Chapter 21

Seismic hazard evaluation for buried pipes crossing unstable slopes

P. Casamichele, M. Maugeri & E. Motta
Department of Civil and Environmental Engineering,
Catania University, Italy

Abstract

Pipelines are often located in seismic unstable slopes where earthquake-induced movements may cause their collapse. Thus the analysis of the soil-pipe interaction plays a large role for the seismic hazard prevision. This paper deals with an analytical approach for the study of the interaction phenomena occurring between pipe and surrounding soil when pipelines cross unstable slopes. Some analytical solutions are presented to evaluate displacement, lateral deflection and bending moment distribution along the pipe. The Newmark method extended to a 3-D analysis was used to evaluate the permanent earthquake-induced displacements in slope due to ground motion. The aim is the prevention of unacceptable conditions for the pipeline and to avoid the pipe failure. Field and laboratory tests confirm that the soil shows a non-linear behaviour even under low loading: so a non-linear behaviour for the soil-pipe interaction should be considered to take into account the plastic response. The non-linear model used in the analysis and the results obtained, with particular reference to the influence of the soil modelling, are presented.

Keywords: seismic hazard, non-linear analysis, pipeline.

1 Introduction

Earthquake-induced movements in slopes may cause severe damage on the structures that are located there. When a pipeline located inside an unstable slope experiences large movement due to a landslide, additional strains and stresses
can compromise the integrity and safety of the structure. Seismic response of buried pipelines depends on various factors, such as the direction of the ground movement, the entity of the earthquake-induced slope motion, and the strength and stiffness of the pipe. The movement-induced stress on the pipe can be linked to the relative displacement between soil and pipe. In an elastic analysis the entity of the load applied to the pipe increases proportionally with the increasing of the relative soil-pipe displacement. In an elasto-plastic analysis, the load increases until the limit yielding value is reached, so the elastic analysis can lead to an uneconomic design.

The soil-pipe interaction analysis has been carried out by many researchers using analytical and numerical solutions. Analytical solutions based on an elasto-plastic approach were proposed by Rajani and Morgenstern [8] and discussed by Motta [6]: they noted that similar interactions are present in a variety of situations such as a laterally loaded pile, a pipeline subjected to fault movement, a pipeline subjected to a landslide movement and so on. Numerical models were proposed by Bruschi et al. ([2] and [3]) with the aim of studying interaction phenomena occurring between pipelines and the surrounding soil in slowly deforming slopes. In their work the soil-pipe interaction analysis was carried out with a finite element discretization after a field and laboratory testing programme.

In this paper the lateral force-displacement response of buried pipes is modelled with the method of the load transfer curves: the load applied to the pipe depends on the relative displacement between soil and pipe $y_s - y$. According to the load transfer functions technique, the ground around the pipeline can be modelled with elastic or elastic-plastic springs for each direction, as shown in Fig. 1.

![Figure 1](image_url)

Figure 1: a) Slope movement and induced pipe load; b) soil-pipe interface.

The evaluation of the seismic response of pipelines has been divided into two steps: i) the prevision of the slope permanent displacements caused by the seismic loading, and ii) the evaluation of stresses and strains in the pipeline induced by the slope movement. The evaluation of the permanent displacements in the slope caused by the seismic action is carried out using the Newmark method [7], while stresses on the pipeline are evaluated with the load transfer
function method by solving a fourth order differential equation. A comparison between the elastic analysis and the elasto-plastic analysis will also be presented.

2 Three-dimensional sliding block method for the permanent displacement evaluation

The seismic stability conditions as well as the failure mechanism of a slope are strictly dependent on the inertial effects and on the shear strength of the soil along the sliding surface. The failure occurs when the inertial effects prevail on the soil shear strength so that, during the seismic motion, the slope is affected by an accumulation of permanent deformations. The seismic slope analysis can be carried out using several approaches such as pseudostatic methods, F.E.M. numerical analysis or simple sliding block methods. In the pseudostatic methods the earthquake load is simply represented by a static force but the analysis does not give any information on the seismic induced displacement. F.E.M. numerical analysis provides detailed information on permanent soil displacement, but it is often very complicated due to the high number of parameters to be introduced in the analysis, and time-consuming. To promote this analysis, the sliding block model introduced by Newmark (1965) can be conveniently used for the prediction of permanent displacements in the slope. The relative displacement between unstable and stable mass occurs when the value of the seismic acceleration exceeds the critical value along the potential failure surface. The value of the critical acceleration should be related to the shear strength of the soil along the potential sliding surface, but also to the failure mechanism assumed in the analysis. If the sliding mass has a limited cross-sectional extension, the boundary effects can be significant and the slope safety factor, as well as the critical slope acceleration, should be evaluated with a 3-D analysis, where the entity and the direction of the seismic acceleration may play a significant role (Kramer and Lindwall, [5]). A simple formulation for the critical acceleration in a 3-D infinite slope was proposed by Casamichele et al. [4].

According to Fig. 2, it is possible to show that the 3-D critical acceleration, for a cohesionless soil, is a function of the sliding mass geometry ratio $H/d$ between the depth and the width of the sliding surface:

$$a_c = g \cdot \left[\left(1 + \frac{K_0 H}{d}\right) \cos i \tan \phi - \sin i\right]$$

where:
- $\phi$ = angle of friction;
- $i$ = angle of the sliding surface;
- $K_0$ = lateral coefficient of earth pressure;
- $H$ = depth of the sliding surface;
- $d$ = width of the sliding mass;
- $k$ = critical seismic coefficient;
- $g$ = gravity acceleration.
Once the value of $a_c$ is determined, the displacement analysis for a 3-D slope can be carried out in the time domain for a given seismic input and a prevision of the amount of the slope movement $y_s$ can be made. The force acting on the pipe is then determined on the basis of the slope earthquake-induced displacement deduced by the Newmark method.

![Figure 2: Three-dimensional analysis of an infinite slope: a) longitudinal section; b) cross section.](image)

### 3 Soil-pipeline interaction

With the increasing of the relative displacements between the soil and the pipe, the load acting on the slope will also increase. Assuming an elastic behaviour for the pipe, the following fourth-order differential equation can be used to evaluate pipe displacement $y$ in the slope movement direction:

$$EIy'''' = P(y)$$

where $P(y)$, the distributed load applied to the pipe, is a function of relative displacement $y_s - y$ between the soil $y_s$ and pipe $y$. $EI$ is the flexural stiffness of the pipe.

### 4 Elastic analysis

In the elastic analysis both the zones where the soil movement is taking place (zone 1) and the stable zone (zone 2) have linear elastic behaviour. However the soil in stable and unstable zones can have different characteristics, so two different load-transfer functions for the soil are assumed in the analysis. A simple schematization of the problem is given in Fig. 3.

Stresses and strains on the pipe are evaluated by the integration of equation (2). The coordinated system is chosen with the $y$-$z$ plane coincident with the boundary between stable and unstable zones, while the origin is on the pipeline axis in the initial configuration. The $x$ axis is coincident with the pipe axis. For zone 1 ($x < 0$) equation (2) can be expressed in the following form:
where $E_{s1}=k_1D$ is the horizontal reaction modulus of the soil in zone 1 [F/L^2].

Figure 3: Elastic analysis: a) initial configuration; b) deformed configuration.

Audibert and Nyman [1] gave guidance for the evaluation of the modulus of a sub-grade reaction for the design of buried conduits. A solution of the differential equation (3) is:

$$y_1 = y_s + e^{X_1} \left( C_1 \cos X_1 + C_2 \sin X_1 \right) + e^{-X_1} \left( C_3 \cos X_1 + C_4 \sin X_1 \right)$$

(4)

where $X_1=x/\lambda_1$ is the dimensionless $x$ coordinate and $\lambda_1=(4EI/E_{s1})^{1/4}$ is the characteristic length for the pipeline in zone 1. For zone 2 ($x>0$) equation (2) can be expressed in the following form:

$$E_l y_2^{IV} + E_{s2}(y_2) = 0$$

(5)

where $E_{s2}=k_2D$ is the horizontal reaction coefficient of the soil in zone 2 [F/L^2]. A solution of the differential equation (5) can be written in the form:

$$y_2 = e^{\beta X_1} \left( C_5 \cos \beta X_1 + C_6 \sin \beta X_1 \right) + e^{-\beta X_1} \left( C_7 \cos \beta X_1 + C_8 \sin \beta X_1 \right)$$

(6)
where \( \lambda_2 = (4EI/E_s)^{1/4} \) is the characteristic length for the pipeline in zone 2 and \( \beta = \lambda_1/\lambda_2 \) is a stiffness ratio parameter. For the best of the boundary conditions the stable zone has been considered infinite, while the unstable zone is finite. With the stable zone infinite, for \( x \to \infty, y_2 \to 0 \) and it can be assumed \( C_5 = C_6 = 0 \). Then equation (6) becomes:

\[
y_2 = e^{-\beta X_1} \left( C_7 \cos \beta X_1 + C_8 \sin \beta X_1 \right)
\]

(7)

For zone 1, in the most general case, the continuity equations at the boundary surface are not sufficient to get the problem determinate and the further conditions that at \( x = -S/2 \) (see Fig. 3) the rotation and the shear force should be zero must be assumed. It is possible to show that for \( S^* = S/2 \lambda_1 \leq 5 \) the values of the six integration constants are the same as those obtained assuming the unstable zone infinite. In this case \( C_3 = C_4 = 0 \). In Figs. 4, 5 and 6, the pipe displacement for different values of the \( \beta \) parameter and of the extension of the unstable zone \( S^* \) are presented.

It can be noted that, for \( S^* = \infty \) and \( S^* = 5 \), the pipe displacement is the same, so it is justified that for \( S^* \geq 5 \) the unstable zone can be assumed as infinite.

5 Elasto-plastic analysis

If one assumes, for the load-transfer function, an elastic-perfectly plastic law, as shown in Fig. 7, the load on the pipe can be expressed as:

\[
\text{For } y_s - y \leq P_{\text{LIM}} / E_s \quad P = E_s (y_s - y)
\]

(8)
For 
\[ y_s - y > P_{LIM} / E_s \] 
\[ P = P_{LIM} \] 
(9)

Then in an elasto-plastic analysis the soil-pipe parameters to be determined are:

- the limit resistance \( P_{LIM} \) [F/L] of the soil at large relative displacement \( y_s - y \);
- the modulus of horizontal soil reaction \( E_s \) [F/L²] that expresses the slope of the elastic behaviour.

![Figure 5: Soil and pipe displacement for \( S^* = 5 \).](image)

![Figure 6: Soil and pipe displacement for \( S^* = 2 \).](image)
A simple schematization of the problem for the initial (a) and the deformed (b) configuration is shown in Fig. 8.

One can distinguish three zones depending on the amount of the relative displacement between pipe and soil $y_s - y$. In zone 1 an elastic behaviour for the soil-pipe interaction is assumed, because, due to the large displacement $y$ of the pipe, the relative soil-pipe displacement is less than the yielding value. That is:

$$\left( y_s - y \right) \leq \left( y_s - y \right)_C = \frac{P_{\text{lim}}}{E_{s1}}$$

(10)

In zone 2 the pipe is at its ultimate load because the relative soil-pipe displacement is greater than the yielding value:

$$\left( y_s - y \right) > \left( y_s - y \right)_C = \frac{P_{\text{lim}}}{E_{s1}}$$

(11)

In zone 3 the pipe is in the stable slope: here it is hypothesized as an elastic behaviour either for the soil, because of its higher shear strength parameters, or for the pipe. The position of the separation surface between zones 1 and 2 is given by the $x$-coordinate $x_p$. In the range $x_p \leq x \leq 0$ a plastic behaviour is assumed for the soil and the differential equation which describes the soil-pipe response can be written in the form:

$$y_2^{IV} = \frac{P_{\text{lim}}}{EI}$$

(12)

$P_{\text{lim}}$ [F/L] being the ultimate soil resistance attributed to the unstable zone and $y_2$ the pipe displacement in zone 2. A model for the evaluation of the maximum lateral resistance of buried pipes has been proposed by Trautmann and O’Rourke [9] and Trautmann et al. [10]. Introducing the non-dimensional parameter:

$$p = \frac{4P_{\text{lim}}}{D \cdot E_{s1}}$$

(13)
where $D$ is the pipe diameter. Equation (12) is written in the following form:

$$y_2 = \frac{p \cdot D}{\lambda_1^4}$$

(14)

The solution is:

$$y_2 = \frac{p \cdot D}{24\lambda_1^4} x^4 + \frac{A_1}{\lambda_1^3} x^3 + \frac{A_2}{\lambda_1^2} x^2 + \frac{A_3}{\lambda_1} x + A_4$$

(15)

where $A_1, A_2, A_3$ and $A_4$ are constants to be evaluated by the boundary conditions.

Posing then $X=x/\lambda_1$, equation (15) becomes:

$$y_2 = \frac{p \cdot D}{24} X_1^4 + A_1 X_1^3 + A_2 X_1^2 + A_3 X_1 + A_4$$

(16)

In the plastic analysis, the unstable zone was considered infinite. This is true if the unstable zone is sufficiently extended ($S^* > 5$). In this way only eight

Figure 8: Elasto-plastic analysis for soil-pipeline interaction: a) initial configuration; b) deformed configuration.
unknown integration constants have to be evaluated. However, a further unknown that should be determined to solve the problem, is the extension $x_p$ of the plastic zone. Its non-dimensional value is:

$$X_{p1} = \frac{x_p}{\lambda_i}$$ (17)

At the dimensionless distance $X_{p1}$, that is at the boundary between the elastic and the plastic zone, the following condition should be ensured:

$$y_s - y_1 = \frac{P_{lim}}{E_{s_i}}$$ (18)

By utilizing this further condition the dimensionless extension $X_{p1}$ can be found as a function of the stiffness ratio $\beta$, as shown in Fig. 9 where $X_{p1}$ is plotted versus the slope displacement $y_s$.

Figure 9: Yielding distance $X_{p1}$ versus slope displacement $y_s$ for $p=0.08$ and $D=1$ m.

6 Analytical solutions

Because the integration constant values depend on the yielding distance $X_{p1}$, once its value is known, stresses and strains on the pipe can be evaluated deriving the displacement expressions for each zone. In the elastic analysis, the results are linearly depending on the soil movement $y_s$. In the elasto-plastic analysis, the results are no longer linearly depending on $y_s$, but are also affected by strength
and stiffness of the soil. In Figs. 10-17 a comparison between elastic and elasto-plastic analysis is presented in terms of pipe displacement, pipe rotation, bending moment and shear force. The results of the plastic analysis are presented in dimensionless form for a value of $p=0.04$ (eq. 13) and a slope displacement $y_s=0.05$ m.

Figure 10: Elastic analysis: pipe displacement.

Figure 11: Elasto-plastic analysis: pipe displacement for $p=0.04$ and $y_s=0.05$ m.

From the figures it can be observed that stress and displacement of the pipe are close to zero for $X_1<-4$ and $X_1>4$. This confirms that the assumption $S^*=\infty$ utilized in the analysis can be considered satisfactory. It is observed that the
stress characteristics strongly reduce with the increasing of the distance from the origin. Stresses and strains on the pipe are influenced by the stiffness ratio $\beta$, particularly at the boundary between the stable and the unstable zone, where bending moment and shear force assume high values if the characteristics of deformability of the stable and unstable zone are very different. The results obtained from the elastic analysis are more conservative than those obtained from an elasto-plastic analysis, particularly in terms of bending moment and shear force. The comparison also shows that the differences between elastic and elasto-plastic analysis are more evident for higher values of the stiffness ratio $\beta$.

Figure 12: Elastic analysis: pipe rotation.

Figure 13: Elasto-plastic analysis: pipe rotation for $p=0.04$ and $y_s=0.05$ m.
Figure 14: Elastic analysis: bending moment.

Figure 15: Elasto-plastic analysis: bending moment for $p=0.04$ and $y_s=0.05$ m.

7 Conclusions

An analytical approach for the assessment of the pipe response in a sliding slope during a seismic event has been investigated by the load transfer function technique. The analysis has been carried on in terms of pipe displacement, rotation, bending moment and shear force. The prevision of the slope permanent displacements, due to the seismic loading, has been carried out using a 3-D sliding block method. The analysis shows that the stresses along the pipe are
larger at the boundary between the stable and unstable zone and they are greatly influenced by the stiffness ratio $\beta$. However, bending moment and shear force rapidly decrease with the increasing distance from the boundary between the stable and unstable zones. Results obtained from the analytical approach are presented in dimensionless plots, for different stiffness ratios and soil strengths.

Figure 16: Elastic analysis: shear force.

Figure 17: Elasto-plastic analysis: shear force for $p=0.04$ and $y_s=0.05$ m.

The comparison between the elastic and the elasto-plastic analysis presented shows that the elastic approach gives values of stress and deformation higher
than those obtained by the elasto-plastic analysis. Such differences are more evidenced for higher values of the stiffness ratio $\beta$ and can be relevant, inducing a non-economical design.

References


