

Boundary Collocation Techniques and their Application in Engineering

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Boundary Collocation Techniques and their Application in Engineering

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The first conference “Trefftz Method” organized in Cracow in 1996.

The persons in the photo are (from left):

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Contents

Preface	xi
Chapter 1	
Introduction	1
1 Boundary collocation versus other modelling methods	1
2 Different names of presented formulation.....	4
3 Short review of present monograph	5
Chapter 2	
General information	15
1 Description of the method – basic versions of boundary collocation.....	15
2 Relation between boundary collocation, boundary integral Trefftz approach and regular boundary element method	18
3 Conditioning of the Trefftz-type formulations	22
Chapter 3	
Analytical trial T-functions applied in the boundary collocation method	29
1 Complete systems of general solutions	29
2 Influence functions with singularities outside an investigated region	34
3 Influence T-functions represented by trigonometric series	37
4 Heuristic T-functions.....	38
5 Special purpose T-functions.....	39
Chapter 4	
Substructuring and T-elements	51
Chapter 5	
Basic numerical investigations of the boundary collocation method	63
1 Simply connected regions	63
2 Doubly connected regions.....	87

Chapter 6	
2D harmonic problems – the Laplace-type equations	95
1 Torsion of prismatic bars. Formulation of the boundary value problem.....	95
2 Application of the boundary collocation method to twisted bars – review	96
3 Torsion of a bar made of different materials	97
4 The Poiseuille flow.....	101
4.1 Formulation of the problem.....	101
4.2 Optimal filling of a cylindrical pipe in gravitational laminar liquid flow	104
5 Singular membrane problem	110
6 The Laplace-type solutions with special T-complete functions	116
6.1 Area with K symmetry axes	116
6.2 Region with a corner	118
6.3 Composite regions with cylindrical rods	120
7 Recent observations.....	125
Chapter 7	
Problems described by two-dimensional, biharmonic equations.....	135
1 Typical problems.....	135
2 Plane creeping flow. Boundary collocation as a modification of the Trefftz method.....	135
3 Special purpose trial functions for 2D creeping flow	140
3.1 Bi-symmetrical axis flow in a cylinder	140
3.2 Symmetrical flow in vicinity of cylinder.....	144
3.3 Flow in vicinity of rotating cylinder.....	146
3.4 Flow around a stiff corner	149
3.5 Stick-slip problem	153
3.6 Flow in vicinity of a rectangular corner, one wall of which moves at constant velocity.....	156
4 Thin plate bending.....	157
4.1 Boundary problem formulation	157
4.2 Construction of special purpose trial functions	158
4.3 Application of the special purpose trial functions. Example	163
Chapter 8	
Two-dimensional problem of elastostatics.....	175
1 Formulation of the 2D elastic problem.....	175
2 Example: tension of square plate with a central hole	177
3 Application of the T-complete Herrera and Kupradze functions	180
4 Optimization of helical spring cross-section	185
4.1 T-complete system	185
4.2 Preliminary numerical investigations	187
4.3 Optimization procedure of helical spring cross-section	188

Chapter 9	
Inverse problems in 2D elasticity	201
1 Introduction	201
2 The Trefftz formulation of the boundary value problem.....	201
3 Three methods of solving inverse problems.....	204
3.1 Optimization of unknown boundary conditions with respect to measurement functional (method 1).....	204
3.2 Direct comparison of calculated and measured stresses (method 2)	204
3.3 Extrapolation of inside solution (method 3).....	204
4 Numerical illustration.....	205
4.1 First example.....	206
4.2 Second example	209
4.3 Third example	214
5 Conclusions	219
Chapter 10	
Three-dimensional problem described by elliptic equations	223
1 General information	223
2 3D potential problems	223
3 3D creeping flow.....	225
4 Spatial (3D) elastostatic problems.....	226
5 Other applications of boundary collocation method to 3D problems.....	229
Chapter 11	
Non-homogenous equations and nonlinear boundary conditions.....	243
1 Non-homogenous harmonic problems	243
1.1 Integral form solution.....	243
1.2 Solutions with radial basis functions.....	246
2 Non-homogenous biharmonic problems	250
3 Boundary collocation method applied to non-linear conditions.....	252
3.1 Non-linear boundary conditions.....	252
3.2 Unknown position of source points or collocation points	253
3.3 Steady problem with variable boundary.....	254
3.4 Surface wave problems	254
Chapter 12	
Physically non-linear, elastic–plastic problems.....	259
1 Scheme of solution.....	259
2 Numerical illustration of the algorithm proposed.....	263
3 Final remarks.....	280
Chapter 13	
Problems described by parabolic equations	283

1	The boundary collocation method in boundary–initial problems.....	283
1.1	Introduction.....	283
1.2	The Ojalvo–Linzer method.....	284
1.3	Method based on the Laplace transformation.....	286
1.4	The boundary collocation method with use of time discretization...	287
1.5	Method based on fundamental solutions of heat transfer equation.....	287
2	The boundary collocation for non-stationary temperature field; comparison with the FEM.....	288
2.1	Formulation of the example.....	288
2.2	The BCM with Laplace’s transformation (BCL).....	289
2.3	The BCM with fundamental solutions as trial functions (BCF).....	293
Chapter 14		
Eigenvalue problems		301
1	Problem formulation and general description of the method.....	301
2	Example. Determination of eigenvalues for a silencer.....	302
Chapter 15		
Final remarks, conclusions and perspectives		311
Index		313

Preface

This book deals with applications of the boundary collocation method – also called the generalized Trefftz approach – to solve problems in engineering and science. The method consists of the use of trial functions, which satisfy the differential equations governing the problem. The unknown coefficients of those functions are determined by the satisfaction of the boundary and connectivity conditions.

As the method involves applying analytical solutions, the approach can be seen as a mixed analytical–numerical method. Discretization is required only on the boundary of the domain under consideration, while the analytical series describes the behaviour of the solution inside.

Hence, the Trefftz type approach can be related to the well known Boundary Element Method, but while the classical BEM formulation applies singular fundamental solutions of the Green's type, the Trefftz trial functions are regular, usually based on series.

The main limitation of the Boundary Collocation or generalized Trefftz method is the necessity of dealing with complete analytical set of, so called, T- or F-functions, which are not always known. This book presents a thorough survey of those functions, which allow for the solution of a wide range of numerical problems. Another important issue is related to the conditioning of the final algebraic set of equations, which, additionally, do not always result in symmetric matrices. The book discusses these problems and presents practical suggestions of how to avoid any ill-conditioning. A detailed investigation of the accuracy and convergence of the numerical solutions is also presented.

The generalized Trefftz method can be widely and efficiently applied to solve numerous linear and non-linear boundary value problems in structural, solid and fluid mechanics. This book illustrates its behaviour through many examples of the type that will be useful to scientists and engineers involved in the development and

application of numerical methods.

The book contains results obtained by the authors and some of their co-workers whose contributions are duly acknowledged throughout the text. We wish to express our gratitude for numerous valuable suggestions first and foremost to the late Professor O.C. Zienkiewicz (1921–2009), one of the originators of the finite element method and propagator of the Trefftz approach, as well as to Professors I. Herrera, J. Jirousek, M. Kleiber and M. Źyczkowski (1930–2005). The very helpful technical assistance of T. Betleja, A. Garlicka, B. Rostworowska, M. Fryc and K. Forma are also acknowledged.

The authors.
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