CHAPTER 4

Effects of creep on new masonry structures

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4.1 Introduction

Creep can affect structures in two ways: deformations typically increase and loads (stresses) can be redistributed among structural components and, within a member, the constituent materials \cite{1}. The effects of creep can be beneficial, neutral, or detrimental for a structure: beneficial, for example through the relief of stress concentrations, detrimental through increasing deformations. The latter can lead to a structure no longer meeting serviceability criteria. Stress redistribution can cause cracking, especially in cases where there is deterioration in strength over time due to environmental factors in that element of the structure which carries increasing load due to creep effects.

Sometimes the two effects work together. Creep buckling is one example. An initial lateral imperfection in a column subject to axial load, or an initial eccentric load, causes an initial lateral displacement of the column. Consequently, there are higher compressive stresses on the inner curvature than on the outer curvature of the column. The side of the column under the higher stress creeps more than the other side, under the lower stress. The creep strains result in increasing lateral displacement. In turn, the secondary moment (the axial load times the lateral displacement) increases, increasing the stress on the inner curvature. Creep increases with the higher stress, so the lateral displacement increases more and more rapidly with the end result being a buckling failure of the column. An initial applied load less than the Euler buckling load for the end constraints of the column can thus cause a buckling failure sometime after load application. For some materials, the stress to initiate such failure can be as low as 60\% of the failure stress in a monotonic
compression test. Binda et al. [2] indicate that for some types of masonry this number may be in the order of 45–50%.

Crack growth caused by creep in tension tests has been recognized as ‘damage’ in the same context as that caused by fatigue [3, 4]. Cracks also grow under compression-induced creep, parallel to the direction of compression. The cause is similar to that in creep buckling in that the presence of the crack disturbs the local stress field, with a higher than average compressive stress parallel to the edge of the crack. Increased creep there accentuates the bending component of the stress and deformation beside the crack, increasing the crack width, and thus the tensile stresses at the crack tip. The crack extends when the stress and energy conditions favor such growth [5]. Creep crack propagation and its cracking rate dependence on bond breakage at the fracture process zone were discussed and modeled [6–8]. Binda et al. [9] remark on the existence of such cracks in the civic tower of Pavia at least 20 years prior to its collapse, and the appearance of such cracks in many historic structures at various ages after construction [2].

It is now well established that masonry creeps. The pioneering work of Lenzner [10, 11] has led to the realization that creep can be expected with any masonry units [2, 12–15] perhaps with the exception of some dry stacked stonework. The potential effects of creep therefore need to be considered in new construction and in rehabilitation. In rehabilitation interventions in historic masonry structures, essentially a new structure has been created, in which one component has already undergone some creep and the new component has yet to creep. Some masonry codes of practice now recognize the effect of creep in terms of increased deformation. Such codes advise designers to use the effective modulus technique to estimate long-term displacements [16, 17]. This technique, however, ignores what may happen in the structure between the initial and long-term states.

4.2 The step-by-step in time approach to modeling time-dependent effects

We demonstrate here, using a simple step-by-step in time technique, that elements in a structure may see increasing, then decreasing, proportions of load over time: that by calculating only the initial and long-term states, the designer may miss a peak stress occurring at an intermediate time. We recognize that masonry is complex, multi-component material: an outer skin of brickwork or blockwork may be filled with grout or rubble masonry, and may contain reinforcing bars. Modern techniques of rehabilitation may involve use of fiber reinforced polymers (FRPs), some of which are known to creep [18]. Alternatively, the epoxy binding the FRP to the underlying masonry may relax over time under the shear stresses transferring load between the FRP and the masonry: such behavior would cause a redistribution of load between these two components.

The step-by-step in time method of creep analysis relies on the applicability of the principle of superposition. In relation to creep, the principle requires that for a material subjected to stresses at different times, the creep strain produced at any
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time due to a previously applied stress is independent of the effects of any other stresses applied before or after that particular stress. The creep response to a set of stresses applied at different times is thus the summation of the creep effects of each stress. We demonstrate the technique and the consequences for load redistribution and increasing deformations in structures through the examples of an axially loaded masonry column and a beam subjected to pure bending. We also demonstrate the effect of damage of masonry on stress redistribution. A wall subjected to both axial and bending loads could be analyzed similarly. The complexity of the analysis can be increased by including plasticity constitutive equations as in [15] or by adopting variable adaptive time-stepping when damage is considered, as recommended by [19]. At the end of the chapter we introduce the concept of using Artificial Neural Networks (ANNs) to predict creep effects, as they can be substantially more accurate than explicit equations that best fit with regression to experimental data.

4.3 Case 1: An axially loaded column

4.3.1 Creep model

Consider a concentric axial load $P$ applied to a symmetric column made of two materials A and B each symmetrically distributed about the column axis. Effects due to eccentricity can therefore be neglected. The two materials have different time-dependent properties. The two materials have cross-sectional areas: $A_A$ being that of material A and $A_B$ that of material B. Equilibrium requires:

$$A_A \sigma_A + A_B \sigma_B = P \quad (4.1)$$

where $P$ is the applied concentric axial load and $\sigma_A$ and $\sigma_B$ are the stresses in materials A and B, respectively.

Compatibility requires the axial strain $\varepsilon$ in the column and each respective material is the same

$$\varepsilon = \varepsilon_A = \varepsilon_B \quad (4.2)$$

Next we assume that each material creeps such that the creep can be expressed as a compliance function in time, where $D_A$ and $D_B$ are the compliances of materials A and B, respectively. This creep function (eqn (4.3)) is shown in Fig. 4.1.

$$\varepsilon_A(t) = D_A(t) \sigma_A = D_A(t) \sigma_A \left( 1 - e^{-t/\tau_A} \right) \quad (4.3)$$

The creep strain $\varepsilon_A(t)$ is obtained by simply multiplying the time-dependent creep coefficient, $(\phi_A(t))$, by the initial strain as in eqn (4.3). $\phi_A$ is the creep coefficient for infinite time. The time constant $\tau_A$ denotes the time when 63% of the creep has occurred. Material B is assumed to creep with a mathematical form
similar to material A, but with \( D_B, \phi_B(t) \) and \( \tau_B \) representing its compliance, creep coefficient, and the 63% creep time, respectively. Other formulations of the creep function as suggested by other researchers [e.g. 11, 12, 20, 21] can be employed in lieu of eqn (4.3) and will have a slight effect in the overall conclusion. Using the equilibrium and compatibility considerations above, the initial stresses in the two materials are:

\[
\sigma_A = \frac{D_B P}{A_b D_A + A_A D_B} \quad (4.4)
\]
\[
\sigma_B = \frac{D_A P}{A_b D_A + A_A D_B} \quad (4.5)
\]

Since both materials are stressed, both will want to creep. We therefore permit a small increment in time to occur from \( t = 0 \) to \( t = t_1 \). In this first time increment, material A will want to creep an amount of strain \( \Delta \varepsilon_{crA} \) as presented in eqn (4.6).

\[
\Delta \varepsilon_{crA} (1) = \sigma_A \phi_A D_A \left( 1 - e^{t_1} \right) \quad (4.6)
\]

Material B will also have a creep increment of similar form. However, the increments in creep strain will be different. Hence, compatibility will be violated. In order to restore compatibility, the material that wants to creep more will have its stress reduced by the one that wants to creep less, while the latter will have its strain increased by the former. Compatibility therefore requires:

\[
\Delta \varepsilon_{crA} (1) + \Delta \sigma_A (1) D_A = \Delta \varepsilon_{crB} (1) + \Delta \sigma_B (1) D_B \quad (4.7)
\]

where the \( \Delta \sigma \)s are the incremental changes in stress. Since there is no increase in the overall axial force, equilibrium requires:

\[
\Delta \sigma_A (1) A_A + \Delta \sigma_B (1) A_B = 0 \quad (4.8)
\]

Figure 4.1: The simple creep function.
The incremental stress changes are therefore

\[
\Delta \sigma_A (1) = \frac{-A_b (\Delta e_{\text{creep}} (1) - \Delta e_{\text{creep}} (0))}{A_b D_A + A_A D_B} 
\]

(4.9)

\[
\Delta \sigma_B (1) = \frac{A_b (\Delta e_{\text{creep}} (1) - \Delta e_{\text{creep}} (0))}{A_b D_A + A_A D_B} 
\]

(4.10)

The stress in A (and similarly in B) at the end of the first time step is therefore

\[
\sigma_A (1) = \sigma_A + \Delta \sigma_A (1) 
\]

(4.11)

The materials, however, want to continue to creep. We therefore invoke superposition since we have made the materials linear viscoelastic. The amount of creep strain that material A would like to creep in the second time step can therefore be expressed as:

\[
\Delta e_{\text{creep}} (2) = \sigma_A \phi_A D_A \left( e^{-\frac{\tau_1}{\tau_A}} - e^{-\frac{\tau_2}{\tau_A}} \right) 
\]

\[
+ \Delta \sigma_A (1) \phi_A D_A \left( 1 - e^{-\frac{(t_2 - t_1)}{\tau_A}} \right) 
\]

(4.12)

The specific creep (creep strain per unit stress) curves for the different stress increments are the same; they just start at each time step, as shown in Fig. 4.2. Assume \(c_1\) is the specific creep that material A would like to creep in time step 1 due to the initial stress. \(c_1\) is also the specific creep for the stress increment \(\Delta \sigma_A\) (4.1) between time \(t_1\) and \(t_2\), whereas the influence of the initial stress in that time interval will be \(c_2\). When multiplied by their respective stresses, the influences are simply added.

Equilibrium and compatibility can be enforced again and the third step considered. This leads to the formula for the \((n)\)th time step where \(n \geq 2\).

\[
\Delta e_{\text{creep}} (n) = \sigma_A \phi_A D_A \left( e^{-\frac{\tau_1}{\tau_A}} - e^{-\frac{\tau_n}{\tau_A}} \right) 
\]

\[
+ \sum_{i=2}^{n-1} \Delta \sigma_A (i-1) \phi_A D_A \left( e^{-\frac{(t_{i+1} - t_i)}{\tau_A}} - e^{-\frac{(t_{i+1} - t_{i+1})}{\tau_A}} \right) 
\]

(4.13)

with the stress in material A at the end of the \(n\)th time step being

\[
\sigma_A (n) = \sigma_A + \sum_{i=1}^{n} \Delta \sigma_A (i) 
\]

(4.14)

With a similar equation for material B, the total strain is

\[
e_A (n) = e_A + \sum_{i=1}^{n} \Delta e_{\text{creep}} (i) + D_A \sum_{i=1}^{n} \Delta \sigma_A (i) 
\]

(4.15)
Example

Consider, for example, a blockwork column (A) filled with grout (B). We consider the following values for the following parameters:

- $A_A = 0.6A_{total}$
- $A_B = 0.4A_{total}$
- $D_A = 1/15 \text{ GPa}^{-1}$
- $D_B = 1/22 \text{ GPa}^{-1}$
- $f_A = 5.0$
- $f_B = 2.5$
- $t_A = 500 \text{ days}$
- $t_B = 1000 \text{ days}$

The blockwork is modeled to creep more and in a relatively shorter time than the grout. The results in Fig. 4.3 show the stress changes in both the blockwork and the grout over time.

Based on their relative stiffnesses, the initial stress in the blockwork is 12.6 MPa while that in the grout is 18.5 MPa. As the blockwork wants to creep faster than the grout, the blockwork initially offloads to the grout. However, as the grout creeps more than the blockwork at later ages, the blockwork will be re-stressed and the grout stress will be reduced during this phase. The effective modulus method [1, 22] is applied to estimate the final stress in the blockwork using eqn (4.16):

$$\sigma_A(\infty) = \frac{D_B(1 + \phi_B)}{A_B D_A(1 + \phi_A) + A_A D_B(1 + \phi_B)}$$

The final stress in the grout can be evaluated similarly. Final stresses of 9.3 and 23.5 N/mm² are determined for the blockwork and grout, respectively. Similar stresses are attained using the step-by-step analysis (Fig. 4.3). However, a designer using the effective modulus method will only detect that creep of the two materials will result in the grout stress rising by 25%, while in reality the grout will be overstressed by 36% from the initial value during an intermediate time period. The model presented here does not include the possibility that the material in which the stress rises (here the grout) might be damaged (degraded) during the overloading period. Typically, as a quasi-brittle material, the grout might be cracked during the overloading, reducing its stiffness. The stress distribution would therefore be...
changed from what is shown here and the blockwork re-stressed to a level higher than shown in Fig. 4.3. The example also demonstrates that the initial and final stress states are not the extremes. Higher stresses occur at intermediate stages.

4.3.2 Effect of coupling creep and damage in concentrically loaded columns

Another interesting effect that is difficult to correlate is the coupling of creep and damage. As mentioned, models are being developed for tensile creep and fatigue [3, 4], but there are other mechanisms which can induce damage. External deterioration can begin on the surface of masonry from actions like freeze-thaw or weathering. Mirza [23] for example, discussed how the resistance of a member in the context of limit states design can decline with time after construction. Valuzzi et al. [24] and Bažant [25] presented methods to account for this coupling effect in finite element modeling of historical masonry. Various damage models are described in the literature [26–28]. We have chosen a simple model as our objective is to show what can happen, rather than to develop a model for a particular case or material. We consider the analysis above but we also introduce a
damage model to account for the effect of damage in one material of the column and couple it with creep. Damage is assumed to accumulate in the form

\[ DM(t) = \sum_{i=t_{\text{start}}}^{t} \left( \eta \right) \left( \frac{t - t_{\text{DM}}}{{\tau_{\text{DM}}}} \right)^n \]  

(4.17)

\( \tau_{\text{DM}} \) is the damage time constant which refers to the time where most damage would occur. The coefficients are taken here as \( \tau_{\text{D}} = 800 \), \( \eta = 0.3 \), and \( n = 10 \). DM(t) represents the level of damage accumulated from the time at which damage starts, \( t_{\text{Dstart}} \) to the time of evaluation \( t \). In the calculations here, damage is assumed to begin at 400 days. The rate of damage accumulation with this model is slow initially, but accelerates over time, as shown in Fig. 4.4. For the sake of showing the trends and effects, quite considerable damage is assumed to occur in a relatively short time in this example. Following [26] and [29], the modulus of the material changes over time with this model as

\[ E_A(t) = (1 - DM(t))E_A(t_{\text{Dstart}}) \]  

(4.18)

where \( E_A(t_{\text{Dstart}}) \) is the material stiffness at the time when the damage begins. When DM is zero, there is no damage and when DM equals 1, the material is unable to bear any load.

![Figure 4.4: The damage model showing the non-linear change of damage ratio with time. Here the blockwork will have a damage factor of about 0.33 after 1000 days with the damage starting at 400 days. Damage initially accumulates at a very low rate but then increases rapidly.](image-url)
The effect of combined creep and damage on the Young’s moduli of both materials is shown in Fig. 4.5, while the stresses variations with time are shown in Fig. 4.6. These stresses are different to those in Fig. 4.3 in that they begin to change quite rapidly at later ages, as the damage accumulates in the outer blockwork, causing redistribution to the stiffer, undamaged grout. It thus becomes possible that the grout could now begin to fail, leading to collapse of the whole structure in time. This problem has been analyzed further elsewhere [30], with consideration of several possible combinations of creep and damage.

4.3.3 Examining the effect of rehabilitation

Much effort has been spent on rehabilitating and strengthening structures, both of historic and of simply practical value. Fiber reinforced polymers have been used on various occasions as they offer distinct advantages over steel in terms of being light weight and thus adding little mass to a structure, and highly durable if protected from sunlight. Some FRPs creep while others do not, unless the stress is a
very high proportion of ultimate [18, 31]. Thus when an FRP is used to strengthen a structure, the longer term consequences should be evaluated. If the structure is historic, creep may well have substantially run its course for the current loading. However, if the FRP is prestressed or changes the stresses in the structure in some other way then creep will start anew. Essentially a new structure has been created and the consequences of time-dependent effects need to be evaluated. One potential effect that needs to be considered is the reduction in load carried by the FRP from flow of the bonding epoxy from the shear that transfers load between the FRP and the underlying substrate. For example, if an FRP strip is applied to a structure to help carry a dead load, or is applied pretensioned to counteract a dead load, then there is the potential for the bonding epoxy to flow [32]. The FRP strip offloads over time. The effect is demonstrated with the next problem. Rigid mechanical anchorage of the strip would be required to avoid the consequence shown.

4.4 Case 2: Composite structural element subject to bending

4.4.1 Development of model

Consider a reinforced concrete beam where a layer of external reinforcement is bonded to the bottom of the beam as shown in Fig. 4.7. The external reinforcement
is added to the model such that the model is general and the effect of any externally applied strengthening material can be considered. We consider the case of pure bending for simplicity. Prestressing or axial dead load can be included. Again, for simplicity, we have assumed there is no compression reinforcement.

Strain compatibility, with plane sections remaining plane, gives

\[ \varepsilon_i = \varepsilon_e \left( \frac{d-kd}{kd} \right) \quad (4.19) \]
\[ \varepsilon_s = \varepsilon_e \left( \frac{1-k}{k} \right) \quad (4.20) \]
\[ \varepsilon_t = \left( \frac{h-kd}{kd} \right) \varepsilon_e \quad (4.21) \]

where \( kd \) is the depth of the neutral axis. Force equilibrium requires

\[ T_i + T_s - C = 0 \quad (4.22) \]
\[ A_i \sigma_i + A_t \sigma_e - \frac{1}{2} bkd \sigma_e = 0 \quad (4.23) \]

Moment equilibrium requires

\[ T_i \left( d - \frac{kd}{3} \right) + T_i \left( h - \frac{kd}{3} \right) = M \quad (4.24) \]
\[ A_i \sigma_i \left( d - \frac{kd}{3} \right) = M - A_i \sigma_i \left( h - \frac{kd}{3} \right) \quad (4.25) \]
Solving the equations provides

$$A_c \sigma_c \left( \frac{E_c}{E} \right) \left( \frac{1-k}{k} \right) - \frac{1}{2} bkd \sigma_c = -A_i \sigma_i$$  \hspace{1cm} (4.26)

$$\sigma_i = \frac{A_i \sigma_i}{\left[ \frac{1}{2} bkd - A_i \left( \frac{E_c}{E} \right) \left( \frac{1-k}{k} \right) \right]}$$  \hspace{1cm} (4.27)

and from the moment equations

$$A_c d \left( \frac{3-k}{3} \right) \left( \frac{E_c}{E} \right) \left( \frac{1-k}{k} \right) \sigma_c = M - A_i \sigma_i \left( h - \frac{kd}{3} \right)$$  \hspace{1cm} (4.28)

Thus at $t = 0$ (no creep)

$$\sigma_i = \sigma_c \left( \frac{E_c}{E} \right) \left( h - \frac{kd}{kd} \right)$$  \hspace{1cm} (4.29)

$$\frac{\sigma_i}{\sigma_c} = \frac{h - kd}{kd} = \frac{A_i}{\left[ \frac{1}{2} bkd - A_i \left( \frac{E_c}{E} \right) \left( \frac{1-k}{k} \right) \right]}$$  \hspace{1cm} (4.30)

leaving the following equation to be solved for $k$:

$$\left( \frac{1}{2} E_c b d^2 \right) k^2 + d \left( A_i E_c + A_i E_c \right) k - \left( A_i E_c h + A_i E_i d \right) = 0$$  \hspace{1cm} (4.31)

Thus the initial stresses in the concrete, FRP, and steel are

$$\sigma_{ci} = \frac{M}{\nu}$$  \hspace{1cm} (4.32)

$$\nu = A_c d \left( \frac{3-k}{k} \right) \left( \frac{E_c}{E} \right) \left( \frac{1-k}{k} \right)$$

$$+ \left( h - \frac{kd}{3} \right) \left[ \frac{1}{2} bkd - A_i \left( \frac{E_c}{E} \right) \left( \frac{1-k}{k} \right) \right]$$  \hspace{1cm} (4.33)

$$\sigma_{ui} = \sigma_{ci} \left( \frac{E_c}{E} \right) \left( h - \frac{kd}{kd} \right)$$  \hspace{1cm} (4.34)

$$\sigma_{si} = \sigma_{ci} \left( \frac{1-k}{k} \right)$$  \hspace{1cm} (4.35)

Now, for $t > 0$, the concrete creeps and

$$E_i(t) = \frac{\varepsilon(t)}{\varepsilon(t)}$$  \hspace{1cm} (4.36)
Further, we need to solve for $k$ as a function of $\sigma_1$ as $\sigma_1$ will reduce with time

$$A_1d \left( \frac{3-k}{3} \right) \left( \frac{E_s}{E_s(t)} \right) \left( \frac{1-k}{k} \right) \left( \frac{A_1\sigma_1}{\frac{1}{2} b k d - A_1 \left( \frac{E_s}{E_s(t)} \right) \left( \frac{1-k}{k} \right)} \right)$$

$$= M - A_1 \sigma_1 \left( h - \frac{kd}{3} \right)$$

(4.37)

$$A_1 \sigma_1 \left[ \left( h - \frac{kd}{3} \right) \frac{1}{2} b k d - A_1 \left( \frac{E_s}{E_s(t)} \right) \left( \frac{1-k}{k} \right) \right]$$

$$= M \left[ \frac{1}{2} b k d - A_1 \left( \frac{E_s}{E_s(t)} \right) \left( \frac{1-k}{k} \right) \right]$$

(4.38)

$$(A_1 \sigma_1 d^2 b) k^3 + 3bd(M - A_1 \sigma_1 h) k^2 + 6A_1 \left( \frac{E_s}{E_s(t)} \right) (M - A_1 \sigma_1 (h - d)) - k$$

$$- 6A_1 \left( \frac{E_s}{E_s(t)} \right) (M - A_1 \sigma_1 (h - d)) = 0$$

(4.39)

We let the FRP stress decline with time as the epoxy creeps (flows) under the shear it is transmitting. We consider a simple classical formula to represent FRP stress relaxation due to binding matrix creep [33]. At the end of the first time step, the stress in the FRP is

$$\sigma_1'(1) = \sigma_0 \left( a + (1-a) e^{-\frac{h}{\Delta \varepsilon}} \right)$$

(4.40)

$$E_s'(1) = \frac{E_s}{\varepsilon(t_{i})}$$

(4.41)

Equation (4.39) is solved for $k'(1)$ and this value is substituted into eqn (4.27) for $\sigma_1''(1)$

$$\varepsilon_1'(1) = \frac{\sigma_1'(1)}{E_s'(1)}$$

(4.42)

$$\Delta \varepsilon_i = \varepsilon_1'(1) \left[ \frac{h - k'(1)d}{k'(1)d} \right]$$

(4.43)

$$\Delta \sigma_i(1) = \frac{E_s}{E_s'(1)} (\sigma_1'(1) - \sigma_{i_1}) \left( \frac{h - k'(1)d}{k'(1)d} \right)$$

(4.44)

$$\sigma_1''(1) = \sigma_1'(1) + \Delta \sigma_i(1)$$

(4.45)
Learning from Failure

With the increment in concrete stress, force and moment equilibrium are checked. If equilibrium is not obtained, we reiterate the calculation of $k$. After $k$ has been computed, $\sigma_c(1)$ is calculated from eqn (4.27)

$$\Delta \sigma_{c1} = \sigma_c(1) - \sigma_c$$

$$\Delta \sigma_{c1} = \sigma_c - \sigma_c(1)$$

for the $n$th step

$$\sigma'_n = \sigma_n + \left[ a + (1 - a) e^{-\frac{\Delta \sigma_c}{\gamma_c}} \right] \sum_{i=1}^{n-1} \left[ a + (1 - a) e^{-\frac{\Delta \sigma_c}{\gamma_c}} \right]$$

$$\varepsilon_n = \varepsilon_0 + \phi \left( 1 - e^{-\frac{\Delta \sigma_c}{\gamma_c}} \right) \sum_{i=1}^{n-1} \left[ a + (1 - a) e^{-\frac{\Delta \sigma_c}{\gamma_c}} \right]$$

$$E_c(n) = \frac{\varepsilon_n}{\varepsilon(n)} E_c$$

The cracked beam curvature at any section $j$ along the beam at any time $t$ denoted $\Psi_{c,j}(t)$ can be computed as

$$\Psi_{c,j}(t) = \frac{M_j}{E_c(\varepsilon) I_{c,j}(t)}$$

where $I_{c,j}(t)$ is the transformed cracked moment of inertia at the $j$th section at time $t$, computed as

$$I_{c,j}(t) = \frac{b^2 [k(t) d]^2}{3} + A \left( E_c(t) \right) \left[ d - k(t) d \right]^2$$

$$+ A \left( E_c(t) \right) \left[ h - k(t) d \right]^2$$

The uncracked beam curvature at any section along the beam at any time $t$ denoted $\Psi_{u,j}(t)$ can be computed as

$$\Psi_{u,j}(t) = \frac{M_j}{E_c(\varepsilon) I_{u,j}(t)}$$

Given the effect of tension-stiffening on beam deflection, an effective curvature can be determined by interpolating between the cracked and uncracked section curvatures using the moment-curvature approach recommended by the CEB-FIP [34] and Ghali and Favre [1] as

$$\Psi_{c,j}(t) = \xi \Psi_{c,j}(t) + (1 - \xi) \Psi_{u,j}(t)$$

where $\xi$ is the tension stiffening coefficient that can be determined using the CEB-FIP [34] method. Here the tension stiffening coefficient $\xi$ was determined such
that the error between the experimentally measured (Section 4.4.2) and model predicted mid-span deflections is a minimum. The mid-span deflection of the beam $\Delta_{\text{mid}}(t)$ can be predicted by integrating the curvature along the span ($S$) or estimated approximately by considering the geometrical relation using three sections at the ends and at mid-span as

$$
\Delta_{\text{mid}}(t) = \frac{S^2}{96} (\Psi_{\text{left}}(t) + 10\Psi_{\text{mid}}(t) + \Psi_{\text{right}}(t))
$$

where $\Psi_{\text{left}}$ and $\Psi_{\text{right}}$ can be assumed to be equal to zero as no end curvature due to restrained shrinkage is expected to occur and $\Psi_{\text{mid}}(t)$ is the mid-span curvature computed as in eqn (4.54).

### 4.4.2 Application to a beam

The model is now applied to reinforced concrete beams with the dimensions shown in Fig. 4.8 and loaded as shown in Fig. 4.9. The material and load properties, including the concrete creep are taken as listed in Table 4.1. Two beams were tested. Both beams have similar properties (cast from the same concrete batch, at the same time and cured in a similar way) one without FRP strengthening strips and one with FRP strengthening strips. The cross-section of the FRP strengthened beam is shown in Fig. 4.8.

![Figure 4.8: Cross-section dimensions of the concrete beam reinforced with one layer of steel reinforcement and externally applied FRP.](image)

![Figure 4.9: Schematic representation of the beam loading set-up.](image)
For the beam without FRP strengthening, the predicted increase in deflection at the center of the beam is compared with experimental data (presented in [35, 36]) in Fig. 4.10.

It can be observed in Fig. 4.10 that the model was capable of predicting the creep deflection of the beam fairly well at early and late times. It is worth mentioning this prediction is achieved using the creep properties listed in Table 4.1 and by adjusting the tension stiffening coefficient to reduce the model error. A tension stiffening coefficient $\zeta = 0.9$ is needed. This high coefficient indicates that the section will be very close to fully cracked in this case ($\zeta = 1.0$ indicates fully cracked section). In fact, the beam has numerous flexural cracks. Now, if we consider the beam strengthened with the FRP strips to have similar material properties, and assume that the only time-dependent effect is the concrete creep, we get the result

Table 4.1: Material properties of reinforced concrete beam.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete compressive strength (MPa)</td>
<td>34.3</td>
</tr>
<tr>
<td>Concrete initial modulus of elasticity (GPa)</td>
<td>21.1</td>
</tr>
<tr>
<td>Reinforcing steel modulus of elasticity (GPa)</td>
<td>200</td>
</tr>
<tr>
<td>CFRP modulus of elasticity (GPa)</td>
<td>165</td>
</tr>
<tr>
<td>Concrete creep coefficient $\phi(t, t_0)$</td>
<td>4.2</td>
</tr>
<tr>
<td>Concrete ultimate creep time $\tau_c$ (days)</td>
<td>2000</td>
</tr>
</tbody>
</table>

Figure 4.10: Predicted versus experimentally measured deflections of reinforced concrete beam including creep effect (Beam 1: no FRP is used).
shown in Fig. 4.11. It is obvious that the predicted mid-span deflection does not meet the measured mid-span deflection.

The significant difference between the measured and predicted deflections in Fig. 4.11 can be attributed to two factors: the tension-stiffening effect and the effect of the creep of the epoxy binding matrix. As installing the FRP strips increased the cracking capacity of the beam, a lower tension stiffening coefficient (representing a cracked section) is required compared to that used in computing the mid-span deflection of the unstrengthened beam. Changing the tension-stiffening coefficient in the case with FRP we obtain the curve shown in Fig. 4.12. It can be observed that predicted deflection in this curve is closely related to the experimentally measured one. The deflection predicted in Fig. 4.12 assumes no FRP binder creep has occurred ($a = 1.0$). If relaxation of stress in the FRP strips due to the creep in the epoxy binding the FRP strip to the concrete ($a = 0.9$ and $t = 600$ days) is included, we get Fig. 4.13. While higher creep coefficient of epoxy at the concrete–FRP interface might be assumed, recent experimental investigations showed that creep of epoxy at the concrete–FRP interface would result in little but fast stress loss in the FRP sheets [37]. Therefore the reduction of FRP stress in the analysis presented here due to creep of epoxy was limited to 90% of the original stress.

Figures 4.12 and 4.13 look very similar, but it is misleading to judge the effect of FRP binder creep from these time-deflection diagrams. This is because the section is close to uncracked and the influence of FRP stress-relaxation on the

Figure 4.11: Predicted versus experimentally measured deflections of reinforced concrete beam (Beam 2: FRP is used) including creep effect only. Tension stiffening coefficient similar to that of Beam 1 is used ($\zeta = 0.9$).
Learning from Failure

The influence of changing the FRP stress-relaxation coefficient from no relaxation (Case 1: \( a = 1.0 \)) to significant relaxation (Case 2: \( a = 0.9 \)) on the transformed inertia is shown in Fig. 4.14.

Figure 4.12: Predicted versus experimentally measured deflections of reinforced concrete beam (Beam 2: FRP is used) including creep effect of concrete and no FRP stress relaxation effect \((a = 1.0)\) and tension stiffening effect \((\xi = 0.27)\).

Figure 4.13: Predicted versus experimentally measured deflections of reinforced concrete beam (Beam 2: FRP is used) including creep of concrete effect, FRP stress relaxation effect \((a = 0.9\) and \(\tau = 600\) days) and tension stiffening effect \((\xi = 0.27)\).
The results shown in Fig. 4.14 demonstrate the fact that the transformed cracked section inertia reduces due to FRP stress relaxation. This reduction would result in a considerable change in the beam deflection if the beam was cracked, particularly if the beam were cracked prior to the application of the FRP strengthening strips. This change (although happening) did not affect the beam deflection in the case study because the beam was uncracked and thus its deflection is dominated by the uncracked section inertia. The step-by-step in time analysis also allows the change in the stress in the concrete top fibers, the reinforcing steel bars, and the FRP strips to be followed. Exemplar result for the change in the concrete stresses is shown in Fig. 4.15.

The stresses shown in the Fig. 4.15 are based on the FRP stress decreasing due to creep of the epoxy at the FRP–concrete interface. With the current model, we force this reduction in stress through our imposition of relaxation in the FRP. However, such a reduction is likely not to be correct over the full time domain if the concrete creeps extensively, as would occur if the concrete were young. Such creep will increase the curvature of the beam resulting in increasing stress in the FRP after the initial decrease. This issue represents a modeling challenge because the concrete and epoxy have different rates of creep. Creep in the epoxy occurs in a considerably shorter time compared with that of creep in concrete. Therefore, the analysis above can be used to model the beam deflection and to give a reasonable estimate of the stresses in the concrete but not in the steel or the FRP. The small
effect on the accuracy of the concrete stresses in this example is related to the relatively small change in the FRP stress over the entire analysis time due to stress relaxation (only 3%) given the geometry of the beam analyzed and the relaxation parameters considered \((a = 0.9 \text{ and } \tau = 600 \text{ days})\). If the creep of concrete is minimal, as would be the case in strengthening an old concrete beam under its own weight, then the FRP stress would be accurately estimated from the above analysis as shown in Fig. 4.16.
4.4.3 Masonry walls subject to axial load and bending

Masonry walls are typically subject to both axial load (example 1) and bending (example 2). Hence to solve for the effects of creep for such loading, it is necessary to combine the two examples above, invoking the need for equilibrium and compatibility in the process. The equations for this situation are being developed. The consequences are well known, in that out-of-plane deformation of the wall will increase, just as the central deflection of the concrete beam in example 2. The stress on the inside curve of the wall will increase while that on the outside will decrease. Two possible long-term consequences are that failure by cracking or crushing may develop on the inner curve of the wall, or the wall may buckle. The situation which should be carefully analyzed is one where a cracked wall, vault, or beam is strengthened in situ, and the load increased. The FRP is likely to off-load itself over time if not anchored mechanically (The FRP strips in the beam as above were anchored with U-shaped FRP sheets).

4.5 New mathematical approaches to modeling creep

The above analysis demonstrated the importance of considering time-dependent analysis in evaluating serviceability of masonry structures. It also showed the importance of predicting creep with a good accuracy. Here we present findings of recent research efforts to enhance the accuracy of predicting creep using means of artificial intelligence. The inspiration for ANNs came from the desire to produce artificial systems capable of performing sophisticated (or perhaps intelligent) computations that mimic the routine performance of the human brain. Artificial neural networks are networks of many simple processors (neurons) operating in parallel, each possibly having a small amount of local memory. Artificial neural networks resemble the brain in two respects: first, knowledge is acquired by the network through a learning process and second, the interneuron connection weights are used to store the knowledge [38, 39]. No closed-form solution for the problem is provided by ANNs. However, ANNs offer a complex, accurate solution based on a representative set of historical examples of the relationship [38]. The units (neurons) are connected by weighted channels which are adjusted on the basis of learning data. Artificial neural networks learn from examples (of known input/output sequences) and exhibit some capability for generalization beyond the training data. Artificial neural networks normally have great potential for parallelism, since the computations of the components are largely independent of each other [39, 40].

The creep deformations of masonry prisms subjected to different stress levels, representing approximately (12, 24, 36, and 48%) of the prisms’ compressive strength, respectively, and exposed to different environmental conditions [12] were used to develop and assess the ANN model. A series of unloaded prisms subjected to environmental conditions similar to their counterparts – loaded prisms – was also measured at the same time intervals to account for shrinkage and
thermal changes. Test results from fourteen testing groups were included in training of the network. A series of multi-layer ANNs for predicting creep performance of masonry structures was developed [41, 42].

The creep prediction neural network consists of an input layer, one hidden layer, and an output layer. The network utilizes a log-sigmoid transfer function and a linear output function. A backpropagation training algorithm was used as the learning rule for the network. A learning matrix including 47 training samples drawn from the 14 testing groups was used in training the network. The Levenberg-Marquardt training criterion [42] was utilized during the learning process of the network with a training goal of achieving a mean square error of 0.0002.

The network was then tested against groups of data that had not been used in training the network. The creep compliance was computed by the network and the output of the network was compared to the measured creep compliance. A comparison between the creep compliance prediction using ANN and a classical model based on regression analysis [12] is shown in Fig. 4.17. While the ANN predictions lie within 15% accuracy, regression analysis prediction lies within only 50% accuracy. Research investigations showed that ANN accuracy can be further enhanced by optimizing the network architecture [42] and by considering time-delaying effects in the model [43].

4.6 Discussion

The step-by-step in time technique demonstrated here allows stresses at intermediate stages to be calculated. However, creep in masonry is a function of the age at loading and the environment [4], so more complex analyses will be needed to simulate reality. The step-by-step method can be used with the specific creep as the input [44] and aging functions can be expressed in integral form [1, 21]. Shrinkage and thermal effects can also be included. However, the number of factors that affect
masonry creep (age of loading, environment, unit type, mortar type, and stress level) suggests that creep for a particular structure will be very difficult to predict. Few data are available, particularly, long-term data. In these circumstances, methods using means of artificial intelligence (e.g. ANNs) which can deal with high levels of stochastic variation may prove useful in predicting both the creep and the range of possible outcomes from time-dependent effects. It becomes obvious that using means of artificial intelligence in modeling creep have two advantages over classical models, first: it allows incorporating a large number of interdependent factors that affect creep without adding further complexity. Second, it provides a systematic approach for model improvement as new data become available.

4.7 Conclusions

The step-by-step in time analysis is a powerful tool to investigate the change of the stresses due to creep over a large time range. Stresses in a material can rise and fall due to the effects of creep (or fall then rise).

Although the effective modulus method using the final creep coefficient can accurately estimate the final stresses in the components of a composite material (e.g. masonry) due to creep, the method may not predict intermediate peak stresses accurately: one needs to know when they will occur.

Rehabilitation with FRPs may introduce additional creep mechanisms with undesirable effects. Particularly, FRP strips may unload a part of any additional dead load applied to a structure after ‘strengthening’. Adequate anchorage must be designed.

The use of the step-by-step time-dependent analysis in creep stress redistribution can be made more accurate by incorporating ANNs where all factors affecting creep deformations can be included in the modeling process.

References


