

# Stochastic Methods in Engineering

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# Stochastic Methods in Engineering

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# Contents

|  |           |
|--|-----------|
| <b>Preface</b>   | <b>xi</b> |
| <b>1 Introduction</b>                                      | <b>1</b>  |
| 1.1 Uncertainty .....                                      | 1         |
| 1.2 Structural analysis and design improvement .....       | 2         |
| 1.3 Reliability .....                                      | 4         |
| 1.4 Evolutionary processes .....                           | 5         |
| 1.5 Statistical comparison .....                           | 7         |
| <b>2 Randomness</b>  | <b>11</b> |
| 2.1 Random variables .....                                 | 11        |
| 2.1.1 Uni- and bivariate case .....                        | 11        |
| 2.1.2 Multivariate systems, random vectors .....           | 16        |
| 2.2 Transformed vector variables .....                     | 19        |
| 2.2.1 Standardization .....                                | 20        |
| 2.2.2 Principal components .....                           | 20        |
| 2.3 Probability .....                                      | 23        |
| 2.3.1 Relative frequency .....                             | 23        |
| 2.3.2 Distribution of random variables .....               | 25        |
| 2.3.3 Markov inequality, Chebyshev inequality .....        | 28        |
| 2.3.4 Distinct distributions .....                         | 30        |
| 2.3.5 Several random variables .....                       | 34        |
| 2.3.6 Discrete random variables .....                      | 39        |
| 2.3.7 Discrete random vectors .....                        | 41        |
| 2.4 Relations between variates .....                       | 42        |
| 2.4.1 Observations on several events .....                 | 42        |
| 2.4.2 Independent random variables .....                   | 45        |
| 2.4.3 Functions of random variables – expected value ..... | 46        |
| 2.4.4 Properties of expected values .....                  | 47        |
| 2.4.5 Determination of mean value and variance .....       | 48        |
| 2.4.6 Approximation of mean value and variance .....       | 49        |

|          |   |            |
|----------|---|------------|
| 2.4.7    | Functions of random variables – distribution .....                  | 54         |
| 2.4.8    | Stochastic simulation .....   | 57         |
| 2.4.9    | Appraisal of the approximation methods .....                        | 61         |
| 2.5      | Aspects of parameter estimation .....                               | 62         |
| 2.5.1    | Sample mean and variance .....                                      | 63         |
| 2.5.2    | Maximum likelihood .....  | 65         |
| 2.6      | Problems .....  | 68         |
| <b>3</b> | <b>Structural analysis by Taylor series expansion</b> .....         | <b>71</b>  |
| 3.1      | Random response .....   | 71         |
| 3.1.1    | Introductory remarks .....  | 71         |
| 3.1.2    | Finite element representation .....                                 | 74         |
| 3.2      | Linear elastic systems .....  | 76         |
| 3.2.1    | Probabilistic solution .....  | 76         |
| 3.2.2    | Computation of derivatives .....                                    | 80         |
| 3.2.3    | Grouping of random input variables .....                            | 82         |
| 3.3      | Large displacements .....   | 89         |
| 3.3.1    | Iterative solution .....  | 89         |
| 3.3.2    | Non-linear analysis .....   | 91         |
| 3.3.3    | Small strains .....   | 93         |
| 3.4      | Path-dependence – elastoplasticity .....                            | 93         |
| 3.4.1    | Incrementation .....  | 93         |
| 3.4.2    | Incremental analysis .....  | 94         |
| 3.5      | Performance of the incremental approach .....                       | 96         |
| 3.6      | Non-linear dynamics .....   | 98         |
| 3.7      | Problems .....  | 100        |
| <b>4</b> | <b>Design optimization, robustness</b> .....                        | <b>101</b> |
| 4.1      | The optimization task .....   | 101        |
| 4.1.1    | Design variables, objective function .....                          | 101        |
| 4.1.2    | Minimization by the steepest descent method .....                   | 102        |
| 4.1.3    | Accounting for constraints .....                                    | 104        |
| 4.2      | Implication of randomness .....                                     | 107        |
| 4.3      | Robustness .....  | 112        |
| 4.4      | Sensitivity of the objective .....                                  | 114        |
| 4.4.1    | Significance of input variables .....                               | 115        |
| 4.4.2    | System suitability for robustness .....                             | 116        |
| 4.5      | Design sensitivity of the response .....                            | 117        |
| 4.5.1    | Linear elastic systems .....  | 118        |
| 4.5.2    | Large displacements .....   | 118        |
| 4.5.3    | Path dependence – incrementation .....                              | 118        |
| 4.6      | Structural compliance optimization of a 25-bar space truss. . . . . | 119        |
| 4.7      | Antenna structure undergoing large displacements .....              | 122        |
| 4.8      | Problem .....   | 124        |

|          |  |            |
|----------|--|------------|
| <b>5</b> | <b>Monte Carlo techniques for system response and design improvement</b> | <b>127</b> |
| 5.1      | Random sampling and statistical synthesis .....                          | 127        |
| 5.1.1    | Outline of algorithm .....   | 127        |
| 5.1.2    | Application of method .....  | 129        |
| 5.2      | The input sample .....   | 137        |
| 5.2.1    | Sampling from univariate distributions .....                             | 137        |
| 5.2.2    | Sampling multivariate input .....  | 139        |
| 5.3      | Relationships between variables .....                                    | 140        |
| 5.3.1    | Input–output systems .....   | 140        |
| 5.3.2    | Simple linear regression .....   | 143        |
| 5.3.3    | Multiple regression .....  | 145        |
| 5.3.4    | Multivariate multiple regression .....                                   | 147        |
| 5.3.5    | Regarding non-linearity .....  | 150        |
| 5.4      | Modification of design .....   | 152        |
| 5.4.1    | Remarks on the objective function .....                                  | 152        |
| 5.4.2    | Design improvement .....   | 153        |
| 5.5      | Exploration of the design space .....                                    | 156        |
| 5.5.1    | Level of design parameters .....   | 157        |
| 5.5.2    | Response surface .....   | 158        |
| 5.5.3    | Latin square .....   | 160        |
| 5.6      | Problems .....   | 160        |
| <b>6</b> | <b>Reliability</b>   | <b>163</b> |
| 6.1      | Definitions .....  | 163        |
| 6.2      | Reliability of assemblies .....  | 167        |
| 6.2.1    | Elements in series .....   | 167        |
| 6.2.2    | Parallel assembly .....  | 170        |
| 6.2.3    | Mixed arrangements .....   | 172        |
| 6.2.4    | Weibull failure statistics .....   | 173        |
| 6.2.5    | Summary .....  | 177        |
| 6.3      | Assessment of system reliability .....                                   | 183        |
| 6.3.1    | Basic variables and performance function .....                           | 183        |
| 6.3.2    | Linear performance function .....  | 189        |
| 6.3.3    | Non-linear performance function .....                                    | 193        |
| 6.3.4    | Transformation to the standard normal space .....                        | 198        |
| 6.3.5    | Summary .....  | 202        |
| 6.4      | Reliability assessment by simulation .....                               | 205        |
| 6.4.1    | Integration by Monte Carlo techniques .....                              | 205        |
| 6.4.2    | Estimation of the failure probability .....                              | 208        |
| 6.4.3    | Antithetic variables .....   | 209        |
| 6.4.4    | Importance sampling .....  | 211        |
| 6.4.5    | Modelling the performance function .....                                 | 212        |
| 6.5      | Optimization and reliability .....                                       | 213        |
| 6.6      | Problems .....   | 217        |

|          |  |            |
|----------|--|------------|
| <b>7</b> | <b>Time variant phenomena</b>                            | <b>221</b> |
| 7.1      | Stochastic processes .....                               | 221        |
| 7.1.1    | Description .....  | 221        |
| 7.1.2    | Autocorrelation .....                                    | 222        |
| 7.1.3    | Stationarity .....                                       | 224        |
| 7.1.4    | Ergodicity .....   | 225        |
| 7.1.5    | An example .....   | 227        |
| 7.1.6    | Cross-correlation .....                                  | 228        |
| 7.1.7    | Summary .....  | 230        |
| 7.1.8    | Discrete representation of process .....                 | 231        |
| 7.2      | Random fields .....                                      | 233        |
| 7.3      | Input–output issues .....                                | 236        |
| 7.3.1    | Instantaneous response to single action .....            | 236        |
| 7.3.2    | Single response to two random processes .....            | 239        |
| 7.3.3    | The time rate of a random process .....                  | 240        |
| 7.4      | Time to failure – lifetime .....                         | 242        |
| 7.4.1    | Hazard rate .....  | 242        |
| 7.4.2    | Empirical approach .....                                 | 244        |
| 7.4.3    | System time to failure .....                             | 245        |
| 7.5      | Problems .....   | 248        |
| <b>8</b> | <b>Material deformation processes</b>                    | <b>251</b> |
| 8.1      | Significance of random input .....                       | 251        |
| 8.1.1    | Extension of viscous rod .....                           | 251        |
| 8.1.2    | Time discrete representation .....                       | 253        |
| 8.1.3    | Stochastic simulation .....                              | 255        |
| 8.1.4    | Taylor series approach .....                             | 256        |
| 8.2      | Finite element formalism for inelastic deformation ..... | 263        |
| 8.2.1    | Quasistatic deformation of inelastic solids .....        | 263        |
| 8.2.2    | Thermal effects .....                                    | 266        |
| 8.3      | Implication of randomness .....                          | 266        |
| 8.3.1    | Stochastic process simulation by finite elements .....   | 267        |
| 8.3.2    | Process analysis by Taylor series expansion .....        | 268        |
| 8.4      | Process design .....                                     | 273        |
| 8.4.1    | Optimization for robustness .....                        | 273        |
| 8.4.2    | Die for cold steady-state extrusion .....                | 275        |
| 8.4.3    | Preform for net-shape hot upsetting .....                | 278        |
| <b>9</b> | <b>Analysis and comparison of data sets</b>              | <b>283</b> |
| 9.1      | Information on the sample .....                          | 283        |
| 9.1.1    | Formal treatment of observation data .....               | 283        |
| 9.1.2    | Exploration of the covariance matrix.....                | 285        |
| 9.1.3    | Distance between observation units.....                  | 287        |
| 9.2      | Reduced representation .....                             | 290        |
| 9.2.1    | Principal component analysis.....                        | 291        |

|  |   |            |
|--|---|------------|
| 9.2.2  | Examples on model reduction.....                          | 292        |
| 9.3  | Comparison of two data sets.....                          | 295        |
| 9.3.1  | Parent population.....                                    | 295        |
| 9.3.2  | Hypothesis testing.....                                   | 295        |
| 9.3.3  | Test on the means.....                                    | 297        |
| 9.3.4  | Test on the covariance matrices.....                      | 303        |
| 9.3.5  | Tests on individual variables, discriminant analysis..... | 306        |
| 9.3.6  | Comparison of group to single observation.....            | 308        |
| 9.4  | Extension to random processes.....                        | 313        |
| 9.4.1  | Testing sets of realizations.....                         | 313        |
| 9.4.2  | Test on the mean vector.....                              | 313        |
| 9.4.3  | One-sample profile analysis on stationarity.....          | 314        |
| 9.4.4  | Two-sample profile analysis.....                          | 316        |
| 9.4.5  | The Kolmogorov–Smirnov test.....                          | 318        |
| 9.5  | Distance between random systems.....                      | 321        |
| 9.5.1  | Preliminaries.....  | 321        |
| 9.5.2  | Confidence region when comparing two<br>mean vectors..... | 322        |
| 9.5.3  | A discussion on bivariate systems.....                    | 325        |
| 9.5.4  | Confidence measures for multivariate samples.....         | 332        |
| 9.5.5  | Application to crashworthiness study.....                 | 338        |
| 9.5.6  | Test case from aerospace engineering.....                 | 343        |
| 9.5.7  | Summary: multivariate vs. univariate treatment.....       | 347        |
| <b>Appendix A Probability distribution of test functions</b> |   | <b>349</b> |
| A.1  | Chi-square ( $\chi^2$ ) distribution.....                 | 349        |
| A.2  | $F$ distribution.....                                     | 350        |
| A.3  | $T^2$ distribution.....                                   | 351        |
| A.4  | Student's $t$ distribution.....                           | 352        |
| A.5  | Beta ( $\beta$ ) distribution.....                        | 353        |
| A.6  | Wilks' $\Lambda$ (Wilks' $U$ ).....                       | 354        |
| <b>Bibliography</b>  |   | <b>355</b> |
| <b>Index</b>   |   | <b>357</b> |



## Preface

This book eventually addresses topics in computational stochastic mechanics. An increasing industrial demand for reliable quantification and management of uncertainty in product performance has occasioned considerable research and development activities in the area, predominantly but not only in aerospace engineering. Companies from other branches claiming for their products high quality within tight limits also indicated a vivid interest to involve computational stochastic methodology in innovative developments.

On this background in 2003 the author started offering lectures on Stochastic Analysis and Optimization at Stuttgart University conceived for the eighth semester, the last of the Diploma Course in Aerospace Engineering. Thereby the students become acquainted with essential notions of probability and enter the domain of computational methods for stochastic structural analysis and design evolving out from conventional, deterministic methodology. The lecture notes have been elaborated and extended to the book at hands.

Following the introduction into the subject of the book, Chapter 2 outlines probability and statistics as far as necessary for the purpose. Chapter 3 deals with probabilistic analysis based on a Taylor series expansion of the finite element equations of the structural system. The methodic development comprises elasticity, large displacements and elastoplasticity as well as an extension to nonlinear dynamics. The subject of Chapter 4 is design optimization. In the presence of randomness the engineer has to compromise between best performance in the mean and minimum scatter; tools are provided supporting the treatment of this task. Chapter 5 discusses statistical Monte Carlo techniques. Different to the analytic Taylor approximation, randomness is simulated directly by sampling the input to the system and registering

the outcome. Chapter 6 addresses the issue of reliability as related to the success or failure of structural members and structural systems. Essential notions are complemented by methods of reliability assessment both, analytic and synthetic in nature. In connection with optimization the requested degree of reliability enters the procedure as a constraint.

Time variant phenomena pose tasks which necessitate further considerations. Chapter 7 gives an elementary introduction into random, or stochastic, processes along with the formalism for their description; analogies to random fields are indicated. Also included is a brief discussion on the life time, the time to failure of operating systems and components. Chapter 8 is concerned with the evolutionary process of material deformation which is of importance in manufacturing as by metal forming operations and for safety studies involving destructive deformation. In addition to analysis in the presence of randomness, process optimization is treated along with the issue of robustness against random fluctuations of the conditions.

The last Chapter is devoted to the statistical assessment and comparison of random systems and stochastic processes. Given data from observations, hypothesis tests examine with a certain probability whether they originate from parent populations exhibiting same characteristics. Attention is paid to the quantification of the distance between random systems.

The book is conceived as an introductory text in stochastic mechanics with emphasis on computational methods. Numerical approaches are presented on the background of the stochastic investigation and design improvement of structures, mechanical parts and of material deformation as an evolutionary process; basic principles are transferable to different disciplines as well. Both stochastic simulation and probabilistic analysis have been founded on methods established in the deterministic context. The reader may thus enjoy the extension of his knowledge and skills in computational mechanics to the area of stochastics. The book should be of interest for master courses in engineering, doctoral students and equally useful for computational analysts in industrial and academic institutions.

The contents of the book are largely based on the authors lecture notes on stochastics including work performed with his team at Stuttgart University in the period between 1997 and 2005; textbooks consulted while writing are listed in the bibliography. The subject of robust design using the Taylor series approach was elaborated in the doctoral thesis of Zhan Kang. Results from statistical micromechanics

modelling have been obtained by Rainer Dattke in his doctoral dissertation within the framework of co-operative research on the strength of ceramics funded by the European Community. Data from laboratory measurements reported to in the book stem from the experimental part of the project carried out at the University of Caen under the guidance of Frederic Osterstock. The statistical investigations related to the analysis and comparison of data sets are the work of Friedrich Rau and Martin Werner associated with the joint European research project SCAT (Statistical Comparison of Analysis and Test). Numerical examples appearing in the text are assigned to the originating project partners from industry or academia.

The author acknowledges all the individual input to the content of the book, and esteems the stimulus of interactions with the students. He is thankful for having the opportunity to enjoy once again the efficient co-operation with the WIT Press publishers regarding the realization of this book project.

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Facing uncertainty.