Integral Equations and their Applications

WIT*PRESS*

WIT Press publishes leading books in Science and Technology.
Visit our website for the current list of titles.
www.witpress.com

WIT*eLibrary*

Home of the Transactions of the Wessex Institute, the WIT electronic-library provides the international scientific community with immediate and permanent access to individual papers presented at WIT conferences. Visit the WIT eLibrary at
http://library.witpress.com

Integral Equations and their Applications

M. Rahman
Dalhousie University, Canada

WIT*PRESS* **Southampton, Boston**

Author:

M. Rahman
Dalhousie University, Canada

Published by

WIT Press
Ashurst Lodge, Ashurst, Southampton, SO40 7AA, UK
Tel: 44 (0) 238 029 3223; Fax: 44 (0) 238 029 2853
E-Mail: witpress@witpress.com
http://www.witpress.com

For USA, Canada and Mexico

WIT Press
25 Bridge Street, Billerica, MA 01821, USA
Tel: 978 667 5841; Fax: 978 667 7582
E-Mail: infousa@witpress.com
http://www.witpress.com

British Library Cataloguing-in-Publication Data

A Catalogue record for this book is available
from the British Library

ISBN: 978-1-84564-101-6

Library of Congress Catalog Card Number: 2007922339

Printed in Great Britain by Athenaeum Press Ltd.

Contents

Preface

While scientists and engineers can already choose from a number of books on integral equations, this new book encompasses recent developments including some preliminary backgrounds of formulations of integral equations governing the physical situation of the problems. It also contains elegant analytical and numerical methods, and an important topic of the variational principles. This book is primarily intended for the senior undergraduate students and beginning graduate students of engineering and science courses. The students in mathematical and physical sciences will find many sections of divert relevance. The book contains eight chapters. The chapters in the book are pedagogically organized. This book is specially designed for those who wish to understand integral equations without having extensive mathematical background. Some knowledge of integral calculus, ordinary differential equations, partial differential equations, Laplace transforms, Fourier transforms, Hilbert transforms, analytic functions of complex variables and contour integrations are expected on the part of the reader.

The book deals with linear integral equations, that is, equations involving an unknown function which appears under an integral sign. Such equations occur widely in diverse areas of applied mathematics and physics. They offer a powerful technique for solving a variety of practical problems. One obvious reason for using the integral equation rather than differential equations is that all of the conditions specifying the initial value problems or boundary value problems for a differential equation can often be condensed into a single integral equation. In the case of partial differential equations, the dimension of the problem is reduced in this process so that, for example, a boundary value problem for a partial differential equation in two independent variables transform into an integral equation involving an unknown function of only one variable. This reduction of what may represent a complicated mathematical model of a physical situation into a single equation is itself a significant step, but there are other advantages to be gained by replacing differentiation with integration. Some of these advantages arise because integration is a smooth process, a feature which has significant implications when approximate solutions are sought. Whether one is looking for an exact solution to a given problem or having to settle for an approximation to it, an integral equation formulation can often provide a useful way forward. For this reason integral equations have attracted attention for most of the last century and their theory is well-developed.

While I was a graduate student at the Imperial College's mathematics department during 1966-1969, I was fascinated with the integral equations course given by Professor Rosenblatt. His deep knowledge about the subject impressed me and gave me a love for integral equations. One of the aims of the course given by Professor Rosenblatt was to bring together students from pure mathematics and applied mathematics, often regarded by the students as totally unconnected. This book contains some theoretical development for the pure mathematician but these theories are illustrated by practical examples so that an applied mathematician can easily understand and appreciate the book.

This book is meant for the senior undergraduate and the first year postgraduate student. I assume that the reader is familiar with classical real analysis, basic linear algebra and the rudiments of ordinary differential equation theory. In addition, some acquaintance with functional analysis and Hilbert spaces is necessary, roughly at the level of a first year course in the subject, although I have found that a limited familiarity with these topics is easily considered as a bi-product of using them in the setting of integral equations. Because of the scope of the text and emphasis on practical issues, I hope that the book will prove useful to those working in application areas who find that they need to know about integral equations.

I felt for many years that integral equations should be treated in the fashion of this book and I derived much benefit from reading many integral equation books available in the literature. Others influence in some cases by acting more in spirit, making me aware of the sort of results we might seek, papers by many prominent authors. Most of the material in the book has been known for many years, although not necessarily in the form in which I have presented it, but the later chapters do contain some results I believe to be new.

Digital computers have greatly changed the philosophy of mathematics as applied to engineering. Many applied problems that cannot be solved explicitly by analytical methods can be easily solved by digital computers. However, in this book I have attempted the classical analytical procedure. There is too often a gap between the approaches of a pure and an applied mathematician to the same problem, to the extent that they may have little in common. I consider this book a middle road where I develop, the general structures associated with problems which arise in applications and also pay attention to the recovery of information of practical interest. I did not avoid substantial matters of calculations where these are necessary to adapt the general methods to cope with classes of integral equations which arise in the applications. I try to avoid the rigorous analysis from the pure mathematical view point, and I hope that the pure mathematician will also be satisfied with the dealing of the applied problems.

The book contains eight chapters, each being divided into several sections. In this text, we were mainly concerned with linear integral equations, mostly of second-kind. Chapter 1 introduces the classifications of integral equations and necessary techniques to convert differential equations to integral equations or vice versa. Chapter 2 deals with the linear Volterra integral equations and the relevant solution techniques. Chapter 3 is concerned with the linear Fredholme integral equations and also solution techniques. Nonlinear integral equations are investigated in Chapter 4. Adomian decomposition method is used heavily to determine the solution in addition to other classical solution methods. Chapter 5 deals with singular integral equations along with the variational principles. The transform calculus plays an important role in this chapter. Chapter 6 introduces the integro-differential equations. The Volterra and Fredholm type integro-differential equations are successfully manifested in this chapter. Chapter 7 contains the orthogonal systems of functions. Green's functions as the kernel of the integral equations are introduced using simple practical problems. Some practical problems are solved in this chapter. Chapter 8 deals with the applied problems of advanced nature such as arising in ocean waves, seismic response, transverse oscillations and flows of heat. The book concludes with four appendices.

In this computer age, classical mathematics may sometimes appear irrelevant. However, use of computer solutions without real understanding of the underlying mathematics may easily lead to gross errors. A solid understanding of the relevant mathematics is absolutely necessary. The central topic of this book is integral equations and the calculus of variations to physical problems. The solution techniques of integral equations by analytical procedures are highlighted with many practical examples.

For many years the subject of functional equations has held a prominent place in the attention of mathematicians. In more recent years this attention has been directed to a particular kind of functional equation, an integral equation, wherein the unknown function occurs under the integral sign. The study of this kind of equation is sometimes referred to as the inversion of a definite integral.

In the present book I have tried to present in readable and systematic manner the general theory of linear integral equations with some of its applications. The applications given are to differential equations, calculus of variations, and some problems which lead to differential equations with boundary conditions. The applications of mathematical physics herein given are to Neumann's problem and certain vibration problems which lead to differential equations with boundary conditions. An attempt has been made to present the subject matter in such a way as to make the book suitable as a text on this subject in universities.

The aim of the book is to present a clear and well-organized treatment of the concept behind the development of mathematics and solution techniques. The text material of this book is presented in a highly readable, mathematically solid format. Many practical problems are illustrated displaying a wide variety of solution techniques.

There are more than 100 solved problems in this book and special attention is paid to the derivation of most of the results in detail, in order to reduce possible frustrations to those who are still acquiring the requisite skills. The book contains approximately 150 exercises. Many of these involve extension of the topics presented in the text. Hints are given in many of these exercises and answers to some selected exercises are provided in Appendix C. The prerequisites to understand the material contained in this book are advanced calculus, vector analysis and techniques of solving elementary differential equations. Any senior undergraduate student who has spent three years in university, will be able to follow the material contained in this book. At the end of most of the chapters there are many exercises of practical interest demanding varying levels of effort.

While it has been a joy to write this book over a number of years, the fruits of this labor will hopefully be in learning of the enjoyment and benefits realized by the reader. Thus the author welcomes any suggestions for the improvement of the text.

M. Rahman
2007

Acknowledgements

The author is immensely grateful to the Natural Sciences and Engineering Research Council of Canada (NSERC) for its financial support. Mr. Adhi Susilo deserves my appreciation in assisting me in the final drafting of the figures of this book for publication. The author is thankful to Professor Carlos Brebbia, Director of Wessex Institute of Technology (WIT) for his kind advice and interest in the contents of the book. I am also grateful to the staff of WIT Press, Southampton, UK for their superb job in producing this manuscript in an excellent book form.