CHAPTER 3

Surge propagation in multiconductor transmission lines below ground

Nelson Theethayi & Rajeev Thottappillil
Division for Electricity, Uppsala University, Uppsala, Sweden.

Abstract

Surge propagation in underground systems has been a subject of interest for many power and telecommunication engineers. Any typical buried electrical installation involves cables (power or telecommunication) and grounding systems. The cables could have multiple shields and multiconductor configuration for the core with either twisted or non-twisted conductors. The grounding conductor system could have counterpoise, rods, grids, etc. Interestingly, all these conductor systems could be modelled as multiconductor transmission line (MTL) systems for the wave propagation studies. In this chapter, we shall see how they can be modelled for transmission line analysis using the Telegrapher’s equations as discussed in Chapters 1 and 2. A brief discussion on crosstalk mechanisms will also be given, even though the analysis for crosstalk in MTL systems is similar to that in Chapter 2.

1 Introduction

Before we begin this chapter, it is good to visualize some practical problems that constitute a buried conductor system. In power and railway systems, we have cables used for bulk power transmission and signalling/telecommunication purposes. Every high-voltage power transmission tower has a long running counterpoise wire [1, 2] in the ground connected to the foot of the tower. The main purpose is to divert the lightning stroke current directly to the counterpoise wire. This is particularly important when the potential at the tower top or at the footing is of interest. Moreover, in the substation, there are complex grounding systems, which include counterpoise wires, buried rods and buried grids/meshes. To get a broader insight into the problem, let us take an example
of a communication system. Lightning strike to communication towers is a usual phenomenon [3]. A schematic diagram showing most of buried conductor systems connected to a communication tower in a communication tower complex is shown in Fig. 1 [3].

In Fig. 1, the tower foundation and building foundation are connected to their respective ring conductors. The ring conductors are provided primarily to minimize the danger of step voltage [4]. Besides, the ring conductor relieves the electrical stress at the tower foundation reducing chances of damage to the foundation. The earth conductor between the tower and building ring conductors prevents ground surface arcs between tower and building, and reduces lightning currents carried by cable shields by sharing a part of it. The follow-on earth conductor takes a part of the lightning current away from the service cables. The radial conductors around the tower are for reducing the percentage of lightning current dispatched to the building via the tower cables and earth conductors. The radial conductors are beneficial only if it can substantially reduce the lightning currents dispatched to the building, especially if the distance between the tower and building is long. The length and design of the radial conductors are to be governed by economy and its efficiency in dissipating lightning currents into the bulk earth. How do we analyse those buried conductor or grounding systems in the event when lightning or switching fault currents are diverted to them? This could benefit in the design of protection and grounding systems. This chapter will discuss as to how to use transmission line concepts to such problems but, of course, under the limits of transmission line approximations as discussed in Chapter 2.

Figure 1: Schematic diagram of grounding conductors considering the tower and building as separate units (not to scale), adapted from [3].
2 Telegrapher’s or transmission line equations for the buried wires

Underground wires can be either bare (counterpoise – representative of grounding conductors) or insulated (representative of cable shields or unshielded cables). We shall come to the analysis of coupling through shields later. Hence, one has two different systems for study as shown in Fig. 2.

In addition to ground conductivity and ground permittivity, there is also insulation permittivity for the insulated cables. The soil is characterized by its conductivity and permittivity. For insulated wires, we have insulation permittivity additionally. The corresponding per unit length transmission line representation of the above system is shown in Fig. 3 [5–9] and the relevant equations are shown below. For bare conductor,

\[
\frac{dV(x,j\omega)}{dx} = -Z_{gb} I(x,j\omega) \quad (1a)
\]

\[
\frac{dI(x,j\omega)}{dx} = -Y_{gb} V(x,j\omega) \quad (1b)
\]

For insulated conductors,

\[
\frac{dV(x,j\omega)}{dx} = -(j\omega L + Z_{gi}) I(x,j\omega) \quad (2a)
\]

\[
\frac{dI(x,j\omega)}{dx} = -j\omega \left( C Y_{gi} \right) V(x,j\omega) \quad (2b)
\]

\[
L = \frac{\mu_0}{2\pi} \ln \left( \frac{b}{a} \right) \quad (3a)
\]

Figure 2: Bare and insulated conductor systems in the soil under study.
The transmission line equations presented here can be extended to the multiconductor transmission line (MTL) system of several buried cables or grounding wires similar to the discussions in Chapter 2. In eqns (1) and (2), $V$ and $I$ are the voltage and currents, respectively. In eqn (1), $Z_{gb}$ and $Y_{gb}$ are the ground impedance and ground admittance of the bare conductor(s). In eqn (2), $Z_{gi}$ and $Y_{gi}$ are the ground impedance and admittance of the insulated conductor(s). Hence, in discussions to follow for ground impedance and ground admittance for buried wires, we define $Z_{gu}$ and $Y_{gu}$ for ground impedance and admittance, respectively, and the subscript ‘u’ is either ‘b’ or ‘i’, as the case may be, for bare or insulated wires. In eqn (2), $L$ and $C$ are the insulation inductance and capacitance, respectively [5], calculated using eqn (3). Lightning first return stroke or switching transients have frequency components ranging from a few tens to a few hundreds of kilohertz and subsequent lightning return strokes could have frequencies up to a few megahertz [10, 11]. For this reason, we shall investigate the ground impedance behaviour up to 10 MHz, beyond which the validity of transmission line approximation for buried wires could be questionable and will be discussed later.

The current wave equation for insulated wires can be written in a convenient form as in eqn (4) so that the total shunt element, i.e. series combination of insulation capacitance $j\omega C$ and $Y_{gi}$ in Fig. 3 can be transformed to a parallel combination of insulation capacitance and modified ground admittance, i.e. $j\omega C \parallel Y_{gi}^P$, which is comparable with above-ground wires’ transmission line equations as discussed in Chapter 2, and researchers like [12] have adopted this.

$$\frac{dI(x, j\omega)}{dx} = -(j\omega C + Y_{gi}^P) V(x, j\omega)$$ (4a)

$$Y_{gi}^P = \frac{-(j\omega C)^2}{j\omega C + Y_{gi}}$$ (4b)

Figure 3: Per unit length transmission line representation for bare and insulated conductor systems in the soil under study.
2.1 Ground impedance for buried wires

In the analysis below, let us assume the radius of the wire as $R_{ab}$. If the bare wire is in the analysis, then $R_{ab} = a$ else $R_{ab} = b$, the outer radius of the insulated wire, and the depth of the wire is $d$. Several researchers have contributed to the development of ground impedance expression for buried wires. The ground impedance expression for buried wires was developed first by Pollaczek [13] in 1926, as shown in eqn (5), which is a low-frequency approximation (similar to that in Chapter 2) in the sense that it can only be used when frequency of incident pulse satisfies $\omega \ll \sigma / \varepsilon$.

$$Z_{g}^{\text{Pollaczek}} = \frac{j \omega \mu_{0}}{2\pi} \left[ K_{0} \left( R_{ab} \sqrt{j \omega \mu_{0} \varepsilon_{g}} \right) - K_{0} \left( \sqrt{R_{ab}^{2} + 4d^{2}} \sqrt{j \omega \mu_{0} \varepsilon_{g}} \right) \right]$$

(5)

Because of the low-frequency approximation one can see that eqn (5) does not include the permittivity of the ground. All limitations associated with Carson’s ground impedance as explained in Chapter 2 for above-ground wires are applicable to Pollaczek’s ground impedance expression for buried wires as well. Saad et al. [14] have analytically showed that an excellent approximation for eqn (5) is given by eqn (6).

$$Z_{g}^{\text{Saad et al.}} = \frac{j \omega \mu_{0}}{2\pi} \left[ K_{0} \left( R_{ab} \sqrt{j \omega \mu_{0} \varepsilon_{g}} \right) + \frac{2}{4 + R_{ab}^{2} (j \omega \mu_{0} \varepsilon_{g})} \right]$$

(6)

Observe that if the infinite integral, e.g. in eqn (5), is eliminated, then those expressions without infinite integrals we shall here refer to as closed form approximations. Wedepohl and Wilcox [15] and Dommel [16, 17] (who presents expressions of A. Semlyen and A. Ametani) have proposed their own closed form ground impedance expressions, and Saad et al. [14] have shown that those expressions are more or less identical to eqn (6) when numerical calculations were compared.

For a wide range of frequencies, as long as the transmission line approximation is valid, Sunde [6] has derived the ground impedance expression as given by eqn (7). It is similar to eqn (5), but the only difference is that full propagation constant of the soil was used in eqn (7).

$$Z_{g}^{\text{Sunde}} = \frac{j \omega \mu_{0}}{2\pi} \left[ K_{0} \left( R_{ab} \gamma_{g} \right) - K_{0} \left( \sqrt{R_{ab}^{2} + 4d^{2}} \right) \right]$$

$$+ 2 \int_{0}^{\infty} \frac{e^{-2d\sqrt{u^{2} + \gamma_{g}^{2}}} \cos(uR_{ab})}{u + \sqrt{u^{2} + \gamma_{g}^{2}}} \, du$$

(7)
The integral term in eqn (7) converges slowly leading to longer computation times and possible truncation errors as the frequency is increased. Further, it was found that the first two Bessel terms in eqn (7) are oscillatory, when frequency is high. However, one can say Sunde’s expression for ground impedance is more valid than Pollaczek’s ground impedance expression in the sense that it uses the full expressions for propagation constant. Recently, Bridges [8], Wait [9] and Chen [18] have independently proposed more complex ground impedance expressions derived from rigorous electromagnetic theory, and in [8, 18] closed form logarithmic approximations for transmission line solution have been proposed. The authors found that even those expressions in appearance, though simple, are oscillatory (perhaps numerical convergence problems) at high frequency, hence not discussed here as it needs further investigation.

Bridges [8] mentions that his complete expression for ground impedance has two modes, namely transmission line modes and radiation/surface wave modes. Moreover, the solution of his ground impedance expressions is based on complex integration theory and adds that pole term in his expression corresponds to transmission line mode and the branch cut corresponds to radiation/surface wave modes [8]. Thus, under the transmission line modes, it can be shown that Bridges’s expression is identical to Sunde’s expression. Further, Wait [9] proposed a quasi-static approximation (under transmission line modes) and is given by eqn (8).

\[
Z_{_g}^{\text{Wait}} = \frac{j \omega \mu_0}{2\pi} (1 + \Theta) \ln \left( \frac{-j1.12}{\kappa R_{ab}} \right) \quad (8a)
\]

\[
\kappa = \sqrt{\varepsilon_0 \mu_0 \omega^2 - j \omega \mu_0 \sigma_g} \quad (8b)
\]

\[
\zeta = 2jkd \quad (8c)
\]

\[
\Theta = \frac{1}{K_0(jkR_{ab})} \left[ K_0(\zeta) + \frac{2}{\zeta} K_1(\zeta) - \frac{2}{\zeta^2} (1 + \zeta)e^{-\zeta} \right] \quad (8d)
\]

Vance [7] proposed one of the simplest closed form approximations for eqn (7) given by eqn (9), where Henkel functions are used instead of Bessel’s functions.

\[
Z_{_g}^{\text{Vance}} = \frac{\mu_0 \omega}{2\pi \gamma E_{\gamma E}} \frac{H_0^1(jR_{ab}/\gamma_E)}{H_1^1(jR_{ab}/\gamma_E)} \quad (9)
\]

Petrache et al. [12] have proposed a logarithmic approximation as shown in eqn (10) for the ground impedance and also claim that it’s the simplest expressions for the ground impedance.

\[
Z_{_g}^{\text{Log}} = \frac{j \omega \mu_0}{2\pi} \ln \left( \frac{1 + \gamma E R_{ab}}{\gamma E R_{ab}} \right) \quad (10)
\]

It can be seen in eqns (9) and (10) the depth of the wire is missing. Models that neglect air–earth interface [7] are known as infinite earth models. Wait [9] has
shown that this condition exists if \[ |2 j \delta \sqrt{\sigma_g \mu_0 \omega^2 - j \omega \sigma_g| >> 1. \] From this condition, it was found that neglecting the wire depth might not be a good approximation for higher frequencies typical for lightning. Moreover, it was found that for any combination of ground material property, the logarithmic approximation (10) and Vance’s approximation (9) for the ground impedance are identical.

Theethayi et al. [19] proposed a modified empirical logarithmic-exponential approximation (11), which is similar to eqn (9) but with an extra term accounting for wire depth taken from Saad et al. expression (6).

\[
Z^\text{Log-Exp}_g = \frac{jo \mu_0}{2\pi} \left( \ln \left( \frac{1 + \gamma_e R_{ab}}{\gamma_e R_{gab}} \right) + \frac{2}{4 + R_{gab}^2 \gamma_e^2} e^{-2d_{gab}} \right) \tag{11}
\]

Note that, in all the expressions above for the mutual impedance between the wires \( R_{ab} \) needs to be replaced by horizontal distance between the buried wires and \( d \) by average depth [15].

Figures 4 and 5 show the comparison of various ground impedance expressions and the deviations between expressions can be seen clearly both in amplitude and argument responses. The example is a wire of radius 2 cm and a depth of 0.5 m.
The consequence of missing depth term in logarithmic expression can be seen in the amplitude, and the consequence of low-frequency approximation in argument response. It can be seen that empirical logarithmic-exponential approximation has some agreement with either Sunde’s or Wait’s expression.

2.1.1 Asymptotic analysis

Now let us see the asymptotic behaviour of the ground impedance for buried wires as frequency tends to infinity (even though such an approach is questionable under transmission line limits). For using the time domain analysis, it is necessary to know the value of the ground impedance at \( t = 0 \). It is thus necessary to confirm if at all the ground impedance pose any singularity as the frequency tends to infinity. Similar to overhead wires, as discussed in Chapter 2, the ground impedance in time domain is referred to as transient ground impedance \([5, 10]\), given by inverse Fourier or Laplace transform as,

\[
Z_g(t; j\omega) = F^{-1}\left\{Z_g/j\omega\right\}.
\]

The value at \( t = 0 \) can be obtained by initial value theorem as

\[
\lim_{t \to 0} Z_g(t; j\omega) = \lim_{\omega \to \infty} Z_g.
\]

Vance’s expression (9) can be rewritten as eqn (12).

\[
Z_{\text{Vance}} = \frac{\mu_0 j\omega}{2\pi jR_{ab}r_g} \left( J_0(jR_{ab}r_g) + jV_0(jR_{ab}r_g) \right) = \frac{\mu_0 j\omega}{2\pi jR_{ab}r_g} \Gamma
\]
Simplifying eqn (12), we obtain eqn (13) and using the asymptotic expansions given in [20] it can be shown that \( \lim_{\omega \to \infty} \Gamma \to j. \)

\[
Z_{\text{Vance}}^{\text{Vance}} = \frac{\mu_0 j\omega}{2\pi j R_{ab}} \Gamma = \frac{\mu_0}{2\pi j} \sqrt{\frac{j\omega}{\mu_0 (\sigma_g + j\omega \varepsilon_g)}} \Gamma
\]

(13)

Thus, under the asymptotic conditions, it can be shown that eqn (13) tends to eqn (14) as frequency approaches infinity. The same is applicable with the logarithmic expression (10). Further, in the empirical expression, as frequency tends to infinity the exponential part approaches zero.

\[
\lim_{\omega \to \infty} Z_{\text{Vance}}^{\text{Vance}} = \left( \frac{1}{2\pi R_{ab}} \right)^{1/2} \frac{\mu_0}{\varepsilon_g}
\]

(14)

A similar expression to eqn (14) has been obtained in Chapter 2 for the overhead wires, the only difference was that in eqn (14) the radius of the conductor \( R_{ab} \) is to be replaced by the height of the conductor above ground. For wire of radius 2 cm at a depth of 0.5 m, the asymptotic value of ground impedance is 948 \( \Omega/m \). The important dimension here is the outer diameter of the conductor. Equation (14) is also applicable for a tubular conductor or for the shield of a cable in contact with the ground. Clearly, a singularity would appear if ground impedance expression corresponding to the low-frequency approximation was used.

### 2.2 Ground admittance for buried wires

Once we have the ground impedance, it is simple to get the ground admittance term. Vance [7] suggests that ground impedance and ground admittance are related to each other approximately by the propagation constant as in eqn (15). This approximation is valid in the sense that most of the currents return with in the soil because of which the ratio of electric to magnetic fields in the ground should be associated with intrinsic impedance of the soil as discussed in Chapter 2.

\[
Y_g = \frac{\varepsilon_g}{Z_g}
\]

(15)

For a wire of radius 2 cm at a depth of 0.5 m, Figs 6 and 7 show both ground impedance and admittance amplitudes and arguments, respectively. The ground admittance appears to be more sensitive to ground conductivity than ground impedance. This is unlike overhead wires where the ground impedance is very sensitive to ground conductivity, as discussed in Chapter 2. The interesting feature is that even though the ground impedance has asymptotic value at a very high frequency, the ground admittance monotonically increases and tends to infinity as the
frequency is increased, i.e. \( \lim_{\omega \to \infty} \sqrt{\frac{Z}{Y}} \). Thus it is clear that surge or characteristic impedance \( \sqrt{Z/Y} \) of the buried bare wire will tend to zero at a very high frequency, unlike overhead wires where it tends to a constant value. For buried insulated cables, the insulation permittivity additionally modifies the behaviour of total series impedance and shunt admittance of the wire. The modified ground admittance for insulated cables in eqn (4b) has an asymptotic value given by eqn (17) as the frequency tends to infinity even though the ground admittance of bare wire tends to infinity. The following arguments support this statement. Petrache et al. [12] have also obtained similar expression as eqn (17). A reason why the total shunt admittance in the current wave equation (2b) tends to infinity as the frequency tends to infinity is that because the ground admittance term \( Y_{gi} \) tends to infinity as frequency tends to infinity.

\[
y_{gi}^p = -\frac{C^2 Z_{gi}}{\frac{C Z_{gi}}{j\omega} + \frac{\mu_0\sigma_{gi}}{j\omega} + \mu_0\epsilon_{gi}}
\]  

\( (16) \)
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Transient simulation packages like Electromagnetic Transients Program (EMTP) [16, 21] ignores ground admittance and uses low-frequency approximation of ground impedance, i.e. neglects ground permittivity for underground cables. For bare wires, one cannot ignore ground admittance at all that is the reason why there are not any models for counterpoises etc., unlike line and cable models in EMTP [16, 21]. The inadequacies of low-frequency approximations for ground impedance have been discussed in the earlier section and also in Chapter 2. To illustrate the importance of ground admittance, let us consider two cases for an insulated wire corresponding to cable transmission line representation shown in Fig. 3. Case 1 is with ground admittance and Case 2 without ground admittance, i.e. \( Y_{gi} \to \infty \). The line propagation constant for Cases 1 and 2 are given by eqns (18) and (19), respectively in Laplace domain \( s \leftrightarrow jo \).

\[
\gamma_1 = \left[ (sL + Z_{gi}) \left( \frac{sC Y_{gi}}{sC + Y_{gi}} \right) \right]^{1/2} \quad (18)
\]

\[
\lim_{\omega \to \infty} Y_{gi}^P \to \left[ \frac{2\pi\varepsilon_{in}}{\ln(b/a)} \right]^2 \frac{1}{2\pi b\mu_0 \varepsilon_r} \sqrt{\frac{\mu_0}{\varepsilon_r}} \quad (17)
\]

Figure 7: Ground impedance and admittance argument response for various ground conductivities for a bare wire of radius 2 cm, buried at 0.5 m depth and \( \varepsilon_{rg} = 10 \).
\[ \gamma_2 = \left[ (sL + Z_g)sC \right]^{1/2} \]  

Observe that when \( Y_g \to \infty \), then \( \gamma_1 \to \gamma_2 \). The system that is being considered is an insulated cable buried at a depth of 0.5 m and having a radius of 2 cm and an insulation thickness of 2 mm. The ground conductivity is 1 mS/m, and the ground relative permittivity is 10, and the insulation relative permittivity is either 2 or 5. The ratio of attenuation factors for Cases 1 and 2, \( a_1/a_2 \), and the velocity ratio of propagation \( v_1/v_2 \) (note velocity is obtained by the ratio \( \omega/\beta \), where \( \omega \) is the angular frequency and \( \beta \) is the phase constant calculated independently, which further is the imaginary part of \( \gamma \) ) are shown in Figs 8 and 9, respectively.

It can be seen from Fig. 8 that the attenuation per meter for Cases 1 and 2 is same only at 100 Hz, and beyond that frequency, attenuation for Case 1 is a few times higher than Case 2. Situation is worse when the insulation permittivity is higher. Incorrect attenuation of currents at various points would predict incorrect rise times for propagating pulses.

![Figure 8: Attenuation ratio for Case 1 (\( Y_g \) included) and Case 2 (\( Y_g \) neglected) for a wire of radius 2 cm, insulation thickness of 2 mm and buried at 0.5 m depth, \( \varepsilon_{\text{rin}} = 2 \) or 5, \( \sigma_g = 1 \) mS/m and \( \varepsilon_{\text{rg}} = 10 \).](image-url)
Now consider the velocity ratio as shown in Fig. 9. The velocity of propagating waves is the same for Cases 1 and 2 only until 10 kHz. Beyond this point, again, the velocity ratio is increasing, that is underestimation of velocity when ground admittance is neglected. Situation is worse when the insulation permittivity is higher. Velocity of waves propagating in Case 2 is much slower than Case 1. The consequence of different velocities between Cases 1 and 2 will result in different time delays. Case 2 is certain to predict incorrect velocities. Changing velocity with frequency will cause dispersion in the propagating waves. For the above problem, Theethayi et al. [19] have shown the difference in the attenuations and velocities clearly with time domain simulations as well for a typical pulse propagation problem.

3 Possible limits of transmission line approximation for buried wires

The discussions until now are the analysis based on the transmission line theory. Some discussion on the limits of transmission line approximation was made in Chapter 2. A critical reader may wonder as to what the limit of transmission line approximation is for the buried system as well. For this, we shall use the same
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philosophies that were used for overhead wires, i.e. using the concept of penetration depth. Assume the wire is buried at a depth of \( d \) meters. The penetration depth at which the currents return in the soil can be calculated using the expression given in Chapter 2 and are shown in Fig. 10 (solid lines) for ground relative permittivity of 10. It should be remembered that the discussion here is applicable for infinite earth model. So if one considers the infinite depth models, the penetration depth is defined not from the surface of the ground but from the surface of the wire because of the axis-symmetric nature of the problem, unlike overhead wires where the depth was measured from the surface of the ground.

For a wire located at a certain depth in the soil, the currents have a return path only in the soil but at various depths depending upon the frequency of the pulse propagating in the wire. Generally, underground wire type systems in the soil will not be at a depth more than 1 or 2 m. As shown in Chapter 2, the penetration depth attains asymptotic value when the frequency is sufficiently high, which then becomes independent of frequency and is only dependant on the material properties of the soil. Further, the maximum velocity of the waves propagating in the soil is determined by its permittivity [7], i.e. \( v_{\text{max}} = 3 \times 10^8 \sqrt{e_{\text{rg}}} \). The velocity of the

![Figure 10: Penetration depth at which the currents return in the soil for buried wires and wavelength in the soil for \( e_{\text{rg}} = 10 \) and for various ground conductivities.](image-url)

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wave propagating in the soil is a function of frequency as given by eqn (20) and the wavelength in the soil at any frequency is given by \( \lambda_{\text{pulse}} = \frac{2\pi v_g}{\omega} \).

\[
v_g = \frac{1}{\sqrt{\frac{\mu_0 \rho_0}{2} \left( 1 + \left( \sigma_{\text{eo}} \omega \right)^2 + 1 \right)}}
\]

Similar to overhead wires, for underground wires, one can impose a condition that the currents should return at a plane measured from the surface of the wire in the soil at depths smaller than the wavelength in soil, i.e. \( \lambda_{\text{pulse}} \geq \delta_{\text{e}} \). The condition \( \geq \) is used instead of \( \gg \), because most of the return current is in the soil and the air-earth interface is neglected due to the infinite earth model. Figure 10 shows the crossover points where the penetration depth (solid lines) equals the wavelength (dashed lines) in the soil for ground relative permittivity of 10. Mathematically, this limit occurs at a frequency approximately given by eqn (21). This limit is possibly the limit for transmission line approximation, because if the incident pulse has frequencies beyond this limiting frequency, clearly, the transmission line approximation is questionable as discussed in Chapter 2 for overhead wires.

\[
f_{\text{TLlimit}} = \frac{\mu_0 \sigma_{\text{e}} \pi (3 \times 10^8)^2}{\sqrt{\varepsilon_{\text{eo}} (\varepsilon_{\text{eo}} + \mu_0 \varepsilon_{\text{e}} [2\pi (3 \times 10^8)]^2)}}
\]

### 4 Coupling to cable core through cable shields

Having seen how the external impedance and admittance for the buried insulated and bare wires influence the externally induced voltages and currents, we now move on to yet another interesting area of how induced voltages are developed in the inner core conductors within the cable, particularly the shielded cables. This is mainly attributed to the fact that fields due to currents in cable shields not only couples the external environment of the cable, i.e. shield–soil insulation medium and insulation–soil medium, but also to the internal conductors of the cable (core) due to shield imperfections. The word imperfections mean that the shield is not a perfect conductor and it could be leaky due to cavities/apertures as in braided shields [5, 7]. To understand the phenomenon of coupling to cable core, one needs to know which parameter of the shield contributes to the coupling phenomenon between the shield and internal conductors. One could have multiple cable shields. Sometimes the outermost one facing the soil is known as armour.

To describe the various parts of multiple shield cables and the twisted pair inner core conductors, we take the example of a typical telecommunication cable used by the Swedish Railways (Banverket) for its telecommunication and signalling applications, as shown in Fig. 11. The outermost metallic tubular/cylindrical conductor is called the armour and the inner metallic tubular/cylindrical conductor is called the shield. Electromagnetic compatibility and power engineers interchange
the words between armour and outermost shield. Perhaps, for them, armour serves as a mechanical protection for the cable as such. In the coming sections, let us investigate whether an MTL can be derived for multiple shield and multiple core conductors for the transient analysis based on transmission line analysis.

To understand the coupling phenomenon between the tubular/cylindrical conductors and the internal conductors, let us start with the concept of transfer impedance [5, 7, 22] and later develop expressions for those phenomena in terms of what are known as tube impedances, first introduced by Schelkunoff [22] and later applied by Wedepohl and Wilcox [15]. We extended the analysis of single core cables proposed in [15] to multicore cables, with a view to applying transient analysis of cables with complex internal conductor system. The analysis is based on tubular shields, because it is reasonable to represent the armour and shield of the telecommunication or power cables for frequencies up to several hundreds of kHz as a solid tube. The leakage effects, in authors’ opinion, due to tube apertures and imperfections are predominant only at high frequencies beyond 1 MHz. Under such circumstances, no generalized expressions exist for tube impedances with imperfections and they have to be determined either through experiments or extensive theory. A simple/approximate method for determining the capacitance matrix by bridge method is shown and is particularly useful for twisted pair cables applicable to most of the telecommunication cables.

Figure 11: A typical telecommunication cable used by the Swedish railways for signalling and telecommunication applications.
4.1 Generalized double shield three-core cable

The discussion presented here is a generalization for a multiconductor arrangement of the cable. By using the analogy presented here, one can apply the same to any practical arrangement of the cables. Let us take the following examples: (a) an RG-58 cable that has single core and braided shield – a discussion on the concept of transfer impedance [5, 7] and its experimental determination are presented; (b) three conductor power cables that have a similar shield as that of RG-58 cable. In the authors’ opinion, the analysis of coupling mechanism between the shield and inner conductor as proposed by Vance [7] is valid only for a particular terminal condition. In Vance’s method, when the inner conductor circuit does not carry any appreciable current, the coupling mechanism between the shield and the inner circuit, due to current in the shield, is represented by distributed series voltage sources in the transmission line formed by the inner circuit. This allows us to eliminate the shield circuit and simplify the problem only to a transmission line due to inner circuit. This, in general, is not true for any arbitrary terminal conditions at the near and far ends of the shield and inner conductor/circuit. On the other hand, the analysis presented in the discussions to follow is valid for any arbitrary terminal conditions on either the shield or inner core conductors of the cable.

4.1.1 Telegrapher’s equations for shielded cables

Consider a cable cut away view as shown in Fig. 12, which shows a three-core cable arrangement. It has two shields, solid tubular/cylindrical with annular cross-section. Let the core conductors carry currents $I_1$, $I_2$ and $I_3$. The currents through

![Figure 12: Generalized three-conductor cable arrangement for studying the coupling between the shields and internal conductors.](image)
the shield and the armour are marked as $I_4$ and $I_5$. In all the cases, let us assume that the external inductance and capacitance matrices are known with the shield as the reference for Fig. 12.

In eqn (22), the voltages $V_1$, $V_2$, $V_3$, $V_4$ and $V_5$ are written in differential form between inner conductors (cores) and the shield, between the shield and the armour and the armour to remote reference, respectively. These voltages are also referred to as loop voltages [15, 16]. The voltages like $V_{1\text{core}}$, $V_{2\text{core}}$, $V_{\text{shield}}$ and $V_{\text{armour}}$ are that of the conductors with respect to remote reference [15, 16]. These voltages are later useful for a transformation by which one can find voltages on any conductor with respect to any given/specified reference.

$$V_1 = V_{1\text{core}} - V_{\text{shield}}$$  \hspace{1cm} (22a)

$$V_2 = V_{2\text{core}} - V_{\text{shield}}$$  \hspace{1cm} (22b)

$$V_3 = V_{3\text{core}} - V_{\text{shield}}$$  \hspace{1cm} (22c)

$$V_4 = V_{\text{shield}} - V_{\text{armour}}$$  \hspace{1cm} (22d)

$$V_5 = V_{\text{armour}}$$  \hspace{1cm} (22e)

The voltage wave equations based on the loop voltages are given by eqn (23).

$$-\frac{dV_1}{dx} = Z_{11}I_1 + Z_{12}I_2 + Z_{13}I_3 + Z_{14}I_4 + Z_{15}I_5$$  \hspace{1cm} (23a)

$$-\frac{dV_2}{dx} = Z_{21}I_1 + Z_{22}I_2 + Z_{23}I_3 + Z_{24}I_4 + Z_{25}I_5$$  \hspace{1cm} (23b)

$$-\frac{dV_3}{dx} = Z_{31}I_1 + Z_{32}I_2 + Z_{33}I_3 + Z_{34}I_4 + Z_{35}I_5$$  \hspace{1cm} (23c)

$$-\frac{dV_4}{dx} = Z_{41}I_1 + Z_{42}I_2 + Z_{43}I_3 + Z_{44}I_4 + Z_{45}I_5$$  \hspace{1cm} (23d)

$$-\frac{dV_5}{dx} = Z_{51}I_1 + Z_{52}I_2 + Z_{53}I_3 + Z_{54}I_4 + Z_{55}I_5$$  \hspace{1cm} (23e)

Since loop voltages are adopted, the currents in eqn (23) are referred to as loop currents $I_1$, $I_2$, $I_3$, $I_4$ and $I_5$. These loop currents are related to the core currents $I_{1\text{core}}$, $I_{2\text{core}}$, $I_{3\text{core}}$ shield current $I_{\text{shield}}$ and armour current $I_{\text{armour}}$ as shown in eqn (24).

$$I_1 = I_{1\text{core}}$$  \hspace{1cm} (24a)

$$I_2 = I_{2\text{core}}$$  \hspace{1cm} (24b)

$$I_3 = I_{3\text{core}}$$  \hspace{1cm} (24c)
The relationship between the conductor currents and loop currents also helps in the transformation by which telegrapher’s equations become in the form similar to MTL systems, as discussed earlier. Now each parameter in the impedance matrix $Z'$ in eqn (23) is a combination of series loop impedances in terms of internal and external impedances. Similarly, each admittance term in $Y'$ in eqn (25) is a combination of external and mutual admittance terms forming the loop admittances. These are discussed in Section 4.1.2.

4.1.2 Transmission line impedance and admittance parameters for shielded cables

Our aim is to arrive at the MTL equations of the form eqn (26) after simplifying eqns (22) and (24) using eqns (25) and (23).

\[
\begin{align*}
-d\frac{d}{dx} &= \begin{bmatrix}
V_{\text{core}} \\
V_{\text{2core}} \\
V_{\text{3core}} \\
V_{\text{shield}} \\
V_{\text{armour}} \\
\end{bmatrix} = \begin{bmatrix}
I_{\text{core}} \\
I_{\text{2core}} \\
I_{\text{3core}} \\
I_{\text{shield}} \\
I_{\text{armour}} \\
\end{bmatrix} X \\
\end{align*}
\]

\[
\begin{align*}
-d\frac{d}{dx} &= \begin{bmatrix}
I_{\text{core}} \\
I_{\text{2core}} \\
I_{\text{3core}} \\
I_{\text{shield}} \\
I_{\text{armour}} \\
\end{bmatrix} = \begin{bmatrix}
V_{\text{core}} \\
V_{\text{2core}} \\
V_{\text{3core}} \\
V_{\text{shield}} \\
V_{\text{armour}} \\
\end{bmatrix} Y
\end{align*}
\]
The impedance parameters of eqn (23) are defined in eqn (27).

\[ Z'_{11} = Z_i + j\omega L_{11} + Z_{\text{Shield-in}} \]  
\[ (27a) \]
\[ Z'_{12} = Z'_{21} = j\omega L_{12} + Z_{\text{Shield-in}} \]  
\[ (27b) \]
\[ Z'_{22} = Z_i + j\omega L_{22} + Z_{\text{Shield-in}} \]  
\[ (27c) \]
\[ Z'_{13} = Z'_{31} = j\omega L_{13} + Z_{\text{Shield-in}} \]  
\[ (27d) \]
\[ Z'_{23} = Z'_{32} = j\omega L_{23} + Z_{\text{Shield-in}} \]  
\[ (27e) \]
\[ Z'_{33} = Z_i + j\omega L_{33} + Z_{\text{Shield-in}} \]  
\[ (27f) \]
\[ Z'_{14} = Z'_{24} = Z'_{44} = Z_{g} \]  
\[ (27g) \]
\[ Z'_{15} = Z'_{25} = Z'_{35} = Z'_{55} = 0 \]  
\[ (27h) \]
\[ Z_{15} = Z_{25} = Z_{35} = Z_{51} = Z_{52} = Z_{53} = 0 \]  
\[ (27i) \]
\[ Z_{35} = Z_{54} = -Z_{\text{Armour-mutual}} \]  
\[ (27j) \]
\[ Z_{55} = Z_{\text{Armour-out}} + Z_{\text{Armour-Earth-insulation}} + Z_{g} \]  
\[ (27k) \]

The admittance parameters of eqn (25) are given by eqn (28). Many of the mutual admittance terms are null as shown in eqn (28b), this assumption may not be valid at very high frequencies.

\[
\begin{bmatrix}
Y'_{11} & Y'_{12} & Y'_{13} \\
Y'_{21} & Y'_{22} & Y'_{23} \\
Y'_{31} & Y'_{32} & Y'_{33}
\end{bmatrix} = j\omega
\begin{bmatrix}
C_{11} & C_{12} & C_{13} \\
C_{21} & C_{22} & C_{23} \\
C_{31} & C_{32} & C_{33}
\end{bmatrix}
\]  
\[ (28a) \]
\[
\begin{align*}
Y'_{14} & = Y'_{15} = Y'_{24} = Y'_{25} = Y'_{34} = Y'_{35} = 0 \\
Y_{41} & = Y_{41} = Y_{42} = Y_{43} = Y_{43} = 0 \\
Y_{54} & = Y_{54} = 0
\end{align*}
\]  
\[ (28b) \]
\[ Y'_{44} = j\omega C_{\text{Sheath-Armour-insulation-capacitance}} \]  
\[ (28c) \]
\[ Y_{55} = j\omega C_{\text{Armour-Earth-insulation-capacitance}} \]  
\[ (28d) \]

In eqn (27), \( Z_i \) is the internal impedance of the conductors, which is a consequence of skin effect phenomena of the core conductors as discussed in Chapter 2. There are some impedance terms in addition to external inductance (discussed later) of the internal conductors in eqn (27). Those impedances, excluding the internal impedances, are the ones that contribute to the coupling between the shield and
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inner conductors and the shield and the armour. Let us discuss them in some detail in a while. Insulation capacitance and inductance calculations were explained earlier, i.e. using eqn (3). The ground impedance and ground admittance terms appear in eqns (27k) and (28d), which has been discussed in the earlier sections.

If \([L]_{3\times3}\) and \([C]_{3\times3}\) are the inductance and the capacitance matrix of the core conductors with respect to the shield, then for arbitrarily located untwisted conductors inside the shield as shown in Fig. 13 (adapted from Paul’s book [23]), those parameters for the core conductors are given by eqns (29) and (30), respectively, in the same notations as discussed by Paul [23].

\[
L_{GG,RR} = \frac{\mu_0}{2\pi} \ln \left( \frac{r_{SH}^2 - d_{G,R}^2}{r_{SH} r_{WG,G,R}} \right) \tag{29a}
\]

\[
L_{GR} = \frac{\mu_0}{2\pi} \ln \left( \frac{d_G}{r_{SH}} \sqrt{\left( \frac{d_G d_R}{d_G + d_R} \right)^2 + \frac{r_{SH}^2 - 2d_G d_R r_{SH}^2 \cos(\theta_{GR})}{\left( \frac{d_G d_R}{d_G + d_R} \right)^2 + \frac{r_{SH}^2 - 2d_G d_R \cos(\theta_{GR})}{\left( \frac{d_G d_R}{d_G + d_R} \right)^2}} \right) \tag{29b}
\]

For the capacitance matrix as explained in Chapter 2, we first estimate the potential coefficient matrix, then invert the potential coefficient matrix similar to the above-ground wires. Note that in eqns (30a) and (30b) the permittivity of the insulation medium is used.

Figure 13: Untwisted parallel conductor arrangement in the shield for MTL parameter estimation for multiconductor cables, adapted from [23].
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\[ P_{GG,RR}^{SR} = \frac{1}{2\pi e_0 i} \ln \left( \frac{r_{SH}^2 - d_{GR}^2}{r_{SH}^2 d_{GR}} \right) \]  
\[ P_{GR} = \frac{1}{2\pi e_0 i} \ln \left( \frac{d_{GR}^2}{r_{SH}^2} + \frac{r_{SH}^2}{4} \right) \]  
\[ C = [P]^{-1} \]

Schelkunoff [22] gives a very good discussion on the surface impedances of hollow solid cylindrical shells. Discussions in this section are applicable only for imperfect conductors. Consider a hollow conductor whose inner and outer radii are \( a \) and \( b \), respectively. The return of the coaxial path for the current may be provided either outside the given conductor or inside it or partly inside and partly outside. Let \( Z_{aa} \) be the surface impedance with internal return and \( Z_{bb} \) with that of external return. This situation has appears to have in effect, two transmission lines with distributed mutual impedance \( Z_{ab} \).

According to Schelkunoff, \( Z_{ab} \) is due to the mingling of two currents in the hollow conductor common to both lines; and since \( Z_{ab} \) is not the total mutual impedance between the two lines, \( Z_{ab} \) is called the transfer impedance from one surface of the conductor to the other [22].

Using magnetomotive intensities associated with the two currents, he derived the surface impedances as given by eqns (31) and (32). The analysis presented here are applicable only to tubular shields.

\[ Z_{ab} = \frac{1}{2\pi e_0 i} \ln \left[ \frac{I_0(a\sqrt{j\omega \mu})K_1(b\sqrt{j\omega \mu}) + I_1(b\sqrt{j\omega \mu})K_0(a\sqrt{j\omega \mu})}{I_1(a\sqrt{j\omega \mu})K_0(b\sqrt{j\omega \mu})} \right] \]  
\[ Z_{bb} = \frac{1}{2\pi e_0 i} \ln \left[ \frac{I_0(b\sqrt{j\omega \mu})K_1(a\sqrt{j\omega \mu}) + I_1(a\sqrt{j\omega \mu})K_0(b\sqrt{j\omega \mu})}{I_1(a\sqrt{j\omega \mu})K_0(b\sqrt{j\omega \mu})} \right] \]  
\[ A = \frac{I_1(b\sqrt{j\omega \mu})K_1(a\sqrt{j\omega \mu}) - I_1(a\sqrt{j\omega \mu})K_1(b\sqrt{j\omega \mu})}{I_1(a\sqrt{j\omega \mu})K_0(b\sqrt{j\omega \mu})} \]  
\[ Z_{ab} = Z_{ba} = \frac{1}{2\pi e_0 i} \frac{1}{\Lambda} \]

Two theorems proposed by Schelkunoff [22] with regard to tube impedances are the following.

Theorem 1: If the return path is wholly external (\( I_a = 0 \)) or wholly internal (\( I_b = 0 \)), the longitudinal electromotive force (voltage) on that surface which is the nearest to the return path equals the corresponding surface impedance (self) multiplied by
the total current flowing in the conductor and the longitudinal electromotive force (voltage) on the other surface equals the transfer impedance (mutual) multiplied by the total current.

**Theorem 2:** If the return path is partly external and partly internal then the total longitudinal electromotive force (voltage) on either side of the surface can be obtained by applying Theorem 1 with the principle of superposition.

Equations (31) and (32) give the tube impedances. In comparison with eqn (27), one can define eqn (31a) as the tube-in impedance of either shield or the armour (31b), as the tube-out impedance of either the shield or the armour, and eqn (32) as the tube-mutual impedance of either the shield or the armour.

Wedepohl and Wilcox [15] have given a very simple approximation as shown in eqns (33) and (34) for the tube impedances without any Bessel functions, and they are valid only if \[(b - a)/(b + a)\] < 1/8. This has been validated for various practical tubular cable shields by the authors and it was seen that approximations (33) and (34) are excellent ones to exact Schelkunoff equations (31) and (32), respectively.

\[
Z_{ab} = \frac{\sqrt{i\omega\mu}}{2\pi a} \coth \left( \sqrt{\frac{i\omega\mu}{2\pi a}} (b - a) \right) - \frac{1}{2\pi a(b + a)} \tag{33a}
\]

\[
Z_{ob} = \frac{\sqrt{i\omega\mu}}{2\pi b} \coth \left( \sqrt{\frac{i\omega\mu}{2\pi b}} (b - a) \right) - \frac{1}{2\pi b(a + b)} \tag{33b}
\]

\[
Z_{ab} = Z_{ba} = \frac{\sqrt{i\omega\mu}}{\pi(a + b)} \frac{1}{\sinh \left( \frac{1}{i\omega\mu} \right)} \tag{34}
\]

It is now shown how to obtain all the important impedance and admittance parameters that are needed to create the voltage and current wave equations (26a) and (26b).

**4.2 An example of RG-58 cable**

This cable, based on standard catalogues, has a single core and a braided copper shield. The capacitance of the core conductor with respect to the shield for the RG-58 cable is about 101 pF/m. This can be obtained using the formula (30) if the geometry of the cable is known. The inductance of this cable using the characteristic impedance of 50 Ω will be 0.255 µH/m. The shield is braided because of which the tube mutual impedance or the transfer impedance will not be the same as discussed in Section 4.1. Vance [7] mentions that there will be two components that contribute to the net transfer impedance. One is due to the diffusion of electromagnetic energy across the thickness of the shield and the other, due to
the penetration of the magnetic field through apertures of the braid. The diffusion part is identical to the case as if the shield were like a solid tube, similar to the tube mutual impedance as shown in Section 4.1. The penetration or the leakage part is usually represented as a leakage inductance term as in eqn (35).

\[ Z_{\text{Shield-mutual}} = Z_T = Z_d + j\omega M_{12} \]  

(35a)

\[ Z_d = r_{\text{dc}} \frac{2n_{\text{bw}} \sqrt{\mu \sigma}}{\sinh (2n_{\text{bw}} \sqrt{\mu \sigma})} \]  

(35b)

The values of \( Z_d \) and \( M_{12} \) are calculated from the shield geometries and the number of carrier wires on the braid and on the weave angles. The terms \( r_{\text{dc}} \) and \( r_{\text{bw}} \) are the dc resistance of the shield and radius of the carrier wire with which the shield is formed. Associated formulas can be found in [7].

Can transfer impedance be measured experimentally? Under the assumption of weak coupling between the inner circuit and the external circuit, it is possible to determine the transfer impedance exactly. Let us get back to our differential equation (23). For the RG-58, the equations reduce to eqns (36) and (37).

\[
\begin{bmatrix}
V_{\text{core}} \\
V_{\text{shield}}
\end{bmatrix} =
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix}
\begin{bmatrix}
I_{\text{core}} \\
I_{\text{shield}}
\end{bmatrix}
\]  

(36)

\[
Z_{11} = Z'_{11} + Z'_{12} + Z'_{21} + Z'_{22}
Z_{12} = Z_{21} = Z'_{12} = Z'_{21} + Z'_{22}
Z_{22} = Z'_{22}
\]  

(37)

Assume a 1 m length of the RG-58 cable, which is electrically small. At one end (near end marked as suffix N) let us short the inner conductor and shield, and inject a current source with respect to reference plane. At the other end (far end marked as suffix F), let us open circuit the inner conductor and short the shield to the reference plane. Thus the voltage at the near and far ends with respect to core and shield currents can be written as

\[
-(V_{\text{coreF}} - V_{\text{coreN}}) = Z_{11}I_{\text{core}} + Z_{12}I_{\text{shield}}
\]  

(38a)

\[
-(V_{\text{shieldF}} - V_{\text{shieldN}}) = Z_{21}I_{\text{core}} + Z_{22}I_{\text{shield}}
\]  

(38b)

The core current is zero, because of which we have,

\[
-(V_{\text{coreF}} - V_{\text{coreN}}) = Z_{12}I_{\text{shield}}
\]  

(39a)

\[
-(V_{\text{shieldF}} - V_{\text{shieldN}}) = Z_{22}I_{\text{shield}}
\]  

(39b)
Also $V_{\text{core}N}$ and $V_{\text{shield}N}$ are the same as they are shorted at the injection point, hence by subtracting the two equations from each other in eqn (39) we have

$$\left( V_{\text{core}F} - V_{\text{shield}F} \right) = (-Z_{12} + Z_{22})I_{\text{shield}}$$

(40a)

$$(-Z_{12} + Z_{22}) = Z'_{12} = Z'_{21} = Z_T = \frac{V_{\text{core}F} - V_{\text{shield}F}}{I_{\text{shield}}} = \frac{V_{\text{core}F}}{I_{\text{shield}}}$$

(40b)

This is the concept used in various transfer impedance measurement methods. Similar to transfer impedance, one can also define the transfer admittance. Experimentally the Triax set up as shown in Fig. 14 is used for the transfer impedance measurement [24]. There could be other more accurate or simple methods as discussed in [5]. To illustrate the Triax measurements, consider an RG-58 cable of 1 m length and make a simple Triax arrangement [24] as shown in Fig. 14. Note that one can make the radius of the tube smaller so as to be in contact with the outer insulation (not shown in Fig. 14) in order to get rid of the external inductance and capacitance between the tube wall and insulation. Now inject a step pulse current at the near end with the connection as shown in Fig. 14 and measure the open circuit voltage at the far end between the inner core and the shield. The ratio of the open circuit voltage and the shield current should give us the required tube mutual impedance or the transfer impedance.

Assume that for using eqn (35) for the RG-58 cable, the values needed (taken from [7]) are $M_{12} = 1 \text{ nH/m}$, $r_{dc} = 14 \text{ m\Omega/m}$, $r_{pe} = 63.5 \mu m$ and the copper conductivity and permeability is chosen. The transfer impedance as a function of frequency using eqn (35a) for the above values are shown in Fig. 15.

Note that one could also reproduce the curve using the equations corresponding to eqns (32) or (34), if the shield thickness or inner and outer radii are known. It is seen that if the shield was tubular of certain thickness, then the transfer
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impedance magnitude decays at high frequencies as shown in Fig. 15, when only the diffusion part is considered. However, if the shield was leaky or braided, then the transfer impedance first decreases at high frequency and then increases as shown in Fig. 15 with both diffusion and the leakage part considered. Tesche et al. [5] mention that the transfer impedance increases at high frequency either as \( \mu f \) or \( \mu \sqrt{f} \), perhaps dependant on the type of the shield. Usually the leakage part is difficult to calculate as it depends on number of carrier wires, their optical coverage, weave angle and, more importantly, they are functions of complete elliptic integrals of first and second kinds. There could be other types of shields that are not braided but similar to the ones shown in Fig. 11, for which no formula exists and controlled measurements can only give the transfer impedance. At low frequencies, the transfer impedance is dominated by only DC resistance of the shield. Similar to transfer impedance one could have transfer admittance, but it exists only if the shield is leaky and at high frequency. The transfer admittance is approximately given by eqn (41). In eqn (41), \( C_1 \) is the capacitance per unit length between the internal conductors and the shield, \( C_2 \) is the capacitance per unit length between shield and the external current return path and \( K \) is a function with complete elliptic integrals and carriers and weave angle, etc. More details can be

Figure 15: Transfer impedance for an RG-58 cable with braided shield, showing the influences of leakage and diffusion part.
found in [5, 7]. As a reasonable approximation, transfer admittance is usually neglected.

\[
Y_T = j\omega C_{12} \\
C_{12} = C_1 C_2 K
\] (41)

4.3 Influence of shield thickness in the coupling phenomena

Let us now consider as to what would be the influence of shield thickness on the transfer impedance characteristics. For simplicity, we shall consider the example of tubular shields with two thicknesses, namely, 127 and 254 µm, made of copper with an arbitrary outer radius of the shield as 1.65 mm. The expressions (32) or (34) can be used for transfer impedance. One would also get the same if the Traix experiments as explained earlier were made and the ratio of open circuit voltage and shield current were determined. The transfer impedances for the two shield thickness are shown in Fig. 16.

The higher the thickness, the less the transfer impedance, as can be clearly seen in Fig. 16. The transfer impedance in frequency domain shown in Fig. 16 is

Figure 16: Transfer impedance for two shield thicknesses assuming a shield to be solid tubular.
vector fitted [25] using ten poles and the time domain response is shown in Fig. 17. The value of transfer impedance in time domain is large, which is a result of convolution.

An interpretation can be provided to time domain ‘transfer impedance’ presented in Fig. 17. It takes some time for the shield current (magnetic field) to penetrate inside. The time, when the steady low value of transfer impedance is achieved in Fig. 17, is nearly the same as the time at which the steady value of the open circuit voltage is achieved in Fig. 18. Stern [26] has provided a time domain formula as a series expansion for expression (35b). The convergence of such a time domain formula depends on shield properties and shield dimensions. Another interesting feature of the transfer impedance is that it predicts the diffusion time [7] required for the development of the internal voltage. The diffusion time constant can be obtained from the shield thickness and shield material properties and is given by eqn (42).

\[ \tau_s = \mu \sigma (\text{shield thickness})^2 \]  

(42)

To demonstrate this, let us take the same example of transfer impedance as above. The internal open circuit voltages, which corresponding to the Triax setup, would have been the product of transfer impedance and shield current for two shield thicknesses.

Figure 17: Transfer impedance in time domain for two shield thicknesses assuming a shield to be solid tubular.
are shown in Fig. 18. Let us assume a step function for injected shield current. It can then be seen that the constant voltage is attained at about times corresponding to diffusion time of the shield and those times are marked in Fig. 18. Note that in generating the curves corresponding to Fig. 18, the frequency domain voltage obtained from the product of transfer impedance and step current in frequency domain were Vector fitted using ten poles. It is readily seen that, higher the thickness, the larger the diffusion time and the less the internal voltages due to low value of transfer impedance.

4.4 A simple measurement for estimating inductance and capacitance matrix elements for internal conductors of cables

Today, very sensitive, high precision and accurate measurement systems like AC bridges are commercially available for measurement of smaller values of capacitance and inductances. Further, there are voltage and current sensors or instruments for measuring small currents and voltages. In this section, we discuss some of the methods to evaluate through experiments, the inductance and capacitance matrix elements for the MTL arrangement of conductors in the cable with respect to the shield based on the method proposed in [5]. This is necessary because there
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could be situations where the medium between the core conductors and the shield is not homogenous and, at the same time, one could have twisted pair arrangement. Under such circumstances practical formulae as discussed earlier, i.e. using eqns (29) and (30), no longer applies.

4.4.1 MTL capacitance matrix estimation
Let there be a system of \( n \) conductors forming an MTL system inside the shield and let the length of the cable be \( L \). Let one of the ends of the cable be referred to as the near end and the other, as the far end. From the transmission line equations, the relationship between the current in the \( i \)th conductor and the voltages on the other conductors are related through admittances as follows.

\[
\frac{dI_i}{dx} = Y_{i1}V_1 + Y_{i2}V_2 + \cdots + Y_{in}V_n
\]  
(43)

For a source with line length much less than the wavelength, i.e., under weak coupling assumptions we can rewrite eqn (43) as

\[
I_{i}(0) - I_{i}(L) = [Y_{i1}V_1 + Y_{i2}V_2 + \cdots + Y_{in}V_n]L
\]  
(44)

In eqn (43), \( I_i(0) \) and \( I_i(L) \) represent the current at the near and far ends, respectively. Let all the lines at the far end be left open circuited. Then, for any conductor, the current \( I_i(L) = 0 \).

Now at the near end, say if we are interested in calculating the self-capacitance of \( k \)th conductor, \( C_{kk} \), then we do the following. Short all the conductors to the shield in the near end except for the \( k \)th conductor where a voltage source \( V_k \) is connected to the shield. This voltage source injects a current \( I_k \) on the \( k \)th conductor which can also be measured. Then based on eqn (44), since all the voltages are zero at the near end excepting on the \( k \)th conductor, we will have

\[
(0) = \frac{I_k}{C_{kk}V_k L}
\]  
(45)

If the voltage source frequency, measured current and applied voltage at the near end is known, then the self capacitance \( C_{kk} \) can be calculated.

Similarly, if we measure the short-circuit current in the \( j \)th conductor with the same voltage source \( V_k \) with respect to the shield at the near end then we will have eqn (46), from which the mutual capacitance \( C_{kj} \) or \( C_{jk} \) can be obtained.

\[
I_j(0) = \frac{I_j}{C_{kj}V_k L} = j\omega C_{kj}V_k L
\]  
(46)

4.4.2 MTL inductance matrix estimation
Similar to eqn (43), we can have the voltage and the currents in the conductor related through the impedance parameters as follows:

\[
\frac{dV_i}{dx} = Z_{i1}I_1 + Z_{i2}I_2 + \cdots + Z_{in}I_n
\]  
(47)
For a source with line length much less than the wavelength, i.e. under weak coupling assumptions, we can rewrite eqn (47) as

\[ V_i(0) - V_i(L) = (Z_1I_1 + Z_2I_2 + \cdots + Z_nI_n) L \]  

In eqn (43), \( V(0) \) and \( V(L) \) represent the voltage at the near and far ends, respectively. Let all the lines at the far end be short-circuited to shield. Then, for any conductor, the current \( V_i(L) = 0 \).

Now at the near end, say if we are interested in calculating the self-inductance of \( k \)th conductor, \( L_{kk} \), then we do the following. Open-circuit all the conductors in the near end except on the \( k \)th conductor, connect a voltage source \( V_k \) with respect to the shield, which injects a current \( I_k \) on the \( k \)th conductor and that is measured. Then based on eqn (48), since all the currents are zero at the near end excepting on the \( k \)th conductor, we will have

\[ V_k(0) = Z_{kk}I_kL = j\omega L_{kk}I_kL \]  

If the voltage source frequency, measured current and applied voltage at the near end is known, then the self-inductance \( L_{kk} \) can be calculated.

Similarly, if we measure the open circuit voltage in the \( j \)th conductor with the same voltage source \( V_k \) with respect to the shield injecting current \( I_k \) on the \( k \)th conductor at the near end, then we will have eqn (50), from which the mutual inductance either \( L_{jk} \) or \( L_{kj} \) can be obtained.

\[ V_j(0) = Z_{jk}I_kL = j\omega L_{jk}I_kL \]  

### 5 Some additional cases of ground impedance based on wire geometry

#### 5.1 Impedance with wires on the ground

Having known the expressions for ground impedance and admittance for wires above and below ground, it would be interesting to see what would be the expressions for the ground impedance for wires on the surface of the ground. This is needed because there are railway systems where cables are sometimes laid beside the tracks either on the ground or in cable trenches. This can be seen in some typical power and communication systems too. Therefore, it is necessary to know the ground impedance expressions for wires on the ground. Sunde [6] has given the expression for ground impedance for the wires on the ground as eqn (51).

\[ Z_{gsbi}^{\text{Sunde}} = \frac{j\omega h_0}{\pi_{\gamma_{\text{g}}}^2 L_{\text{ab}}} \left[ 1 - \gamma_{\text{g}}R_{\text{ab}}K_1(\gamma_{\text{g}}R_{\text{ab}}) \right] \]  

\[ \pi_{\gamma_{\text{g}}}^{\text{Sunde}} = \frac{J_{\gamma_{\text{g}}}h_0}{\pi_{\gamma_{\text{g}}}^2 L_{\text{ab}}} \]
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In the above expressions, the Bessel’s function is used. An expression for ground impedance for wires on the ground can be obtained by using the ground impedance expressions for wires below ground, i.e. using eqn (11) with depth $d$ approaching zero.

$$Z_{g_{sbn}}^{Log} = \frac{j \omega \mu_{0} c}{2\pi} \ln \left( \frac{1 + \gamma_g R_{ab}}{\gamma_g^e R_{ab}^{e}} \right) + \frac{2}{4 + R_{ab}^{e}} \right)$$

(52)

What is interesting to note is that there is not any exponential term in eqn (52). It was seen that the logarithmic term corresponds to ground impedance of wires buried at infinite depth in the soil [7]. The second term in eqn (52) modifies or corrects the infinite depth ground impedance model to obtain the ground impedance expression for the wires on the surface of the earth. Therefore, the authors refer eqn (52) as modified logarithmic formula. A comparison between the Sunde’s formula (using Bessel function) and the modified logarithmic formula is shown in Fig. 19. The example is a wire of radius 2 cm, ground conductivity is varied from 10 to 0.1 mS/m, and the ground relative permittivity is 10. It is seen

![Figure 19: Comparison between Sunde and modified-logarithmic formula for the ground impedance of wires on the surface of the earth.](image-url)
that eqn (52) is a good approximation for eqn (51) and does not involve Bessel function. It should be remembered that if there are many parallel wires on the ground surface, the mutual ground impedance expressions could be obtained by substituting in eqn (52) the radius of the wire with horizontal distance between the wires, the same analogy as was applicable for the wires below ground. The ground admittance can be obtained using the propagation constant and the relation (15).

5.2 Mutual impedance with one wire above ground and the other below the ground

The mutual ground impedance expressions between an overhead wire and buried wire can be derived from Sunde’s method [6] but it involves infinite integrals as shown in eqn (53). In eqn (53), \( d_{kl} \) is the horizontal distance between above ground and buried conductors. The height of the overhead wire is \( h \) and the wire depth is \( d \). The authors feel that simplified expression for eqn (53) in terms of logarithms or exponentials should be derived to avoid infinite integrals. It is a subject for future study.

\[
Z_{\text{gabbi}} = \frac{j \omega \mu_0}{\pi} \int_0^\infty \frac{e^{-\left(\frac{u-d+\sqrt{u^2+d^2}}{\sqrt{u^2+d^2}}\right)}}{u+\sqrt{u^2+d^2}} \cos(d_{kl}u)du
\]  

(53)

6 Some examples

We have seen various expressions for impedance and admittances for the bare and insulated wires. We have also seen how to model the coupling through cable shields. In this section, we take some examples, which will describe the solutions in time or frequency domain and also a practical example of how crosstalk phenomena can be minimized using shielded cables.

6.1 Time domain simulation of pulse propagation in bare and insulated wires

When large transient currents or voltages propagate in bare wires or insulated cables there will be non-linear arcing or breakdown mechanisms occurring within the soil or insulation medium. The mechanisms of soil ionization in grounding systems and insulation breakdown in cables are common during power system transients/faults. To include such non-linear effects only time domain solution of transmission line equations helps. An efficient way of solving numerically lossy transmission line equations in time domain is by using the finite difference time domain (FDTD) method with recursive convolutions as discussed in Chapter 2. The only difference is that a vector fitting [25] of ground admittance in addition to ground impedance is to be made. The discussions presented here can be found in
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Theethayi et al. [19] as well. Further, we will have constant term in addition to the exponential terms while fitting the ground impedance and admittance terms which has to be used in the FDTD equations as discussed in Chapter 2.

As discussed in the beginning of this chapter, the transmission line equations, either eqns (1) or (2), when transformed to time domain will appear as eqn (54).

\[
\frac{\partial V(x,t)}{\partial x} + \int_0^t \zeta(t-\tau) \frac{\partial H(x,\tau)}{\partial \tau} d\tau = 0 \tag{54a}
\]

\[
\frac{\partial H(x,t)}{\partial x} + \int_0^t \eta(t-\tau) \frac{\partial V(x,\tau)}{\partial \tau} d\tau = 0 \tag{54b}
\]

Note that in eqn (54), the terms \(\zeta(t)\) and \(\eta(t)\) are given by eqns (55a) and (55b), respectively.

\[
\zeta(t) = F^{-1}\left(\frac{Z(j\omega)}{j\omega}\right) \tag{55a}
\]

\[
\eta(t) = F^{-1}\left(\frac{Y(j\omega)}{j\omega}\right) \tag{55b}
\]

Let us assume that eqns (55) or (56), when fitted with Vector fitting, say, up to 10 MHz, will give eqns (57) and (58).

\[
\zeta(t) = Cz_0 \delta(t) + \sum_{i=1}^{n} Cz_i e^{-\alpha_i t} \tag{56a}
\]

\[
\eta(t) = Cy_0 \delta(t) + \sum_{i=1}^{n} Cy_i e^{-\beta_i t} \tag{56b}
\]

To demonstrate the final time domain expression, let us discuss considering only eqns (54a) and (56a). Substituting eqns (56a) into (54a), we have

\[
\frac{\partial V(x,t)}{\partial x} + \int_0^t \left[Cz_0 \delta(t-\tau) + \sum_{i=1}^{n} Cz_i e^{-\alpha_i (t-\tau)}\right] \frac{\partial H(x,\tau)}{\partial \tau} d\tau = 0 \tag{57}
\]

Let us split eqn (57) as

\[
\frac{\partial V(x,t)}{\partial x} + \int_0^t \left[Cz_0 \delta(t-\tau)\right] \frac{\partial H(x,\tau)}{\partial \tau} d\tau + \int_0^t \left[\sum_{i=1}^{n} Cz_i e^{-\alpha_i (t-\tau)}\right] \frac{\partial H(x,\tau)}{\partial \tau} d\tau = 0 \tag{58}
\]
In eqn (58), the second term with a delta function can be written as a convolution given by eqn (59).

\[
\int_0^t [Cz_0 \delta(t - \tau)] \frac{\partial I(x, \tau)}{\partial \tau} \, d\tau = [Cz_0 \delta(t - \tau)] \otimes \frac{\partial I(x, \tau)}{\partial \tau} \tag{59}
\]

\[
[Cz_0 \delta(t - \tau)] \otimes \frac{\partial I(x, \tau)}{\partial \tau} = F^{-1} [Cz_0 j\omega I(x, j\omega)]
\]

\[
F^{-1} [Cz_0 j\omega I(x, j\omega)] = Cz_0 \frac{\partial I(x, t)}{\partial t} \tag{60}
\]

From eqn (61), it is clear that \(Cz_0\) has dimensions similar to line series inductance and term \(C_y\) to shunt capacitance. Thus, the final set of transmission line equations in time domain for pulse propagation that needs to be solved is given by eqn (62).

\[
\frac{\partial V(x, t)}{\partial x} + Cz_0 \frac{\partial I(x, t)}{\partial t} + \left[ \sum_{i=1}^{n} Cz_i e^{-\alpha_i(t-\tau)} \right] \frac{\partial I(x, \tau)}{\partial \tau} \, d\tau = 0 \tag{62a}
\]

\[
\frac{\partial I(x, t)}{\partial x} + C_y \frac{\partial V(x, t)}{\partial t} + \left[ \sum_{i=1}^{n} Cz_i e^{-\beta_i(t-\tau)} \right] \frac{\partial V(x, \tau)}{\partial \tau} \, d\tau = 0 \tag{62b}
\]

It is interesting to see that eqn (62) appears similar to the expressions for overhead wires and the solutions to these equations using the FDTD method are straightforward, as discussed in Chapter 2, using the recursive convolution.

To demonstrate the differences one would obtain between the application of the FDTD method to eqn (62) and solutions using direct frequency domain solutions, as discussed in Chapter 2, we take an example of a bare wire 1 km long and buried at a depth of 0.5 m and having a radius of 7.5 mm. A current source having a peak of 1 A with the shape given by \(I_0(t) = 1.1274 (e^{(1 \times 10^9 t)} - e^{(4 \times 10^9 t)})\) is injected one end, and the voltage at the injection point, 100, 200, 300, 400 and 500 m from the injection point by two different methods are shown in Fig. 20 for a ground with \(\sigma_g = 1 \text{ mS/m}\) and \(\epsilon_{rg} = 10\). The differences between the two methods are nominal, which could be due to the numerical errors inherent in both the methods.

### 6.2 A practical crosstalk problem

In Chapter 2, we have seen the crosstalk mechanisms with the MTL conductor system in the presence of finitely conducting ground. Here, we shall consider crosstalk to shielded wires based on weak coupling analysis, i.e. the generator wire circuit currents (source) is not influenced by the current in the shielded cable.
Note that the analysis to be presented is applicable to electrically short lines as discussed in Chapter 2. Shielded cables are used to protect the signal wires from external interference and also to provide a return path or reference (e.g. the common coaxial cable type RG-58). First, we consider a perfect shield without any direct field penetration inside, and then find out how connecting the shield ends to the ground influence the crosstalk (Fig. 21). For simplicity, the ground is assumed to be perfectly conducting. The cable length is assumed to be electrically small so that the circuit analysis is valid. Let us now consider crosstalk to the shielded wire in the set up shown below. This situation is treated in detail by Paul [23].

First consider the capacitive coupling as discussed in Chapter 2. The shield is assumed to be perfect. Therefore, there is no ‘direct’ coupling between the generator wire and the receptor wire. The coupling capacitance between them is a series combination of $C_{GS}$ and $C_{RS}$ (Fig. 22)

$$C_{12} = \frac{C_{RS}C_{GS}}{C_{RS} + C_{GS}} \quad (63)$$

Figure 20: Calculated voltages using the transmission line solution and the FDTD method for a bare wire buried at a depth of 0.5 m, 1 km long and radius of 7.5 mm. The ground medium has $\sigma_{g} = 1$ mS/m and $\varepsilon_{rg} = 10$. 

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$$C_{12} = \frac{C_{RS}C_{GS}}{C_{RS} + C_{GS}} \quad (63)$$
The near end and far end crosstalk due to capacitive coupling is given by eqn (64).

\[
NF = + RRVV j C I RR w \tag{64}
\]

Where \( I_2 \) is approximately given by eqn (65) (assuming currents and voltages in the receptor and shield circuit do not influence the currents and voltages of the generator circuit, that is, weak coupling with the generator circuit).

\[
I_2 \approx \frac{V_s}{R_g + R_L} \tag{65}
\]

If the screen is connected to the ground at either end, the screen voltage becomes zero everywhere (low-frequency approximation) and the capacitor \( C_{RS} \) is shorted out. There is no capacitive crosstalk in this case, \( V_{F\,Cap} = V_{N\,Cap} = 0 \) when screen is connected to ground at one end or both ends.

Next, we will consider the inductive coupling. The disturbing current in the generator circuit produces a flux in the circuit formed by the shield and the ground plane. This flux induces a current in the shield which produces a flux opposing the original flux in the space between the shield and ground plane. That is, the flux linking with the circuit formed by the receptor wire and the ground plane is also reduced.
In electrically small cables, there can be a shield current flow only if both ends of the shield are connected to the ground. Therefore, to reduce inductive coupling, both ends of the shield are to be connected to the ground. The equivalent circuit for the inductive coupling of the shielded wire is given in Fig. 23. In Fig. 23,

- \( M_{GR} \) – Mutual inductance between the generator circuit and the receptor circuit.
- \( M_{RS} \) – Mutual inductance between the receptor circuit and the shield circuit.
- \( M_{GR} \) – Mutual inductance between the generator circuit and the shield circuit.
- \( R_s, L_s \) – Resistance and self inductance of the shield.

The direction of the disturbing current is from near end to the far end. It produces a voltage \( joM_{GS}I_2 \) in the shield and drives a current \( I_s \) in the direction shown. In the absence of the shield, the voltage induced by \( I_2 \) in the inner wire (receptor wire) would have been \( joM_{GR}I_2 \). The shield current \( I_s \) produces a flux that opposes the flux produced by \( I_2 \). Therefore, the induced voltage in the receptor wire would be reduced by an amount equal to \( joM_{RS}I_2 \). Note that the two voltage sources in the receptor wire oppose each other. In the absence of shield current, \( I_s \), the voltage \( joM_{GS}I_2 = 0 \). Shield current is given by eqn (66).

\[
I_s = \frac{joM_{GS}I_2}{R_s + joL_s}
\]

The near end inductive crosstalk voltage is given by eqn (67).

\[
V_{N}^{IN} = \frac{R_N}{R_N + R_f} \left( joM_{GR} - \frac{\omega^2 M_{GS}M_{RS}}{R_s + joL_s} \right) I_2
\]

Substituting eqn (66) in eqn (67), we have

\[
V_{N}^{IN} = \frac{R_N}{R_N + R_f} \left( \frac{joM_{GR}}{R_s + joL_s} - \frac{\omega^2 M_{GS}M_{RS}}{R_s + joL_s} I_2 \right)
\]

Figure 23: Equivalent circuit for inductive crosstalk to shielded cable.
Under weak coupling assumptions, the mutual flux linking the receptor circuit and the shield circuit is about the same as the mutual flux linking the generator circuit and the receptor wire circuit, leading to $M_{GR} \approx M_{GS}$. Similarly, the magnetic flux in the shield-ground circuit due to shield current is the same as that magnetic flux produced by an equal current in the receptor wire leading to $L_s \approx M_{RS}$. Applying these two conditions in eqn (68), we get

$$V_N^{IN} = \frac{R_N}{R_N + R_F} \frac{j\omega M_{GR} I_2}{R_s + j\omega L_s}$$

(69)

In a similar manner, we can find the crosstalk at the far end, as

$$V_F^{IN} = -\frac{R_F}{R_N + R_F} \frac{j\omega M_{GR} I_2}{R_s + j\omega L_s}$$

(70)

The effect of the shield appears as a multiplying factor (71) on the crosstalk without shield, where $\tau_s = L_s/R_s$ is the shield time constant.

$$SF = \frac{R_s}{R_s + j\omega L_s} = \frac{1}{1 + j\omega \tau_s}$$

(71)

Shielding Factor (SF) can be approximated as

$$SF = \begin{cases} 
1 & \text{for } \omega < 1/\tau_s \\
\frac{R_s}{j\omega L_s} & \text{for } \omega > 1/\tau_s
\end{cases}$$

(72)

For $\omega < 1/\tau_s$,

$$|V_N^{IN}| = \frac{R_N}{R_N + R_F} \frac{j\omega M_{GR} I_2}{R_s + j\omega L_s}$$

$$|V_F^{IN}| = -\frac{R_F}{R_N + R_F} \frac{j\omega M_{GR} I_2}{R_s + j\omega L_s}$$

(73)

That is, inductive crosstalk increases by 20 dB/decade increase in frequency up to about $f = 1/2 \pi \tau_s$, or $R_s = \omega L_s$.

For $\omega > 1/\tau_s$,

$$|V_N^{IN}| = \frac{R_N}{R_N + R_F} \frac{M_{GR} R_s}{L_s} I_2$$

$$|V_F^{IN}| = -\frac{R_F}{R_N + R_F} \frac{M_{GR} R_s}{L_s} I_2$$

(74)
Equation (74) is independent of frequency. That is, inductive crosstalk remains constant for $f > 1/2 \pi \tau$, where $\tau$ is the time constant.

Total crosstalk is the sum of capacitive and inductive crosstalk. If the shield is grounded, at least one end, the capacitive coupling contribution is zero, and the inductive coupling is reduced by the shield only if the shield is grounded at both ends and the frequency of the interfering signal is greater than the shield cut-off frequency.

7 Concluding remarks

In this chapter, we have seen how the transmission line theory can be extended to solve pulse propagation problems in buried conductors, bare as well as insulated. Two of the key concepts for including the effect of finitely conducting ground are the ground impedance and ground admittance. After a review of different expressions for these two quantities, we have presented a set of new simple expressions that can represent them. These two new expressions do not involve any complicated infinite integrals and hence computationally very efficient, at the same time retaining the accuracy. It was pointed out that ground admittance is very sensitive to ground conductivity, whereas for overhead conductors (chapter 2) it was ground impedance that was sensitive to ground conductivity. The popularly used transient simulation packages like EMTP/ATP ignores ground admittance and the pitfalls of using these programs for the analysis of underground systems have been pointed out. For the given conductivity and permittivity of ground, the frequency up to which transmission line methods could be used for buried systems without significant errors is derived. An MTL method for buried shielded cables is presented. This model is capable of calculating the voltages and current developed internal to the cable due to induced external currents in the shield under all possible terminal conditions (short circuit to open circuit). Numerical examples of pulse propagation in buried bare wires and cables have been presented in time domain and discussed.

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References


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