Domain Decomposition Techniques for Boundary Elements

Application to Fluid Flow

Advances in Boundary Elements Series

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Contents

Preface

The Finite Difference Method (FDM), the Finite Element Method (FEM) and the Finite Volume Method (FVM) currently exist as the principal numerical techniques used to solve fluid flow boundary value problems. Although there are analytical solutions for some simplified cases, mainly in one dimension, in many real-world applications, where an accurate prediction of the flow and transport processes is required, the above-mentioned numerical methods have provided significant contributions.

On the other hand, the Boundary Element Method (BEM), whilst being recognised as a very efficient tool in various types of engineering applications, e.g. heat transfer, electrostatics, wave scattering and propagation, stress analysis, crack propagation etc., has not been extensively used in fluid flow problems.

The BEM relies on transforming the governing partial differential equations (PDEs) through Green's second identity into an equivalent system of integral equations, which are in turn evaluated over the boundaries of the domain. Necessarily, the BEM requires the fundamental solution for the governing PDE in closed form. In the case when this is not possible, or is very difficult to be achieved, the terms of the original PDE that are not considered when the fundamental solution is derived would appear in a domain integral. For the engineering applications noted above, there exist fundamental solutions in a closed form and therefore in absence of body loads the integrals are evaluated only over the boundaries of the problem domain. This provides an opportunity for the use of the BEM relative to domain methods as only the boundary of the problem domain needs to be discretized, thereby saving time in preparation of the model input data. The advantage of the BEM is most pronounced for model domains extending to infinity, e.g. wave radiation and scattering. Using the BEM only the near-field finite surface needs to be discretized since a fundamental solution that naturally obeys the radiation condition exists, consequently avoiding the need for discretization of an infinite domain. These features, combined with the second-order accuracy of the BEM, have allowed the BEM to remain a favourite engineering analysis/designing tool for certain types of problems.

When the domain integrals cannot be avoided, there are several options available. The most commonly used technique in the past was to employ domain integration. Whilst being quite accurate this approach can also be time consuming for large problems, as the integration is not just over the boundaries but also over the domain of interest. An alternative is to convert the domain integrals into integrals over the boundary, by using, for example, the dual reciprocity (*Nardini and Brebbia*, 1983)

(DRM) or the multipole reciprocity (MRM) methods (*Nowak and Brebbia*, 1989; *Nowak and Partridge*, 1992). The dual reciprocity method appears to be the preferred approach nowadays when transforming the domain integrals to the boundary and its application will be explained in more detail in several chapters in this book.

One of the earliest works on the application of boundary integral equation methods (BIEM) to the flow in porous media is due to *Liggett* (1977). Other early efforts include *Butterfield and Tomlin* [1972] and *Lafe et al.* [1981], who have used the so-called zoning technique to solve anisotropic problems. When the domain of the problem is non-homogeneous, as is often the case in groundwater problems, the BEM formulation requires the use of sub-domains. *Liggett and Liu* (1983) mentioned that the BIEM could be used for non-homogeneous porous media with a large number of sub-regions: '… the BIEM could be broken into sub-regions as small as elements. If such division proves necessary, however, the finite element method would be a better choice.' That the BEM would loose its attractiveness if the number of sub-domains becomes large was certainly the view of the majority of the BEM users in the past. This view is easy to understand considering that the most important feature associated with the BEM is that the discretization requirements are restricted to the boundary of the problem. However, for certain applications, e.g., large problems, the BEM has a high CPU demand due to integration over the boundaries and solution of fully populated system matrix. Therefore, for very large problems, the main advantage of the BEM seems to be lessened and alternative BEM formulations are typically sought, e.g., the fast multipole BEM, BEM with domain subdivision, or other alternatives.

Among the first efforts on the use of BEM with a large number of sub-regions, resulting in a mesh that looks like a FEM mesh, is the work of *Taigbenu* (1990) who called the numerical scheme the Green element method (GEM). The GEM uses the fundamental solution of the Laplace equation and accounts for the remaining terms of the governing equation using domain integration. The advantages of this approach relative to the conventional BEM are a consequence of the fewer number of integrations over elements evaluated per source node, the global coefficient matrix is sparse, and the GEM more readily accommodates flow and medium inhomogeneities. The disadvantages are related to the complications associated with the mesh generation, common for all domain methods, and since the GEM ensures only one degree of freedom per node, the flux at the internal node is expressed in terms of the primary dependent variable by a difference expression thereby compromising the second order accuracy commonly associated with the BEM. *Taigbenu* (1991) demonstrated that the accuracy can be improved with the use of higher interpolating function (quadratic) while still performing the boundary and domain integrals analytically. The method has been applied to 1D and 2D heterogeneous flow problems, Helmholtz and Boussinesq equations, diffusion and advection-diffusion problems using linear and quadratic interpolating functions (*Taigbenu*, 1995) and also to the nonlinear unsaturated flow using linear interpolating function (*Taigbenu and Onyejekwe*, 1995).

Škerget and collaborators developed another BEM sub-domain approach, which is known as the Boundary-Domain Integral Method (BDIM). This approach has been used to solve convection-diffusion problems (•*agar et al*., 1994) using the fundamental solution of the convection-diffusion equation with constant coefficients, resulting in domain integrals with a convective term due to the perturbation velocity field. The formulation has been tested for high values of Pe (»10^6) and shows good accuracy. The BDIM has also been applied to transient non-linear convection-diffusion problems (*Škerget and Rek*, 1995) using the parabolic diffusion fundamental solution. The BDIM has the advantage over the GEM in that the fluxes are obtained directly from the integral equations, thereby preserving the inherent accuracy of the BEM. The advantage of the GEM with respect to BDIM is that for a same domain discretization the GEM would result in a smaller system matrix.

Another BEM sub-domain approach is the Dual Reciprocity Method - Multi-Domain (DRM-MD) approach, which has been introduced by *Popov and Power*. This formulation was a logical step after Popov and Power noticed substantial improvement in the performance of the DRM when used with sub-domains. The improvement relates to the accuracy and stability of the method. It also eliminates the problem of distribution of internal DRM nodes, usually used to improve the DRM approximation, since the distribution of the internal nodes in the DRM-MD is not any more an issue. The initial problem solved using this formulation was the flow of a mixture of gases through a porous media. The DRM-MD has also been applied to linear and non-linear advection-diffusion problems, driven cavity flow governed by the Navier-Stokes equations, flow of polymers inside mixers with complex geometries, flow through fractured porous media, two phase flow in porous media, etc.

We must emphasize that the sub-domain techniques in the BEM are nowadays finding its place in the toolbox of numerical modellers, especially when dealing with complex 3D problems. We see their main application in conjunction with the classical BEM approach, which is based on a single domain, when part of the domain needs to be solved using a single domain approach, the classical BEM, and part needs to be solved using a domain approach, BEM subdomain technique. This has usually been done in the past by coupling the BEM with the FEM, however, it is much more efficient to use a combination of the BEM and a BEM sub-domain technique. The advantage arises from the simplicity of coupling the single domain and multi-domain solutions, and from the fact that only one formulation needs to be developed, rather than two separate formulations based on different techniques.

There are still possibilities for improving the BEM sub-domain techniques. However, considering the increased interest and research in this approach we believe that BEM sub-domain techniques will become a logical choice in the future substituting the FEM whenever an efficient solution requires coupling of the BEM with a domain technique.

The Editors
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