line model, the phenomena which are not included in the solution of the classical transmission line model but could be only taken into account by a full-wave solution. It is not only possible to obtain the correct behavior of high-speed interconnects in high-frequency ranges, but also to distinguish between the phenomena affecting the solution at such frequencies: finite size, radiation, mode conversion, frequency-dependent losses in conductors and dielectrics, excitation of parasitic modes.

Starting from a fluid model, a transmission line model is also derived to describe the propagation along interconnects made by metallic CNTs. Although simple, this model takes into account complex phenomena related to the quantistic behavior of such nanostructures, with a suitable definition of the transmission line model parameters. This tool is extremely useful to compare the performances of CNT interconnects and conventional ones for future nanoelectronic applications.

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References


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