PHYSICAL MODEL AND DESIGN RULES FOR THE OPTIMIZATION OF SOLAR CHIMNEY SYSTEMS

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ABSTRACT

Natural ventilation is a basic quantity to reach comfort in passively acting buildings. It delivers not only fresh air to breath but can also be used to temper the room if the indoor temperature is above the outdoor one. Driving forces are temperature differences (buoyancy) and wind. However, both may be weak in hot and especially humid locations.

A solar chimney uses solar radiation to heat up the exhausted air and to increase buoyancy, thus could help to improve that situation at least during the daytime. Nevertheless, the implementation of solar chimneys is quite rare. It may be that the idea to use heat to cool and ventilate a building seems strange. The literature reports about the potential of solar chimneys, characteristics like volume flow and temperatures are measured or simulated. Though, the findings of these publications are based on a special geometry and provide not enough detailed information about the optimized shape (width, height, length, etc.) of solar chimneys. To overcome that situation, this paper presents general design rules for the geometry of solar chimney systems that could be adapted to existing or newly erected buildings. A solar chimney system is assumed as composed by three components: An absorber as the main part to reach a higher temperature, a stack extension on top for further acceleration of the exhausted air, and a stack at the bottom connected to the (lower) storeys that should be ventilated.

A full set of equations of fluid mechanics and thermodynamics is presented and describes the physical behaviour of the air in the system. These equations are coupled with each other and can be solved in iterations, also with a simple spreadsheet. As a compromise between accuracy in results and complexity of the simulation method, the physical model is based on some simplifying assumptions like turbulent flow, friction on walls, air as an incompressible medium, and immediate heat transfer from the absorber to the air. Nevertheless, the outcomes are in accordance with the findings from the literature, as the model seems to reflect the physical behaviour adequately. The main results are volume flow, velocity and temperature, allowing the optimization of the geometry of the chimney system. An applicable list of design rules for solar chimneys is finally presented as well as proposals for their integration in typical apartment buildings in hot and humid locations.

Keywords: application in apartment buildings, buoyancy, design rules for geometry of solar chimneys, friction, hot and humid Climates, natural ventilation, physical model, solar chimney.

1 NATURAL VENTILATION IN HOT AND HUMID CLIMATES

1.1 Functions of natural ventilation and specialities in hot and humid climates

Natural ventilation has the main purpose to remove CO₂ (and replace it by O₂), humidity and heat (and thus cooling with outdoor air). Driving forces for natural ventilation are the differences between indoor and outdoor in pressure (wind) and temperature (buoyancy).

In hot and humid climates, sufficient ventilation is especially difficult to reach. Because of the small changes in temperature between day and night, there is nearly no temperature difference between in- and outdoor. For many of the locations, wind velocity is quite low supplementary thus both principles do not work strongly.
1.2 Vernacular architecture

The first priority was a comfortable temperature indoor during the night for sleeping. During the day, people stayed outside. That could be realized best with a light construction that does not store the heat of the day and allows that indoor temperature drops quickly with outdoor temperature (see also chapter 5 for thermal behaviour).

It is known from practical experience that even if there was a vertical opening (like the eye in a staircase and an opening in the roof) offering a ventilation stack for buoyancy, the air rested on the bottom of the building and does not move upwards. As a consequence, vernacular architecture concentrated on the wind as driving force. Walls (and even floors!) were open for a maximum of air movement, using woven structures as the principal ones.

In cold climates, each oven produced hot air in a chimney. As a result, strong buoyancy was created, and the smoke was removed out of the building. This idea to use a heat source (like the sun) to produce hot air in a ventilation stack was not developed in hot climates, may be because of the lack of material (glass) and experience, or maybe because of the odd concept of heating something to cool a building. Supplementary, the sun as heat source works only during the day and not during the night, when the need was the strongest.

1.3 Today’s conditions and needs

Architecture today is different and widely not following the principles of vernacular architecture. Walls become tighter and not permeable for air movements, windows have a glazing, also for (sound) protection. Especially in towns, buildings are placed near together and multi-storey, creating obstacles for wind. Technical HVAC equipment undertakes increasing tasks of ventilation and tempering of the rooms. On the other hand, there are a lot of countries where inhabitants have not enough financial means to pay for the supplementary energy demand and power cuts may happen. As a result, inhabitants live often under worse conditions than their ancestors in vernacular architecture.

But there are also chances: the existence of a wider range of building materials like glazing and thermal insulation allows the construction of solar chimneys. Especially for residential buildings in locations with a rural character, such equipment can help a lot, especially since it does not cause permanent energy costs. People stay more and more in their apartments also during the day, when the system works best.

This paper is thought as a contribution for passive optimization of buildings. Solar chimneys could be a reasonable supplement for today’s buildings in hot and humid climates. The given rules of thumb for design shall help to introduce them to contemporary architecture.

2 PRINCIPLES OF A SOLAR CHIMNEY

2.1 Absorber

To facilitate the model, the absorber is assumed with a rectangular cross-section (assume other shapes as a rectangle with an identical area!) with a width W and a depth D (Fig. 1). Three sides of the absorber are assumed as adiabatic (no heat flow). They could be integrated into the building or not and are assumed as with thermal insulation and coloured black to absorb incoming solar radiation. The fourth side is oriented to the sun and has a glazing that separates the air volumes inside and outside of the absorber.
The sun is shining through the glazing onto the absorbing walls. They absorb this radiation and heat up finally the air in the absorber. Because of buoyancy, the air in the absorber moves up and is accelerated according to the passed height difference.

2.2 Stack extension

It may be that the air at the end of the absorber has still a temperature difference with the outdoor temperature, which means that the acceleration potential is still not completely used. A simple vertical stack extension without glazing can serve with its height difference to accelerate the air further (Fig. 2). It is assumed to be rectangular with a depth d, a width w and vertical with a height h. All four walls are opaque and oriented to outdoor conditions and thus outdoor temperature. To avoid quick thermal losses and thus temperature reduction as well as an early end of the acceleration, the walls of the stack extension have a thermal insulation.

2.3 Stack

In a multi-storey building, there are also floors below the absorber. A supplementary stack can be added on the bottom of the absorber connecting downwards to the floor to ventilate (Fig. 2). It is assumed as rectangular with width w’, depth d’ and height h’. The stack is
2.4 The complete solar chimney system: Construction and operating principle

The absorber is often integrated into sloped roofs: that’s why it may have a slope. Length $L$ and slope result in a height difference $H$ (Fig. 2).

The accelerating forces act in the absorber and in the stack extension. But there are also forces hindering the air movement:

- The friction of moving air on the walls (growing with the square of the velocity!).
- The entrance of the air into the absorber.
- The bendings from the stack to the tilted absorber and to the stack extension.
- The widening or redirecting between the stack and the absorber, and the absorber and the stack extension.

Forces that act accelerating (buoyancy) and inhibiting (friction) reach quickly their balance and the final steady-state with a constant velocity. An example is a human falling with a parachute, reaching a final, limited velocity in spite of the fact that gravity tries to accelerate permanently. Thus, an increase in the absorber height will become more and more ineffective. The investigations here and in the literature show that the main effect is reached after 3 to 4 m height difference. Typical velocities are around 0.4 m/s; therefore, the whole system is passed in less than one minute. Because there is no further absorption of solar radiation in the stack extension, there will be a critical height where the velocity decreases. Melting temperature difference and friction would decrease and finally stop the air flow – one should be careful with the height of the extension so that the optimal height is met and not increased further. As a conclusion, a stack extension would be most effective if the absorber has only a height of less than 3 m. In this case, the absorber probably reaches the final temperature already but cannot use the resulting acceleration potential. The stack extension serves then as a supplementary acceleration path.

3 MATHEMATICAL DESCRIPTION OF A SOLAR CHIMNEY

Thermodynamics is used to calculate the temperature difference between absorber/stack extension and the outside. This is the driving force. Fluid mechanics is used to estimating the losses in energy by friction and changes of form and direction which hinders acceleration. In the absorber, the energy from the absorbed solar radiation is transferred to the air causing a difference in density and an acceleration of the air. A higher velocity transports a higher air volume along the absorber, where the same energy is distributed to a bigger air volume. The resulting air temperature in the absorber becomes smaller. Supplementary, there are thermal losses through the walls and the glazing of the absorber and the extension to the deeper outdoor temperature. Thus, the thermal model and the fluid mechanics do influence each other, and the equations cannot be solved separately. An iterative solution method is necessary and can be realized with spreadsheet software.

3.1 Preparations

The hydraulic diameter of a non-circular cross-section of a stack $D_{\text{hyd}}$ is defined as the diameter of a circular one with the same quantities in fluid dynamics. The described solar chimney model assumes rectangular cross-sections, for the absorber $D_{\text{hyd}}$ is exemplarily:
D_{hyd} = 2*W*D / (W+D). [m] (1)

Fix quantities of air are summed up in Table 1. The relation between temperature T [K] and density [kg/m³] is described by the specific gas equation (eqn (2)), where for example, values are 1.20 kg/m³ at 20°C (393.15 K) and 1.16 kg/m³ at 30°C (403.15 K).

\[ \rho = \frac{p}{(R_s*T)}. \text{[kg/m}^3\text{]} \] (2)

With those information quantities for the accelerating part absorber + stack extension can be described. The density difference between the outdoor air (with T_e) and the average temperature in the accelerating part absorber + stack extension (with T_{acc}) is:

\[ \Delta \rho = -\left[ \frac{p}{R_s} \left( \frac{1}{T_e} - \frac{1}{T_{acc}} \right) \right]. \text{[kg/m}^3\text{]} \] (3)

The corresponding height difference is H+h. The average cross-section of both units is:

\[ \overline{A} = \frac{(W*D*L + w*d*h)}{L + h}. \text{[m}^2\text{]} \] (4)

For fluid mechanics, it is important to know what kind of flow exists: a laminar one, or a turbulent one. This can be decided by determining Reynolds number:

\[ \text{Re} = \rho*v*d_{hyd} / \eta. \text{[\text{-}]} \] (5)

The literature reports that well-working solar chimneys have a d_{hyd} of at least about 0.35 m, the air moves in it with about 0.4 m/s. With a temperature of 30°C the density is 1.16 kg/m³. That results in Re = 8120, what is clearly above the range of laminar flows, as threshold Re = 2320 is used. If the chimney would be bigger in size or higher in velocity, Re only increases furthermore. The flow can be assumed always as a turbulent one.

The air flow is influenced by friction on the inner surfaces in the different parts of the solar chimney. Even a turbulent flow forms a thin laminar layer at the surfaces. In this layer, the velocity is small and thus the friction becomes also minor. If the roughness of the surfaces is smaller than the thickness of this layer (hydraulically smooth surface), the flow can run smoothly, an advantage that should be used. Equation (1) gives a formula to estimate the thickness of this layer:

\[ d_{laminar} = 32.8 \frac{d_{hyd}}{(\text{Re} * \lambda^{0.5})}. \text{[m]} \] (6)

\[ \lambda = 0.0055 + 0.15* \left( \frac{1000}{d_{hyd}} + \frac{50}{\text{Re}} \right)^{1/3}. \text{[\text{-}]} \] (7)

Table 1: Fix quantities of air.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific heat capacity</td>
<td>c</td>
<td>1000</td>
<td>Ws/kgK</td>
</tr>
<tr>
<td>Air pressure (normal conditions)</td>
<td>p</td>
<td>101325</td>
<td>Pa</td>
</tr>
<tr>
<td>Specific gas constant (dry air)</td>
<td>Rs</td>
<td>287.1</td>
<td>Ws/kgK</td>
</tr>
<tr>
<td>Dynamic viscosity (at 300 K)</td>
<td>h</td>
<td>0.00002</td>
<td>Pa s</td>
</tr>
</tbody>
</table>
Equation (6) contains the (Darcy) friction factor \( \lambda \) that is dependent on Reynolds number as well as the real roughness \( k \) [mm] of the surfaces. If we assume that the surfaces are hydraulically smooth, eqn (6) delivers with \( d_{\text{hyd}} = 0.35 \), \( \text{Re} = 8120 \) and \( \lambda = 0.03 \) (see eqn (7) with \( \text{Re} = 8120 \)) a value for \( d_{\text{laminar}} \) of about 8 mm. To reach a stack with hydraulically smooth walls, the roughness \( k \) of the surfaces of the stacks has to be below \( d_{\text{laminar}} / 4 \), which means below 2 mm. For stacks made of metal, that can be assumed but for bricked stacks (especially without plaster), we would lie more in the transition range to rough surfaces with remarkably higher friction. Thus, the inner surfaces of a solar chimney should be prepared in a way that the roughness does not exceed 1 to 2 mm, so that the surfaces can be assumed still as hydraulically smooth.

Different estimation formulas for the friction factor \( \lambda \) exist and an approximation is given in eqn (1). All investigations and results presented here assumed a roughness of 2 mm in the whole chimney system that should be achievable with a careful realization in any case.

Supplementary, each change in the shape and direction of the stack causes friction on the transferring medium. Its strength is described by the resistance coefficient \( \zeta \). Values for \( \zeta \) (or also written as \( K \)) are constant and can be found in the literature [2].

Because \( \zeta \) describes a transition in a stack, it has to be related to one of the two related cross-sections. By definition, it is always related to the smaller cross-section. Table 2 shows the 4 positions that are possible in the investigated solar chimney system. Bending can occur in the transitions stack-absorber as well as in the one absorber-stack extension. The investigation here found as the optimal proportion between width and depth of a stack a value of about 3. Out of that ratio, eqn (10) was estimated on basis of eqn (2). For any outlet \( \zeta \) is always equal to 1 (no friction).

3.2 Basis equations – Fluid mechanics

In general, Newton’s law of motion is described by eqn (11) and the different terms are described in eqn (1).

\[
\text{Acceleration of mass} = \text{accelerating forces} - \text{inhibiting forces} \quad [\text{kg}\cdot\text{m}/\text{s}^2] \quad (11)
\]

\[
F = F(\text{acc}) - F(\text{inhibit})
\]

a) Accelerating forces (buoyancy)

\[
F(\text{acc}) = \Delta \rho \cdot g \cdot A_0 \cdot (H + h).
\]

(12)

Table 2: \( \zeta \)-values for different changes in the stacks with rectangular cross-section.

<table>
<thead>
<tr>
<th>Sharp inlet from free air to a rectangular stack</th>
<th>Narrowing from cross section A1 to A2</th>
<th>Widening from cross section A1 to A2</th>
<th>Bending</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Diagram of stack transitions" /></td>
<td><img src="image" alt="Diagram of stack transitions" /></td>
<td><img src="image" alt="Diagram of stack transitions" /></td>
<td><img src="image" alt="Diagram of stack transitions" /></td>
</tr>
<tr>
<td>( 0.5 )</td>
<td>( 0.5 \cdot \left(1 - \frac{A_2}{A_1}\right) ). eqn. (8)</td>
<td>( (1 - \frac{A_1}{A_2})^2 ). eqn. (9)</td>
<td>( 0.43 \cdot \sin(\alpha) ). eqn. (10)</td>
</tr>
</tbody>
</table>
With gravity constant \( g = 9.81 \, \text{m/s}^2 \). Note that buoyancy is proportional to the height difference, that’s why in eqn (7) is \( H \) and not \( L \)!

b) Inhibiting forces (friction and changes in direction and shape)

\[
F(\text{inhib}) = \frac{\rho}{2} \left[ \sum_i \frac{\lambda_i}{4} v_i^2 \cdot (\text{wetted area})_i + \sum_j \xi_j \cdot (\text{cross section})_j \right].
\]

with

\[
\begin{align*}
\rho & \quad \text{density of air at temperature } T_e = T_{\text{outdoor}} \, \text{(assumed as identical to } T_{\text{indoor}}) \, [\text{kg/m}^3] \\
i & \quad \text{sum over stack, absorber and stack extension} \\
v & \quad \text{velocity} \, [\text{m/s}] \\
(\text{wetted area} & \, \text{diameter} \times \text{length}, \text{for stack } 2\times(w'+d')h', \\
& \text{for absorber } 2\times(W+D)h, \\
& \text{for stack extension } 2\times(w+d)h \, [\text{m}^2] \\
\lambda & \quad \text{friction coefficient, see eqn (7) } [-] \\
j & \quad \text{sum over all changes in size and direction} \\
\xi & \quad \text{resistance coefficient (see ch. 3.1.6) } [-] \\
(\text{cross-section for stack } w'*d', \text{for absorber } W*D, \text{for stack extension } w*d \, [\text{m}^2] \\
\end{align*}
\]

Both terms in the bracket in eqn (13) describe losses in mechanical energy by friction, the first one on the walls of stack, absorber and stack extension, while the second one regarding the changes in size and direction.

Note that for gases Newton’s friction’s law must be used. Here the inhibiting force increases with the second power of the velocity \( v \), thus very strong!

c) A solution for final (steady) state and volume flow

At the final state, there is no acceleration and the left part of eqn (11) is zero. Thus, eqns (12) and (13) could be set identical:

\[
F(\text{acc}) = F(\text{inhib}) \quad (14)
\]

Inside the whole solar chimney system, the volume flow \( V \, [\text{m}^3/\text{s}] \) must be constant, since it is related to the velocity \( v \):

\[
V = v_i \times (\text{cross section})_i \, [\text{m}^3/\text{s}] \quad (15)
\]

Some transformations lead to:

\[
V = \sqrt{\frac{C_1}{C_2}} \, [\text{m}^3/\text{s}] \quad (16)
\]

with

\[
C_1 = 2 \times \frac{\Delta \rho}{\rho} \times g \times (H + h) \times \bar{A} \, [\text{m}^4/\text{s}^2] \quad (17)
\]

\[
C_2 = \frac{1}{2} \left[ \lambda_{\text{stack}} \times \frac{(w' + d')h'}{(w' + d')^2} + \lambda_{\text{absorber}} \times \frac{(W + D)h}{(W + D)^2} + \lambda_{\text{stack extension}} \times \frac{(w + d)h}{(w + d)^2} \right].
\]
3.3 Basic equations – Thermodynamics

3.3.1 Absorber
Heat gains arise from the solar radiation \( I \) [W/m²] that passes through the glazing of the absorber with a g-value and strikes on the (black painted) inner surfaces of the walls of the absorber, where a part of \( \alpha_s \) (absorption coefficient) is absorbed as a heat gain \( q_{\text{gain}} \) (eqn (19) and Fig. 3). The surface temperature of the absorber is increasing to \( T_{\text{abs}} \) and consequently heating up the air in the absorber to \( T_{i,\text{abs}} \). The temperature difference \( T_{\text{abs}} - T_{i,\text{abs}} \) causes a heat transfer \( q_{\text{loss}} \) from the absorber to the air that has to get over the heat transfer resistance \( R_{si} \) (eqn (20) and Fig. 3).

\[
q_{\text{gain}} = I \cdot g \cdot \alpha_s \quad \text{[W/m²]} \tag{19}
\]

\[
q_{\text{loss}} = \frac{1}{R_{si}} \cdot (T_{\text{abs}} - T_{i,\text{abs}}) \quad \text{[W/m²]} \tag{20}
\]

\( R_{si} \) is for standard conditions for internal walls generally assumed with 0.13 m²K/W. Yet in the absorber, the air is not stationary and has to be adapted to the air velocity in the absorber. \( R_{si} = 1/(\alpha_s + \alpha_c) \) is determined by the heat transfer coefficients \( \alpha_s \) and \( \alpha_c \) for radiation and convection. With \( \alpha_s = 0.9 \), long wave emission factor \( \varepsilon = 0.9 \) and \( T \) about 300 K, the value for \( \alpha_s \) can be estimated with \( \alpha_s = 4^{*}\alpha_s^{*}T^{3} \) (\( \sigma \) is Stefan–Boltzmann constant \( 5.67 \times 10^{-8} \) W/m²K⁴) to 6 W/m²K. \( \alpha_c \) depends on the air velocity, from own investigations as well as in the literature it is known that the velocity in the absorber is about 0.35 m/s. With the estimation formula \( \alpha_c = 2 + 12 \cdot v^{0.5} \) results \( \alpha_c = 9.1 \) W/m²K and finally \( R_{si} = 0.06 \) m²K/W, less than the standard value.

The air of the absorber on their part loses energy by two processes (see Fig. 4):

a) Losses through the glazing to the cooler outdoor air with \( T_e \). The standard value for \( U_g \) must be adapted to the air velocity in the absorber in the same way by:

\[
U_{g,\text{adapted}} = \frac{1}{1/U_g - 0.13 + 0.06} \quad \text{[W/m²K]} \tag{21}
\]

The heat flux density is:

\[
q_1 = U_{g,\text{adapted}} \cdot (T_{i,\text{abs}} - T_e) \quad \text{[W/m²]} \tag{22}
\]

b) Heating up the air that transfers the absorber upwards. The energy that is necessary to heat up a mass \( m \) of a substance by a temperature difference is determined by

- \( \sigma \) Stefan–Boltzmann constant
- \( \varepsilon \) long wave emission factor
- \( T \) surface temperature
- \( \alpha_s \) absorption coefficient
- \( \alpha_c \) convection coefficient
- \( \varepsilon \) air velocity
- \( U_g \) heat transfer coefficient
- \( R_{si} \) resistance
- \( I \) solar radiation
- \( g \) g-value
- \( U_g \) heat transfer coefficient
Q = m * c * (temperature difference). We do not have stationary air but a mass flow [kg/s] that runs through the cross-section W * D, hence the corresponding heat flux density is:

\[ q = \frac{\dot{m} * c}{W * D} (T_{i,abs} - T_e) \text{ [W/m}^2\text{]} \] (23)

\( \dot{m} \) can be replaced by \( \rho * V \) leading to:

\[ q = \frac{V * \rho * c}{W * D} (T_{i,abs} - T_e) \text{ [W/m}^2\text{]} \] (24)

The total heat loss \( q_{loss} \) is \( q_1 + q_2 \), with:

\[ q_{loss} = q_1 + q_2 = U_{total} * (T_{i,abs} - T_e) \text{ [W/m}^2\text{]} \] with (25)

\[ U_{total} = U_{g,adapted} + V * \frac{\rho * c}{W * D} \text{ [W/m}^2\text{K]} \] (26)

In the final steady-state, heat gains and heat losses must be in balance:

\[ I * g * a_s = \frac{1}{R_{si}} * (T_{abs} - T_{i,abs}) = U_{total} * (T_{i,abs} - T_e) \text{ [W/m}^2\text{]} \] (27)

Transferring these equations to \( T_{abs} \) and \( T_i \), resp., leads to the final steady-state temperatures on the wall, as well as the air in the absorber whereby it is assumed that:

- The wall of the absorber is very thin and has no storage mass, therefore it heats up immediately to its final steady-state temperature,
- The air in the absorber forms an extended layer and it has the same temperature along its thickness D.

\[ T_{i,abs,steady-state} = T_e + \frac{I * g * a_s}{U_{total}} \text{ [°C]} \] (28)

\[ T_{abs, steady-state} = T_e + I * g * a_s * \left[ R_{si} + \frac{1}{U_{total}} \right] \text{ [°C]} \] (29)

The time-dependent temperature of the air in the absorber can be described following the model of an infinitely extended plate that is heated by a heat source and has losses to the surrounding [3] (A is the area of the absorber W*L):

\[ q = \frac{1}{A} * m * c * \frac{dT_{i,abs}}{dt} = \frac{1}{A} * \rho * A * D * c * \frac{dT_{i,abs}}{dt} \]
\[
\rho \cdot D \cdot c \cdot \frac{dT_{\text{abs}}}{dt} = q_{\text{gain}} - q_{\text{loss}} \cdot [W/m^2]
\] (30)

Using eqns (19) and (25), the heat transfer equation is:

\[
\rho \cdot D \cdot c \cdot \frac{dT_{\text{abs}}}{dt} = I \cdot g \cdot a_s - U_{\text{total}} \cdot (T_{\text{abs}} - T_e) \cdot [W/m^2]
\] (31)

The solution of this equation is [3]:

\[
T_{i,\text{abs}} = T_e + \frac{I \cdot g \cdot a_s}{U_{\text{total}}} \cdot \left(1 - \exp\left(-\frac{U_{\text{total}}}{\rho \cdot D \cdot c} \cdot 1\right)\right). \quad [\degree C]
\] (32)

In case that there is no stack extension, combining the eqns (32), (15) to (18) and (3) allows calculating with an iterative solution algorithm:

- The volume flow V and velocities \(v_{\text{stack}}\) as well as \(v_{\text{absorber}}\) in the stack and in the absorber.
- The time that the air needs to pass through the stack and the absorber \((t_1 = h'/v_{\text{stack}} \text{ and } t_2 - t_1 = L/v_{\text{absorber}})\).
- The temperature on top of the absorber \(T_1\).
- The accelerating temperature difference \(T_{\text{acc}} = 0.5 \cdot (T_1 - T_e)\).

3.3.2 Stack extension

In the stack extension (if existent) the air is not heated furthermore by solar radiation but perhaps accelerated furthermore by buoyancy. The reduction of the temperature of air in the stack extension depends on the area \(A_{\text{ext}}\) and average of the U-value \(U_{\text{ext}}\) of the walls of the stack extension to outdoor air, as well as the time \(t_3 - t_2\) the air needs to pass through the stack extension (Fig. 5).

The behaviour of temperature in time is described by eqn (3):

\[
T_{i,\text{ext}} = T_1 - \left(T_1 - T_e\right) \cdot \left(1 - \exp\left(-\frac{U_{\text{ext}} \cdot A_{\text{ext}}}{\rho \cdot D \cdot c \cdot w \cdot h}\right)\right). \quad [\degree C]
\] (33)

Combining eqns (33), (32), (15) to (18) and (3) allows calculating with an iterative solution algorithm (see also Fig. 2):

- The volume flow V and velocities \(v_{\text{stack}}\), \(v_{\text{absorber}}\) and \(v_{\text{ext}}\) in the stack, the absorber and the stack extension.

![Figure 5: Heat losses from stack extension to outdoor air.](image-url)
The time that the air needs to pass through the stack, the absorber and the stack extension 
\( (t_1 = h'/v_{\text{stack}}, t_2 - t_1 = L/v_{\text{absorber}}, t_3 - t_2 = h/v_{\text{ext}}) \).

- The temperatures on top of the absorber \( T_1 \) and the stack extension \( T_2 \).

- The accelerating temperature difference \( T_{\text{acc}} = 0.5 * (T_2 - T_0) \).

3.4 Limitations of the chosen physical model and comparison with the literature

The coupling between fluid mechanics and thermodynamics is more complex than here assumed. Especially, there is a velocity profile in the cross-section of the chimney with a thin layer of stationary air on the walls, whereas we assume a constant velocity in the whole cross-section. Thus, the effects of deeper temperatures near the glazing and resulting counter flow cannot be displayed in the model. The model used in fluid mechanics is based on the assumption that the flow is turbulent. With increasing length of the stack inside of the buildings, the flow becomes lazier and reaches the transition range to laminar flow where different models must be used. The equations used hold for incompressible mediums (liquids) and effects of compressible mediums (like gases) are not included. But investigations in advance showed identical results for both conditions. It is reasonable that with the low velocity of up to 0.4 m/s the effect of compression in gases does not have a dominant role and that gases behave like an incompressible substance.

The absorber is assumed to be infinitely thin and reaching its final temperature under solar radiation immediately. The effects of a more inert process of heating up and cooling down because of a real storing mass are not included. But extending the model also to dynamical behaviour of the materials that form the chimney could be interesting! It could help to optimize the chimney in a way that it could act also after sunset using the heat stored in its walls for further acceleration of air during night time, for instance, when people want to sleep with comfortable temperatures. The model used for thermodynamics assumes that the air in the chimney is an infinite plate with a certain thickness and at the same temperature. Effects of non-homogeneous temperature distributions are not included.

The presented model can be used to optimize the geometry of solar chimneys. It does not allow optimizing the materials used in the walls. For that, a more complex model would be necessary.

Literature that reports about optimization of geometry and material of solar chimneys is scarce. In general, it describes a defined chimney and the found quantities by measurements and simulations [4]–[9], the recommendations that could be derived from the presented model are in satisfying accordance with it.

4 RESULTS – DESIGN RULES FOR SOLAR CHIMNEY SYSTEMS

4.1 Absorber only

It is recommended to optimize the geometry of the absorber in the first step and only then the one of the stack extension and the stack. But it may happen that the slope and the length of the absorber are given with the geometry of the building (‘s roof).

With an assumed solar radiation of 500 W/m², the temperature of the absorbing wall reaches about 55°C, heating up the air in the absorber. The air also has thermal losses through the glazing and a maximal steady-state temperature (eqn (29)) is reached in few seconds. It is only about one degree higher than the inlet temperature but that is enough to accelerate the
air to up to 0.4 m/s. The dimensioning of the absorber should exploit this potential of maximized temperature as much as possible (>= 99%).

The depth D has an optimal value. If it is too small, the volume flow can’t expand optimally. If it is too big, there is a risk of counterflow along the cold glazing. This is not included in the simple formulas here. The optimal depth D seems to be about 0.15 to 0.5 m. An increase in depth D above 0.3 m only delivers a small increase in the volume flow. Thus, recommendations for the optimal dimensions are a depth D of 0.15 to 0.3 m, a width W of 0.5 to 1.2 m, a height H of 4 m and single glazing. The resulting flow rates are about 1300 to 1600 m³/h and m² absorber, thus for a D 0.3 m and a W 1.2 m about 500 m³/h. The minimal width W should be 0.4 to 0.5 m to allow free volume flow. The recommended ratio W/D is about 2 to 3, delivering the best volume flow per cross-section (Fig. 6).

To reach the maximal velocity, a minimal height H of 3 m, better 4 m is necessary (Fig. 7). A further increase in the height increases the volume flow only weakly. Even for a higher solar radiation, the minimum height is 3 to 4 m. An increase in the solar radiation from

![Figure 6: Calculated volume flow in dependence of the absorber depth. The range with the highest increase is for depths between 15 and 30 cm. The arrow shows the risk of counterflow that could happen for too deep absorbers.](image)

![Figure 7: Calculated volume flow of an absorber in dependence of the absorber height (left) and the solar radiation (right). The recommended minimal height H is 3 to 4 m. A further increase in the height increases the volume flow only weakly and an increase in the solar radiation increases the volume flow but less than linear.](image)
500 to 800 W/m² increases volume flow by about 17%. There is no improvement with double glazing; a single one behaves best (for cold climates that might be different!).

4.2 The absorber and the stack extension

The transition between the absorber and the stack extension should be as smooth as possible. Each narrowing, widening or redirecting increases friction. In an ideal case, the stack extension has the same depth and width as the absorber and both are vertical.

The U-value of the walls of the stack extension should be at least 1.5 W/m²K, worse values decrease the volume flow remarkably (Fig. 8a). The increase of the volume flow decreases with the stack extension height, then only the first meters are helpful and a too high stack extension can reduce or stop the volume flow (Fig. 8b).

For absorbers with a small height difference <= 3m (absorber nearly horizontal on a barrel roof), a stack extension with thermal insulation delivers remarkable improvement!

The optimal absorber has only a single glazing to receive the maximum of solar radiation and should be oriented with its slope to maximal solar radiation. But on the other side, this causes higher losses and a combination with a well-insulated stack extension is the best way for further acceleration of the air. This combined system delivers the best results if the total height of both systems H+h is about 8 to 10 m. The best use of the advantages of both systems would be an absorber where the maximal temperature is reached (and not higher) plus a stack extension for further acceleration. A combination of 2 m absorber and 6 m stack extension behaves better than an absorber of 8 m.

4.3 The stack in a building

To ventilate also storeys that are located below the absorber and the stack extension a stack in the building is added to transfer air (see Figs. 2 and 13 (c)). Such a stack would have a length of about 3, 6, 9, 12 m and so on for an increasing number of storeys. Obviously, the volume flow of the whole chimney system will decrease with an increasing length of the stack. This influence (caused by friction) is remarkable and leads to minor velocities of less than 0.15 m/s. A smaller velocity leads to a smaller Reynolds number and thus to a higher friction coefficient $l$ (eqn (7)), in this way, the whole systems inhibits itself and the volume flow decreases remarkably (Fig. 9).

Figure 8: The volume flow in dependency of the U-value of the walls of the stack extension (a, left). U-value should be 1.5 W/m²K or better to guarantee a high volume flow. Increase of volume flow with increasing height of the stack extension (b, right). The first meters have the highest effect, too high extensions should be avoided.
Further smaller Reynolds numbers indicate that the flow characteristic is in the transition range turbulent - laminar with very unstable conditions. The models presented in this paper do not represent this physical behaviour and consequently, the results are not completely accurate. Following all practical experience, the potential of a stack (without remarkable temperature difference) is exploited after a few storeys. But it seems to be possible to ventilate buildings with solar chimneys up to two to three storeys.

5 APPLICATION OF SOLAR CHIMNEYS IN RESIDENTIAL BUILDINGS

5.1 Existing residential buildings in Banda Aceh, Indonesia

Traditional architecture in hot and humid countries like Indonesia shows a light construction, large (roof) overhangs to keep facades in shadow and plenty of openings for natural ventilation (Fig. 10).

The city of Banda Aceh, Indonesia was hit by a Tsunami in 2004. After the disaster, the city was reconstructed and a wide variety of contemporary residential buildings was erected. That process was accompanied by scientific investigations. Sari [10] reports about the different types of new residential buildings, the temperature measurements and surveys checking out how the inhabitants perceived the thermal comfort in the building, in addition to detailed transient simulations to compare the different building typologies in a quantitative manner.

In general, it can be stated that the predominant building type has one or two storeys. The construction can be classified into three different types: Light, medium and heavy (Fig. 11). All three types exist but the most common is the heavy one, since it is preferred by the inhabitants because of the reasons of stability and a reference to a modern style of living (concrete).

Figure 10: Traditional house in Banda Aceh with overhang and ventilation openings [10].
5.2 Thermal behaviour of residential buildings with different constructions

Based on the measurements and simulations in [10], the schematic thermal behaviour of buildings with light (e.g. fibre cement board), medium (e.g. light bricks and plywood) and heavy (e.g. concrete) construction can be derived and is shown in Fig. 12. It must be mentioned that the graph shows the potential behaviour if the building and its concepts for shadowing and natural (cross) ventilation are optimized. Real buildings behave partly much worse if they are not optimally constructed [10].

It should be noted that the outdoor temperature typically does not follow simple a sinus behaviour. There is a well sinus-shaped basic swing in temperature, but because of the very strong solar radiation there is a rapid and strong further increase in temperature with a peak at about 2 pm. Temperatures stay high until sunset around 6 to 7 pm. During the whole night, we note a rapid drop in temperature.

The light construction follows widely directly the outdoor temperature. The resulting deep temperatures during night time were positively voted in the survey described in Ref. [10] – people can sleep in comfort. The disadvantage of higher temperatures during the day was not

Figure 12: Outdoor temperature and schematic temperature behaviour of a room with light, medium and heavy construction (based on findings in Ref. [10]).
voted very negatively, it seems that inhabitants are more habituated to higher temperatures when they are active and awake. The heavy construction shows the influence of thermal storage mass. Temperatures are balanced with values of a few degrees below the outdoor temperature during the day (a clear advantage if one wants to work in the building) with a peak around 4 to 5 pm but also remarkably higher temperatures during the night and early morning, a clear disadvantage if one wants to sleep. The last effect was voted quite negative but it seems that the problem is solved with AC during night. The medium construction lies between both extremes.

The literature recommends unanimously for hot and humid climates a light construction. The discussion here above shows that this holds for residential buildings and the wish for lowest possible temperatures during the night when people want to sleep. For usage during the day, a heavy construction is of advantage since it delivers the lowest temperatures. That could be an advantage especially in offices that are not engaged during night time.

5.3 Field of application for solar chimneys

Solar chimneys increase natural ventilation during daytime if the sun is shining. But the same chimney system can be used during the night time as a passive ventilation shaft using the temperature difference between indoors and outdoors.

Buildings with heavy construction need high ventilation especially during night time to remove the heat that is stored in the construction, here vertical ventilation shafts would be the appropriate system. During daytime the temperature indoors stays below outdoor temperature, ventilation should be reduced to the level that is necessary for the \( \text{CO}_2 \) removal, thus ventilation shafts should be held more closed.

Light (and medium) constructions show during the day indoor temperatures equivalent to outdoor temperatures. Because of the missing temperature difference indoor – outdoor, the basic ventilation concept is based on wind-driven cross ventilation to reach deep temperatures during the night. During daytime, a solar chimney could reinforce ventilation to remove heat gains and to deliver cooling air movement around the building’s users.

5.4 Possible arrangements of solar chimneys in existing or new buildings and potential energy savings

Buildings in hot and humid climates have in general sloped roofs because of heavy rains. It would be quite easy to integrate solar chimneys directly in such roofs (Fig. 13). For a chimney of 90 cm width and 30 cm depth, a height difference of about 3 m would be enough to produce an airflow of nearly 300 m\(^3\)/h. For a big living room of about 20 m\(^2\) that would result in a four to fivefold air exchange, a very good precondition for tempering the room and producing air movement around the people.

6 CONCLUSIONS

The investigation shows that solar chimneys can be integrated in the design of newly constructed buildings if the number of storeys is not more than 2 to 3. For rural areas and outer quarters in cities that corresponds often to the real situation. The paper delivers design rules for solar chimneys. They are rough but detailed enough to guarantee a well-functioning of the systems and avoid complex simulations. The limited space that solar chimneys need would allow also their implementation into existing buildings as part of their refurbishment.
Solar chimneys could improve user’s comfort without active systems which would need a high effort for installation, maintenance and financing of energy costs. Such passive and low-tech systems use specific quantities of the local climate and deliver widely comfort without negative impact on the necessary sustainable development. An alternative mechanical ventilation system (still without cooling!) needs about 0.6 Wh electricity per m³ transported air. A solar chimney that moves 300 m³/h for about 12 hours daily would thus save per year 300 * 12 * 365 * 0.6 / 1000 = 788 kWh electricity.

REFERENCES


[10] Sari, L.H., Thermal and Environmental Assessment of Post Tsunami Housing in Banda Aceh, Indonesia, Heriot-Watt University, School of the Built Environment, December 2011.