Mechanics of Real Fluids

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The texts of the papers in this volume were set individually by the authors or under their supervision.

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This book is dedicated to the loving memory of
Sir James Lighthill, F.R.S. who gave the author tremendous
inspiration and a love for fluid mechanics science.

A fluid mechanics memoir

Fluid mechanics is an important branch of applied mathematics. It has enormous applications in our real world problems. Since it originated about three centuries ago it is considered old, but because of the new development in new diverse directions it is modern. On the basis of the classical theories with solid foundation, researchers in this field have advanced the subject tremendously. Many excellent treaties on fluid mechanics are available in the literature. There are many books in this area that were written by many pioneers in the subject. We can name a few books which are written during the last two centuries and which are still useful as reference texts for the scientists, engineers and applied mathematicians.

The Modern Development in Fluid Dynamics, published by the Clarendon Press, Oxford in 1938, was an outstanding treaty under the editorship of Sydney Goldstein. *Laminar Boundary Layers*, the fluid motion memoir, published by the Clarendon Press in 1963, under the editorship of L. Rosenhead was another excellent book authored by some pioneers in fluid mechanics. *High-speed Flow* edited by Leslie Howarth is another excellent book.

The Hydrodynamics authored by Sir Horace Lamb is considered a masterpiece of fluid mechanics. *The Introduction to the Homogeneous Turbulence* published by Cambridge University Press in 1953 was written by G.K. Batchelor and it is considered as a very good book in the fluid mechanics field. Recently Sir James Lighthill wrote an excellent book on *Informal Introduction to Theoretical Fluid Mechanics* published by the Clarendon Press in 1979. This book describes the fundamental theoretical development of fluid flow problems in the real world and is considered an outstanding masterpiece for the young scientists and applied mathematicians. *The Dynamics of Upper Ocean* written by O.M. Phillips and published by Cambridge University Press is another outstanding book in the water related science. *Modern Fluid Dynamics* published by Van Nostrand Company, London in 1968 written by N.J. Curle and H.J. Davies is one of the best

students' paperback editions. G.B. Whitham's book on *Linear and Nonlinear Waves* is a very good book on water wave problems. This fluid mechanics memoir is a compendium of works by many pioneering authors and research works of the author since he started publishing scientific papers in reputed journals. I have borrowed some physical concepts from my book *Water Waves: Relating Modern Theory to Engineering Applications* published by the Clarendon Press in 1995.

This book, I hope, will be suitable for the young scientists, graduate students, applied mathematicians and professional engineers. Theory is explained clearly and some applications are manifested in the book. The theory part is heavily borrowed from the standard textbooks but the applications part is completely new and hopefully the reader will appreciate my effort.

M. Rahman

Contents

Preface

Fluid mechanics is one of the most major areas of successful applications of mathematics. It can be considered as one of the branches of applied mathematics. The idea of writing this important book springs from Lighthill's *An Informal Introduction to Theoretical Fluid Mechanics* published by the Clarendon Press in 1986. Although fluid motion is concerned in both gas and liquids, as both of them are fluids, this book mainly deals with the motion of liquids in general, and water in particular. The theory of fluid mechanics has grown so considerably in recent years that study of the mechanics of fluids is important in many aspects of our real life.

In our real world all creatures live immersed in fluids (air or water) and their capability of motion through it is of crucial importance for their life style. As we know, systems of circulating fluid offer important means for distributing things where they are needed. As for example, the blood circulation in our body is vital. Similarly, the ocean is another great circulation system practically equally important to man. Energy stored as potential energy, chemical energy or heat energy becomes converted into kinetic energy in a water turbine, a gas turbine or a steam turbine, in each case by means of fluid flow action on rotating blades. Such flow is studied in order to improve the efficiency of turbines, which may also, in many cases, depend upon effective fluid motion for transferring heat quickly from one part to another in such an engine. Electric power generated by tides is the application of motion of water through the turbine. The design of structures intended to resist strong winds, river erosion, or violent sea motions requires an understanding of the forces exerted by winds, currents or waves upon stationary structures. So, comprehensive knowledge of flow of fluids is very important in all these cases. These are very complex problems and can be tackled only by advanced knowledge of boundary layer flow and turbulence because the fluid motion is usually propagated in a random fashion.

For complex fluid flow problem we must take recourse to the laboratory experiment or field experiment in association with the theory. Most fluid

motions are much too complex and a computer is essential to find solutions, but still the problem is so complex that even if the largest and fastest of the modern computers may fail to obtain the correct result. Great progress with the effective study, and the effective computations have been made, however, it is realized that such progress required creative input on a continuing basis both from theory and experiment.

The study of fluid flow, especially the theory of water waves, has been the subject of intense scientific research since the days of Airy in 1845. As we have described above, it is of great practical importance to scientists and engineers from many disciplines in gaining insight into the complex systems of fluid motions in oceans.

Chapter 1 briefly outlines the content of the book and gives an overview of the specification of the fluid motion. Described in Chapter 2 are the basic equations of fluid motion from the view point of general fluid dynamics. Developments of Euler equations of motion for inviscid fluids have been described from a mathematical view point due to the fact that Euler equations form the backbone for the study of water wave motion. This chapter also considers two important concepts with regard to vortex kinematics and vortex dynamics. Some examples of practical interest are solved using the theory developed in this chapter. Navier-Stokes equations, only, are cited for examples as equations for viscous fluid motion. The philosophy behind the source, sink, singularities and circulation of water particles is explained accompanied by some examples. Physical interpretations of velocity potentials and stream functions are clearly explained.

Chapter 3 contains the concept of mechanics of real fluids. We describe the motion in axially symmetric 3-D bodies; pressure distribution and drag forces on a sphere are evaluated. Some exact solutions of Navier-Stokes equations are considered in this chapter. Very slow motions of fluids as manifested by Stokes and Oseen are explained with examples. Chapters 4 deals with the two-dimensional fluid motion in laminar boundary layers. Boundary layer equations for a variety of problems are discussed. The Von Misses transformations and Pohlhausen's method are discussed. Concepts of momentum and energy integrals are clearly explained. Boundary layer thicknesses such as displacement $\ddot{a}_1$, momentum $\ddot{a}_2$ and energy $\ddot{a}_3$ are defined in integral forms. Flow in laminar wakes and jets are also considered.

Chapter 5 is devoted to the development of similarity technique and the perturbation method in fluid mechanics. Natural convection flow along a vertical plate is considered and its solution technique discussed by similarity analysis. The book concludes with Chapter 6 which is devoted to the theoretical development of turbulent flow. Some interesting theoretical examples are solved for the benefit of the graduate students who are working in the field of turbulence.

Some knowledge of vector calculus including the integral theorems such as Green's theorem, Stokes's theorem and divergence theorem is assumed

on the part of the reader. Isotropic tensor calculus is used sparingly in some chapters. A familiarity with the Bessel functions, Legendre polynomials and hypergeometric functions is also expected.

Matiur Rahman, 2011
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Dr Seyed Hossein Mousavizadegan deserves my special thanks for his assistance in drafting and designing some figures for this book. I also appreciate the assistance from Dr Adhi Susilo, Mr Rezaul Abid, Ms Rhonda Sutherland and Mrs Karen Conrod for helping me in drafting the figures contained in this book.

It is my great pleasure to acknowledge the kind permission granted by Professor Hubert J. Davies to reproduce Figures 5.4, 5.16, 6.5 and 7.2 from *Modern Fluid Dynamics*, Vol. 1 by N.J. Curle and H.J. Davies, published by Van Nostrand, London in 1968 into my book *Mechanics of Real Fluids*.

I am extremely thankful to Terry Edwards for sending me the formal letter of granting me permission to reproduce the figures 22, 23 and 29 from the book *An Informal Introduction to Theoretical Fluid Mechanics by James Lighthill, Clarendon Press, Oxford, 1986* into my new book, *Mechanics of Real Fluids by Matiur Rahman*. It is my great pleasure to acknowledge the IMA and its Executive Director, Dr David Youdan for granting me permission to use the illustrations from Lighthill's book. The Oxford University Press (the Clarendon Press) deserves my grateful appreciation for allowing me free permission to use some of the materials and illustrations contained in this book. Special thanks are given to Ms Shelagh Phillips, permission assistant at the Oxford University Press for taking troubles to verify the material included in this book and for her kind permission to use them in my new book.

I am also grateful to the referees for their kind reviews which help improve the contents of the book considerably, and to WIT Press, Southampton UK for their efforts in producing such a superb book.