

Plasticity of Cold Worked Metals

A Deductive Approach

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A Deductive Approach

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To Roberta,
whose love made this work possible

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Preface

Plasticity Theory is a basic tool of structural analysis used to evaluate the ultimate strength and the post-elastic behaviour of ductile structures. Modern computing codes make use of it, often by exploiting sophisticated computational analysis and advanced kinematics for large deformation. Yet, all this valuable effort has its Achilles' heel: the evolution law of the yield surface, also referred to as the work-hardening rule.

The work-hardening rules most in use to model the post-elastic behaviour of elastic-plastic materials are, essentially, the kinematic rule, the isotropic rule or some combination – more or less linear – of these. None is capable of reproducing the dramatic changes that the elastic domain of real materials can exhibit when plastically strained. Other work-hardening rules are also available. They are invariably more complicated than the former and possibly even less adequate to represent the post-elastic behaviour of a material when it comes to general three-dimensional elastic-plastic deformation processes. Still, these processes are well within the reach of many commercial computer codes; which in principle should enable us to make any general elastic-plastic analysis we may care to do. The lack of realistic work-hardening rules, however, sets serious doubts on the validity of the results that can thus be obtained. As a consequence, we often have to confine our applications to particular classes of processes, reduce the range of admissible deformation and make frequent recourse to experimental validation.

The present book approaches the work-hardening problem in a new and deductive way. It starts from a few elementary, hardly questionable facts and works out their inescapable consequences, following them through until a practical solution is obtained. The analysis it presents will show that plastic yielding of ductile metals is essentially virgin yielding – no matter how or how strong the material was cold worked originally.

The book will also provide the reader with a definite tool to predict all possible subsequent yield surfaces of an elastic-plastic material, once its virgin surface is given. This means that the whole family of subsequent yield surfaces of a material is completely defined by its virgin yield condition. Such a result will be obtained in a purely deductive way, without introducing any special assumption about the way in which plastic deformation affects the evolution of the elastic range.

The emphasis of the book is on the class of materials that goes under the name of von Mises materials. Most metals belong to this class. It should not be difficult, however, to extend the main concepts of the book to other ductile materials, should the need arise.

Of course, it is one thing to determine the family of all possible subsequent yield surfaces that an elastic-plastic material can ever exhibit if properly cold worked, and quite a different thing to find out which of these surfaces should be associated to which cold working process. To do this, we need to know the evolution rule of the parameters upon which the surfaces of this family depend. An evolution law that is fairly general and fits to practical applications will be proposed in Ch. 7. The problem here is, however, a constitutive one. As a consequence, the realm of deductive reasoning must be abandoned, to give way to the many, different and sometimes contradictory experimental data obtained from different materials, or even from different experiments on the same material. As any experimentalist in this field knows only too well, the main difficulty here originates from the fact that the evolution of the yield surface is essentially strain history dependent. As such, any attempt at finding a reasonably simple evolution rule requires us to first of all extract the most representative variables from the jungle of all possible strain histories.

The theory developed in this book will enable us to spot these variables. The evolution law proposed in Ch. 7 will then try to strike a balance between the complexity caused by the strain history dependence, on the one hand, and the need for a practical tool to be handled by numerical codes on the other. The adequacy of the proposed law is evaluated in Ch. 9, where the predictions from the present theory are compared with the results taken from some well-known experimental works.

The present work grew gradually over a span of more than 10 years. On various occasions, I communicated parts of it at scientific meetings and conferences. As the material grew, however, it became evident to me

that it worked like the cogwheels of a clock: once split apart, the ticking would come to a stop. In the end, I surrendered and settled for this monograph.

The book is addressed to research workers and advanced students in Mechanical Engineering, with a standard background in Continuum Mechanics and Plasticity Theory. These readers will find in it all the material they need for the applications. In particular, Appendix B is especially written to facilitate practical handling of rotation tensors. Such tensors find extensive use in the analysis of subsequent yielding, as the latter is closely linked to acquired anisotropy of the elastic limit.

I tried to organize the text so that it could also be of use to software engineers aiming at embodying the new theory in their computing codes. In doing so, they could use the work-hardening rules I present in this book, or exploit the general results concerning the family of all possible yield surfaces to propose better and perhaps simpler work-hardening rules.

The experimental scientist will benefit from this book as well. The general analysis it presents will serve him as a guide through the maze of experimental data. It will provide a new and most effective tool to give order, sense and perspective to the experimental findings and to direct new experimental research. The latter is still much needed in spite of so many years of valuable experimental activity, which unfortunately was partly marred by the lack of adequate understanding of the theoretical basis of subsequent yielding.

Aiming at an essential booklet, I confined the references to those I actually used in developing the theory it contains. They include the lucid and thoughtful treatise by Lubliner [1] and the excellent monograph by Paul [2]. In them the reader will find all the necessary background and much more. The reader is also referred to Lubliner's book, as well as to any recent textbook on Plasticity Theory, for a more complete coverage of the state of art of the discipline. It should be said, however, that the theory to be presented here is largely independent of previous works on the same topic.

Finally, but not least of all, I would like to express my thanks to the many friends and colleagues who encouraged me to undertake this work and gave me their suggestions and advice. My former student, Dr Giorgio Carta, was most helpful in reading the whole manuscript and providing many of the calculations needed to write Ch. 9. Dr Michele Brun kindly helped me to correct some formulae in the first part of the

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A. Paglietti
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