Flow and force characteristics of steady 2D wing section at low Reynolds numbers

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Abstract

The flow and force characteristics of steady 2D wing section is discussed at low Reynolds numbers ($Re \approx 10^3$) in comparison with those at high $Re$ numbers ($Re > 10^5$). At low $Re$ numbers the wing has a thick boundary layer. The lift-to-drag ratio of a 2D wing reduces to 10, and its maximum lift also reduces. The thinner wing section gives a larger lift-to-drag ratio as well as maximum lift. It also gives a larger slope of the lift curve, which sometime reaches the value $2\pi$. The flow is separated from the wing surface even at very low angles of attack. Hence, the 'stall', which is recognized as a variation point of the slope of the lift curve, is not related to the flow separation. The lift continuously increases or keeps a constant value with respect to the angle of attack, even after the 'stall' angle. Computational fluid dynamic methods based on the Navier–Stokes equation are most promising in order to analyze the detailed pressure distribution and the flow pattern at these low $Re$ numbers.

1 Boundary layer

Steady performance of a wing is strongly affected by variation of the Reynolds (Re) number. The relation is well understood between flow and force of a steady 2D wing at high Re numbers ($Re > 10^5$) [1]. Viscous effects are very limited within a thin boundary layer along the wing surface where many small vortices exist. The flow in the boundary layer is mostly turbulent. The boundary layer is so thin that the pressure is constant across the boundary layer and the pressure distribution along the wing surface is equal to the one along the outside surface of the boundary layer. Calculations neglecting the viscous flow, which is called as potential theory, are, therefore, effective to predict the flow and the force of the wing. The nonlinear Navier–Stokes equation is approximated to a linear Laplace equation when compressibility and viscosity of the flow are neglected. The calculation procedure becomes much easier by using the Laplace equation. This boundary layer model, however, cannot be applied when the boundary layer becomes thick. For example, when the flow is separated from the wing surface at a high angle of attack, the viscous
flow region becomes very large. The lift, then, suddenly reduces and this reduction is called as a stall. The stall cannot be analyzed by potential theory. Note that the flow separation and the stall exactly correspond to each other at the high Re number.

Insect wings operate at a low Re number, around $10^3$. In contrast to the high Re number flow, the thickness ratio of viscous flow to wing chord is very large for the 2D wing at low Re number. The flow in the boundary layer is laminar. The thickness ratio of the boundary layer to the wing chord follows the theoretical proportionality $\delta/c \propto 1/\sqrt{Re}$ for a thin flat plate. The thickness ratio of boundary layer of insect wings is, thus, 10 times larger than that of the high Re number wings ($Re > 10^5$), such as airplane wings or some fish tails. A typical flow around a 2D wing calculated using a computational fluid dynamics (CFD) method is shown in Fig. 1 at the low Re number [2]. The length of arrows show the averaged magnitude of flow speed, and the direction of arrows correspond to the averaged direction. The thick viscous flow region is observed, which is indicated by the small averaged flow speed. The ‘boundary layer’ is not a layer any more. This difference of the thickness ratio of viscous flow is the key phenomenon to understand various differences of wing characteristics between the high Re number and the low Re number wings. The difference of the thickness ratio of viscous flow requires a change of the calculation method, too.

2 Calculation method

It is no longer valid that at the low Re numbers the steady pressure distribution along the outside surface of the boundary layer can be approximated by that on the wing surface, because of the thick boundary layer. This means that the handy calculation method of potential flow has difficulty in calculating the detailed pressure distribution on the wing. CFD methods based on the Navier–Stokes equation must be used, since they include the high viscosity for this low Re number flow. The CFD method, however, requires a complicated calculation procedure, and its solution is sensitive to the structure and size of calculation mesh or to the chosen turbulence model. The CFD can predict the qualitative characteristics of pressure distribution for various wing section shapes moving with constant forward speed at a low Re number. It is, however, difficult to predict the quantitative evaluation for this problem at the present moment. Instead, the CFD has the superiority of predicting the detailed flow pattern around the wing, which is useful in understanding the measured pressure distribution, and which is difficult to obtain by experiments. It is enough to know the reasonable value of local flow speed for predicting the flow pattern.
The pressure is, however, proportional to the square of local flow speed and its prediction requires the value of local flow speed two times more accurately than that for the prediction of flow pattern.

## 3 Wing section shape and lift-to-drag ratio

Wing performance is primarily evaluated by the lift-to-drag ratio. The other indices such as maximum lift are only of minor importance for the wing performance. It is known that the lift-to-drag ratio of 2D wing sometimes reaches values around 100 at high Re numbers. Such large lift-to-drag ratios are experienced when using wing sections having a streamlined shape with a round leading edge and a sharp trailing edge and with 10–15% thickness ratio. The thickness ratio mainly affects the maximum lift: it does not affect very much the lift-to-drag ratio. The thicker wing gives the larger maximum lift at high Re numbers as shown in Fig. 2 [1].

When the Re number reduces to the region of $10^4$, the value of lift-to-drag ratio of a 2D wing reduces as shown in Fig. 3. Now there is only a small effect of the thickness ratio on the maximum lift. The transition of boundary layer from laminar to turbulent flow easily occurs on the wing surface. There are various types of thin or thick wing sections available to obtain the larger lift-to-drag ratios by using this transition of boundary layer at $\text{Re} \approx 10^4$.

The lift-to-drag ratio of 2D wing at $\text{Re} \approx 10^3$ reduces down to 10, and its maximum lift reduces too. A comprehensive study indicated the following characteristics for a good wing section at $\text{Re} \approx 10^3$. The thinner wing section gives the larger lift-to-drag ratio as well as the maximum lift as shown in Fig. 2. Even the flat plate is a good wing section. The camber is also effective to obtain the larger lift-to-drag ratios. The corrugated plate has a larger lift-to-drag ratio than a flat plate of the same bending stiffness [3, 4]. The insect wing has all these characteristics of wing
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Figure 3: Variations of lift-to-drag ratio of an NACA0009 wing for Re numbers (re-plot from [4, 7]).

Figure 4: Vortices on 2D wing surface (courtesy of Prof. S. Sunada): water flows from right to left (Re = 4 × 10^3, angle of attack = 10°, wing section; flat plate with 5% thickness ratio).

shapes, and is verified as a good wing section under its flight condition of Re number (Re ≈ 10^3). Detailed relation between force and a section shape for steady low Re-number wing is given in this book by Azuma [5].

4 Flow pattern and aerodynamic force

A flow visualization experiment for a thin wing moving at a low Re number shows that vortices are intermittently shed from the wing surface near the leading edge, stay near the wing surface for a while, and go downstream to the wake as shown in Fig. 4. These vortices are observed even at very low angle of attack with constant forward speed (free stream velocity) for thin wings.
Hence, the flow is always separated from the wing surface. These time-dependent vortices cause periodically unsteady flows. The steady aerodynamic force is the averaged value of the unsteady flow. Hence, the term ‘steady wing’ refers to the steady forward speed alone at low Re numbers. The slope of the lift curve $dC_ℓ/dα$ at low Re numbers is smaller than at high Re numbers except for a thin plate as shown in Fig. 2. In addition, the slope of the lift curve at low Re numbers has a variation similar to the stall at high Re numbers. The lift at low Re numbers continuously increases or keeps a constant value with respect to the angle of attack, even after the so-called ‘stall’ angle. In addition, the ‘stall’ is not related to the flow separation, because the flow is always separated at low Re numbers. It was pointed out for a thin flat plate that the paths of vortices shed from the leading edge correspond with the variation of lift curve slope. These vortices approach to the rear wing surface again before the ‘stall’. After the ‘stall’, they go downstream in the wake with keeping some distance from the wing surface. The figure also shows that the lift curve slope of the thin flat plate has the value of $2\pi$, which is equal to that obtained by potential theory [6].

5 Conclusion

Insect wings operate at a low Re number, around $10^3$. At these Re numbers, the flow and force characteristics of steady 2D wing section are different from those at high Re numbers ($Re > 10^5$). The recent studies by experiments and calculations make clear the superiority of insect wing section shape, such as a thin, cambered, or corrugated wing, even within the framework of steady 2D aerodynamics.

Nomenclature

\begin{align*}
C_ℓ & \quad \text{2D lift coefficient, } C_ℓ = \frac{ℓ}{\frac{1}{2}ρv^2c} \\
ℓ & \quad \text{N/m lift per unit span} \\
Re & \quad \text{Reynolds number, } Re = \frac{vc}{V} \\
α & \quad \text{rad angle of attack}
\end{align*}

For all other parameters, refer to the common nomenclature of this book except for the above.

References

Insect flight aerodynamics: structure of the leading edge vortex and selection pressures responsible for the use of high lift aerodynamic mechanisms in insects

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Abstract

In this paper I deal with an apparently simple question: what is the structure of the leading edge vortex (LEV) insects use for high-lift aerodynamics? Flying insects have been shown to use a range of unconventional aerodynamic mechanisms to generate the high lift forces they exploit in flight. In free-flying insects rotational mechanisms, wake capture mechanisms, and clap-and-fling mechanisms have all been demonstrated, and the associated flows have been visualised, but in all those insects for which we have good free-flight data the normal mode of flight involves the use of separated flows and LEV, or dynamic stall mechanisms. Early analyses suggested that the LEV used by insects was a conical spiral vortex, analogous to that formed over a delta wing (a Werle–Legendre vortex or open bifurcation). However, recent research with free and tethered flying insects has demonstrated that this assumption was false. In fact the LEV forms from an open U-shaped separation, similar to the dynamic stall vortex formed on high aspect ratio wings in the buffet region immediately post-stall. Stability of the LEV is not an issue, because insects operate at Strouhal numbers such that a LEV would naturally be expected to form, and remain stably above the wing for the duration of the downstroke. Whether a LEV does form depends on the angle of attack the insect selects.

1 Structure of the leading edge vortex

The performance of flying insects is so good that it implies aerodynamic forces too large to be accounted for by conventional steady-state aerodynamics [1, 2]. Experiments with large-scale mechanical flapping models and with tethered insects have successfully replicated at least some of the insect force generating abilities [3–8]. In both mechanical models and hawkmoths, the leading edge vortex (LEV) formed above the wing during the downstroke accounted for most, if not all, of the lift enhancement required for insect flight.
A vortex held above a wing has long been known to be capable of enhancing lift [9–18]. A transient LEV can be formed over a conventional aerofoil (such as at the ubiquitous NACA0012) by sudden changes in flow velocity [10] or pitch [9] and aerodynamic experiments in unsteady flows have shown that at its peak the vortex can increase the lift coefficient by as much as an order of magnitude above the steady state value for a given aerofoil [9–11, 13]. Studies with model insects [3, 4, 7, 8] suggest that LEVs can be quite stable over model insect wings, and may produce a two fold-increase in lift.

Three distinct categories of insect LEV structure have been described on the basis of studies with real insects and mechanical flapping models. The LEV was first described, on the basis of model experiments, by Maxworthy [6]. Maxworthy’s description of the structure of the LEV is complex; the LEV is continuous with the wingtip vortices at both outer and inner wingtips on both wings. Its structure is strongly three-dimensional with an axial flow along the core from wing root to wingtip. At the outer wingtip this axial flow trails off into the wingtip trailing vortices, which form a vortex ring connecting the two wingtips. A second trailing vortex connects the two wing-roots in a secondary vortex ring but this secondary ring ‘becomes very distorted under the straining field of the larger [vortex loop] and rapidly loses a clear identity’ (see p. 57, fig. 12 and accompanying text in [6]). Maxworthy showed that the LEV was responsible for a substantial enhancement of lift, in fact for the majority of the lift. Using a quasi-steady approximation he estimated that in the presence of the LEV a lift coefficient of 6.8 could be sustained, providing more than twice the lift required to support the weight of the Chalcid wasp, Encarsia formosa, he modelled.

The second description of LEVs (and the first visualisation of LEVs generated by real insects) was by Luttges’ group, in a series of studies with tethered dragonflies [19, 20] and with tethered hawkmoths [21]. The Luttges group were also able to reproduce the features of the flow field on mechanical flappers [19, 20, 22]. The presence of LEVs is clearly revealed in high-resolution smoke visualisations (summarised in [21]), of both hawkmoths and dragonflies. The structure differed from Maxworthy’s [6] results in two ways. First, the visualisations showed that the LEV was continuous across the thorax, so there were no root vortices. Second, in contrast to Maxworthy [6], Luttges’ simultaneous flow visualisations from top and side views with both dragonflies and hawkmoths show little evidence of spanwise flows at any stage in the wingbeat. Luttges [21] argued that the absence of spanwise flow was a genuine, and general feature of the flows his insects generated—‘the flow structure of the vortices is largely two-dimensional while in the presence of the wing (or wings) that produce them’ (p. 454 in [21]).

The third description of an insect bound LEV came from the detailed analysis by Ellington’s group. In direct contrast to Luttges and colleagues, they emphasised the three-dimensional nature of the LEV [5, 7, 8, 23]. In tethered hawkmoths Willmott et al. [23] and Ellington et al. [5] showed by smoke visualisation, that a small LEV was present on the downstroke at flight speeds from 0.4 to 5.7 m/s, at positions from 0.25R (one-fourth wing length), outboards, and the LEV was larger at higher speeds. Detailed analysis of the flow visualisation images at 1.8 m/s showed that the LEV was absent at 0.25R, visible at 0.5R (midwing), and larger at 0.75R. The LEV broke away from the surface close to 0.75R and rolled up with the wingtip vortices (see Fig. 3 and accompanying text in [23]). No analysis of the flow over the centreline was published, but the authors suggest that the absence of evidence for a LEV at 0.25R implies that the LEV grows along the wing in a conical structure (Luttges’ results showing a LEV over the thorax were dismissed as unnatural on the basis that he observed high lateral forces during some sequences when the moth was struggling to escape the tether).

Ellington and colleagues describe the LEV as ‘a conical spiral, enlarging as it is swept along the wing by an axial (spanwise) flow.’ Ellington et al. [5] state that ‘The conical, spiral vortex
of the flapper is, in fact, remarkably similar in form to that over delta wings.' Implying that Ellington et al. [5] believed the LEV to originate, like the vortices over a delta wing, from a surface bound focus. Willmott et al. [23] were unable to obtain images from the tethered moth with smoke streams within the LEV itself. More detailed interpretations, relying on results with a large-scale mechanical flapper (10 times life size; [7, 8]), showed that at 0° of flap (wings parallel) at 0.25R the LEV was present, but with a diameter just under 2 cm (implying a 2 mm diameter on the hawkmoth, which may explain why it was not detected by Willmott et al. [23]), at 0.5R LEV diameter was 3.5 cm while at 0.63R LEV diameter was about 4 cm [8]. The stability of the flow field around the hawkmoth flapper was assumed to be analogous to that of the LEVs over delta wings, with vortex growth limited by the removal of vorticity through a spanwise axial flow along the vortex cores [5, 7, 8]. Spanwise flow, circulation and vortex diameter were all estimated from qualitative smoke visualisations; axial flow by measuring (from video sequences) the time taken for isolated blobs of smoke to move along the wing. The helix angle was estimated from top-view photographs approximately perpendicular to the wing surface. Combining the helix angle within the LEV and the axial flow velocity along the LEV provides an estimate of the swirl, from which the circulation can be derived. From that estimate of circulation van den Berg and Ellington calculated that the LEV could produce up to two-thirds of the total lift required for flight. An obvious problem with this form of interpretation is that the swirl angle in the smoke within the LEV depends not just on the axial and rotational velocities but also on the history of previous motion of the smoke—smoke lines are streaklines and are not equivalent to streamlines in an unsteady flow, so these estimates need to be viewed with some caution.

Studies by Dickinson and colleagues [3, 4] of mechanical model wings scaled to Reynolds numbers appropriate for hovering Drosophila revealed a spiral LEV with similar structure to that found on Ellington’s flapper. However, the results were equivocal about the influence of spanwise flow as a LEV stabilising factor. Chordwise fences placed on the leading edge of the wing to prevent spanwise flow reduced the size of the LEV, but did not render it unstable, and an acrylic wall placed to reduce the tip vortex to a minimum increased the size of the LEV suggesting that vorticity was accumulating within the LEV. Nevertheless, in both cases the LEV remained stable, raising some doubt as to whether it is necessary for LEV stability for vorticity to be removed from the LEV during the course of the downstroke.

Recently smoke visualisations resolving the flows within the LEV have been achieved with three species of dragonflies (Sympetrum sanguineum, Aeschna grandis and Aeschna cyanea; [24]), with hawkmoths (Manduca sexta; [25, 26]), and with the butterfly Vanessa atalanta [27].

Srygley and Thomas [27] visualised the flow around the butterfly Vanessa atalanta in free flight shortly after take-off in the windtunnel. Detailed analysis showed that the LEV extended from wingtip to wingtip across the thorax. Figure 1 is a composite of smoke visualisations of Vanessa atalanta in free flight in the windtunnel showing the LEV at a series of positions from midline out towards the wingtip. The LEV appears to be similar in structure from the midline out along the wing until the point where it inflects into the wingtip vortex, with which it is continuous. Srygley and Thomas [27] defined the topology of the LEV using the rigorous descriptive framework provided by critical point theory [28, 29]. In the case of Vanessa, the flow topology consists of an open U-shaped separation with a free-slip critical point (a 3D focus) over the centreline (and therefore no surface bound foci).

Thomas et al. [24] used similar techniques to analyse the flows around the wings of three species of free-flying dragonflies in take-off flight, in steady cruising flight and in accelerated or manoeuvring flight in the windtunnel. In all over 8500 smoke visualisations of dragonflies were obtained, providing more data on the flows around the wings of dragonflies than all other flying animals combined. Figure 2 is a composite of tethered flow visualisations of dragonflies, stepping
Figure 1: Composite of top-side views of the Red Admiral butterfly *Vanessa atalanta* flying in the windtunnel. Smoke visualisations are according to the methods of Srygley and Thomas [27]. The images show smoke in a series of planes stepping out along the wing towards the wingtip. The LEV is present, as indicated by the smoke hitting close to the attachment line on the top surface of the wings, and the LEV is of similar diameter in each case until it inflects into the wingtip vortex.

out from the midline along the wing. The resolution in these images is higher than previously obtained, and details of the structure of the LEV and of the flows within it can be clearly seen. Thomas *et al.* [24] used the framework of critical point theory to describe the dragonfly LEV; 'The LEV is an open U-shaped separation, continuous across the thorax, running parallel to the wing leading edge and inflecting at the tips to form wingtip vortices. Air spirals in to a free-slip critical point over the centreline as the LEV grows. Spanwise flow is not a dominant feature of the flow field—spanwise flows sometimes run from wingtip to centreline or vice versa—depending on the degree of sideslip. LEV formation always coincides with rapid increases in angle of attack, and the smoke visualisations clearly show the formation of LEVs whenever a rapid increase in angle
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Figure 2: Composite image of top-side views of the LEV over the wings of tethered dragonflies (*Aeschna grandis*). The insect has been shifted sideways through the smoke so the images show that the LEV is present on the midline, and at all stations along the wing out to the wingtip, where it inflects to join the wingtip vortex. The structure is strikingly similar to that generated by the butterfly in spite of the radical difference in wing morphology. More detailed analysis and methodology are presented in [24].

of attack occurs. Kinematics are configured so that a LEV would be expected to form naturally over the wing and remain attached for the duration of the stroke. However, the actual formation and shedding of the LEV is controlled by wing angle of attack, which dragonflies can vary through both extremes, from zero up to a range that leads to immediate flow separation at any time during a wing stroke.’

The first quantitative analysis of the LEV in real insects was provided by Bomphrey et al. [25, 26], who used digital particle image velocimetry (DPIV) to measure the structure of the flow field around the wings of tethered hawkmoths flying in the windtunnel. Bomphrey et al. [25, 26] showed that an LEV formed over the wings, and extended continuously across the thorax, matching the LEV structure identified in free-flying butterflies by Srygley and Thomas [27] and in free-flying dragonflies by Thomas et al. [24]. Bomphrey et al. [25, 26] were able to measure the strength of the LEV and showed that it alone produced lift capable of supporting 60% of the hawkmoths weight at 3 m/s flight speed. Bomphrey et al. were also able to put an upper limit on the extent of any spanwise flow and showed that it must have been less than 1 m/s in their experiments at 3.5 m/s flight speed, and less than 0.4 m/s at 1.2 m/s flight speed—limiting any spanwise flow, if present, to less than one-third of the freestream velocity in either case. Figure 3 shows the structure of the LEV over the midline of *Manduca sexta* as measured by DPIV, together with 3D DPIV of the flow over a locust, an insect which uses conventional attached flow aerodynamics (Fig. 4).
Leading-edge vortex runs across the thorax with a free-slip critical point above centreline

Figure 3: Digital particle image velocimetry of the LEV over the hawkmoth *Manduca sexta*. The LEV is present over the midline and has a structure matching that in butterflies and dragonflies—an open U-shaped separation with a free-slip critical point above the midline.

The DPIV and free-flight flow visualisation results are clear. In dragonflies, butterflies and hawkmoths the LEV is continuous across the thorax, at least at the end of the downstroke. No data are available yet for diptera or hymenoptera, and these must be the key targets for future DPIV and flow visualisations. There is no physical reason why insects should not be capable of
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Figure 4: Smoke visualisation of the locust *Sistocerca gregaria* in tethered flight. In contrast to the previous insects the flow shows no evidence of any coherent separated flow structures or LEVs. Locusts appear to fly by using conventional aerodynamics.

generating other LEV structures such as the Werle-Legendre conical spiral LEV which has been observed in Ellington’s and Dickinson’s mechanical flappers, or indeed the Maxworthy LEV with both wingtip and wingroot vortices. Indeed, given the ability of dragonflies and butterflies to use a wide variety of aerodynamic mechanisms in flight [24, 27], it would be a surprise if we have even come close to covering the range of aerodynamic mechanisms insects are capable of using, and it is somewhat surprising that insects as different in morphology as butterflies and dragonflies (which exhibit the extremes in aspect ratio within the insects) use a common structure for the LEV. This may perhaps imply that there are significant advantages to the open U-shaped separation as an LEV structure, but what those advantages are remains to be determined.

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References


