Using maximum entropy to develop explicit formulae for friction factor calculation in pipe flow

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Abstract

This paper uses the maximum entropy model to develop explicit formulae to calculate the friction factor for three flow regimes present in the Moody Diagram without iterations for four different problems. The developed formulae calculate the friction factors for the critical point flows, for the smooth turbulent flows and for the laminar flows. The development of the friction factor formulae is based on the maximum entropy model. This development can be regarded as a conceptual model, but not completely, because of the relationship between the Reynolds number (Re) and the entropy parameter (M) determined by curve fittings accomplished with accurate experimental data. The developed friction factors formulae can be used to calculate the discharges, head losses and diameters of pipes for the steady state and for the extended period simulation. It is concluded that the developed formulae to calculate the discharges, the head losses and the diameters are correct and have the potential to be also used in real hydraulic networks. It is also concluded that the developed formulae made easier the calculation of the friction factor for the three flow regimes.
Keywords: maximum entropy, friction factor calculation, four different problems, steady state, pipe flow, moody diagram.

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1 Introduction

When a fluid flows from one point to another inside a pipe, there will always be a head loss (liquid or gas). This head loss is caused by the friction of the fluid with the inner surface of the pipe wall and by turbulences of the fluid flow. So, the greater the roughness of the pipe wall or the more viscous the fluid, the greater the head loss.

In order to establish laws that may govern the head losses in conduits, research and studies have been carried out for around two centuries. Currently the most precise and universally used expression for analysis of flow in pipes, which was proposed in 1845, is the well-known Darcy-Weisbach equation. However, it wasn't early found an accurate way to determine the friction factor (f). Only in 1939, almost 100 years after the Darcy-Weisbach equation, it was definitely established a law to determine the friction factor for the steady state, through the Colebrook-White equation. The determination of the friction factor is a difficult problem to solve for both steady state and for the transient state.

The Colebrook-White equation has been considered as the most precise law of resistance to flow and it has been used as a referential standard, but in spite of this and the whole theoretical fundamentalism and base associated to it, it has a feature which is inconvenient to some people: it is implicit in relation to the friction factor, that is, the unknown f is present at both sides of the equation, without possibility of being isolated from the others quantities presented at the equation. Its resolution requires an iterative process. It has given rise to many researchers, almost all over the world, to strive themselves in finding explicit equations, which could be used as alternatives to the Colebrook-White equation, to calculate the friction factor. Some more compact and simple, easier to be memorized, but with large deviations, others less compact and complex, more difficult to be memorized, but with minor deviations and some others matching simplicity and accuracy, with errors well reduced, in relation to the friction factor calculated with the Colebrook-White equation (Diniz and Souza [2]).

The concept of entropy was used to substantiate the connection between the deterministic and probabilistic worlds, the latter being unfamiliar to hydraulic engineers Chiu in Moraes [1].

Entropy is the cumulative probabilities function that measures the information generated and transmitted by an event, through the weighted sum by the probability of how many times an event has occurred.

According to the principle of entropy, in equilibrium state, a system tends to maximize the entropy on the entropy previously contained. By maximizing the entropy, it is estimated that the most likely event is the one that will happen. This principle can be used to model the most probable distribution of states of a system Chiu in Moraes [1].

From the concepts of entropy and information theory, Chiu in Moraes [1] rewrote the entropy equation and developed equations for open-channel flows from a conceptual form to the velocity distribution profile, shear stress distribution and sediment concentration distribution. Chiu in Moraes [1] used the method of listing the hypothesis with the highest probability of occurrence, that is, the method of maximizing the entropy functional for the development of these equations.

Chiu in Moraes [1] used the maximum entropy model and presented a relation between the entropy parameter (M) and the velocity distribution profile of any given flow in an open-channel cross section. From this relation it is possible to obtain the parameters relative to the friction of various formulas such as Universal, Chézy or Manning. This relation is valid for all flow conditions, from laminar to turbulent. Chiu in Moraes [1] concluded that the definition and demonstrated usefulness of the entropy parameter (M) as a new hydraulic parameter indicate the importance and value of the information given by the location and magnitude of maximum velocity in a cross section.

This paper uses the maximum entropy to develop explicit formulae to calculate the friction factor for four different problems. These formulae calculate the friction factors for three flow regimes present at Moody diagram without iterations: critical point flows, where the Reynolds number (Re) equals 3000 and smooth turbulent flows. Actually, the Reynolds number varies from 2500 to 4000 in the critical zone, but because of the experiments accomplished by McKeon et al. [3] and taken into account in this paper, the critical zone was characterized as a single point where the Reynolds number equals 3000 as just commented. The developed formulae also calculate the laminar flows as it will be shown ahead.

2 Literature review

Chiu et al. [4] presented a new velocity distribution equation based on the probability and entropy concepts as an alternative to the existing power law and universal law equations for pipe-flow studies. The probability concept was used to formulate the velocity distribution and the maximum entropy principle was used to derive the probability law governing the velocity distribution. The new equation, with only two parameters (the entropy parameter (M) and the maximum velocity), is applicable to the entire flow field in a pipe, from laminar to turbulent and regardless whether the pipe is smooth or rough. A new equation for the friction factor (f) was also derived based on the new velocity distribution equation, the flow shear stress and the universal head loss formula. This equation relates the friction factor to the velocity distribution through the entropy parameter (M). A statistical method was also presented to analyze and quantify the uncertainty in the velocity distribution due to uncertainties in the related parameters or variables.

Wang and Duan [5] used entropy as an indirect index to measure the reliability of gas distribution networks. The relative entropy which is the ratio of actual entropy and maximum entropy was used as the reliability constraint to find the optimal diameters of a real gas distribution network from a reliability and economy point of view with different diameter combinations. To obtain the optimal diameters it was used an improved genetic algorithm with a new set of suitable problem-specific genetic operators and fitness functions. The use of a real gas distribution network as example proved that the total investment cost is a little larger in the new design scheme based on reliability constraint, but both the economy and reliability of the gas distribution network are overall considered, so the optimal scheme is more close to the practice.

Jamasb et al. [6] prepared an optimization problem using genetic algorithm and developed a computer program and linked it to EPANET to calibrate the model. The problem optimization was prepared by using the genetic algorithm tool box in MATLAB7. The authors used the pipe friction factor (Hazen William Coefficient), nodal consumptions, combinations of both and pipe diameters as variables to be determined and applied the method to a hydraulic network for different consumption scenarios. Furthermore, by studying the common operating conditions in the example hydraulic network demonstrated that, synchronized adjusting demands and roughness as decision variables and using the observations related to the fire fighting condition (with maximum demand) lead to more precise results in calibration of model and system simulations. The developed program offers a powerful approach to decrease the effects of uncertainties.

Kim [7] developed an alternative approach, namely the address-oriented impedance matrix method (AOIMM), to the conventional methods of transient computation and parameter calibration for a heterogeneous pipeline network for a more robust calibration of a heterogeneous and multi-looped pipe network system. A genetic algorithm was incorporated in address generation of the impedance matrix for leak location as a search engine. Simultaneous calibration of wave speed, friction, leakage location and quantity is one distinct feature of the AOIMM, which means that the impact of various factors can be effectively evaluated. The potential of the proposed calibration algorithm was demonstrated when it was applied to a fictitious heterogeneous pipe network system. The computational costs of the AOIMM were substantially lower than those of numerical approaches and the address generation module allows for the adaptive calibration of leakage as well as for efficient allocation of memory storage. Transient analysis shows that the AOIMM results match those of simulations of MOC (Method of Characteristics) in several flow regimes.

Lambert et al. [8] investigated experimentally the effects of biofilm growth on pipe roughness with the variation of growth conditions of velocity, diameter and water source. Velocity was shown to be the dominant factor in determining the roughness of biofilms and their resistance to fluid shear. It was also analyzed the change to the pipe velocity profile once biofilm had developed. It was shown that Von Karman constant is much lower than the normal 0.40 in biofouled pipes and that it varies with velocity. A modified Colebrook-White equation accounting for this variation in the Von Karman constant was able to explain the variation in friction factor with Reynolds number in a biofouled pipe, which may provide a path to develop an accurate biofilm prediction equation.

Zhuang et al. [9] developed a four-stage framework for optimal planning of regional water distribution system. Most work of the first two stages is left to the decision makers of municipal planning bureau and water utilities. Dijkstra Algorithm is used at the third stage to find the least cost for layout of transmission mains between adjacent subregions. At the fourth stage, a model for diameter (the decision variables) optimization of regional water distribution systems was proposed with the objectives of minimal investment and operation costs and maximum entropy reliability. The shuffled frog-leap algorithm (SFLA), linked to EPANET as the hydraulic engine, was adopted to solve the model. The developed framework was applied and demonstrated on planning of a real regional water distribution system in China. The demonstrated case could serve as an example to other similar planning of regional water distribution systems.

Simpson and Elhay [10] presented a method for the computation of the Jacobian matrix formulae which must be used in order to fully account for the friction factor's dependence on flow when the Todini and Pilati method is applied with Darcy-Weisbach head loss formula. The authors proposed this method because the widely used Todini and Pilati method uses the Hazen-Williams head loss equation, where the Hazen-Williams coefficient is assumed to be independent of flow and because the widely used EPANET, while solving the pipe network equations in which the head loss is modeled by the Darcy-Weisbach formula, treats the friction factors as independent of flow when computing the Jacobian. With the correct Jacobian matrix, the Todini and Pilati implementation of Newton's method has its normally quadratic convergence restored. The method was demonstrated on an example network and showed an improvement over the accuracy obtained when not fully accounting for the dependence of the friction factor on flow in the computation of the Jacobian. The application of the new technique to the example network showed a smaller final error for the same number of iterations.

3 Method

Diniz and Souza [2] presented four explicit formulae to calculate the friction factor for four different problems without iterations for all flow regimes present at Moody diagram, including the critical zone. Now, it will be presented four explicit formulae to calculate the friction factor also without iterations for the same four problems, but only for three flow regimes present at Moody diagram and using the maximum entropy model. The calculated friction factors are used to calculate the discharges, head losses and diameters of pipes for the steady state and for the extended period simulation. The four formulae are going to be present in the form of block diagrams.

It will also be present four formulae to calculate *the entropy parameter (M)*. The formulae to calculate the entropy parameter were obtained through curve fittings and three formulae to calculate the friction factor (f) were developed through mathematical operations on another formula developed by other authors.

3.1 Block diagram for problem 1

The first formula is used to calculate the head losses of pipes given its discharges, lengths, diameters, the kinematic viscosity of the fluid that flows in the pipes and the gravity acceleration.

There are some remarks to be made about fig. 1. To calculate the entropy parameter (M), it is used a single formula that is valid for the laminar, the critical point and the smooth turbulent flows. For the laminar flows, “M=0”, for the critical point, “M=0,5” and for the smooth turbulent flow, “M>0,5”. Still regarding the formula to calculate M, it is important to observe that this formula is composed of two parts: a natural logarithm part and a sigmoid function part. The natural logarithm part was obtained from a curve fitting and the sigmoid function was inserted because of the critical point, in other words, the transition between the laminar flows and the turbulent flows. It is also important to notice that the friction factor formula to calculate the laminar, the critical point and smooth turbulent flows is the same. Another remark is that the formula to calculate the friction factors for the laminar, the critical point and smooth turbulent flows don’t depend upon the pipe roughness, as it should be expected. The remarks made in this paragraph are valid for the entropy parameters (M) and friction factors (f) formulae of the other three problems to be shown ahead.

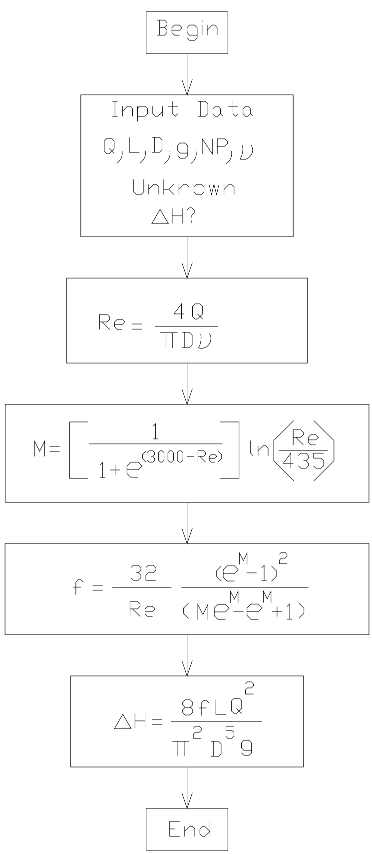

Figure 1: Block diagram for problem 1.

Notice that the Reynolds number (Re) is defined in fig.1. When the Reynolds number “Re=3000”, M is calculated for the flow in the critical point, when “Re>3000”, M is calculated for the smooth turbulent flows and when “Re<3000”, M is calculated for the laminar flows. It is important to remark that “3000” is the Re value for the critical point and this value is written in the formula to calculate M. The formula to calculate the friction factors for the laminar, the critical point and smooth turbulent flows was developed by Chiu et al. [4].

3.2 Block diagram for problem 2

The second formula is used to calculate the discharges of pipes given its head losses, lengths, diameters, the kinematic viscosity of the fluid that flows in the pipes and the gravity acceleration.

It is important to observe that “$Ref^{1/2}$” is defined in fig. 2. When “$Ref^{1/2}$=528”, M is calculated for the flow in the critical point, when “$Ref^{1/2}$>528”, M is calculated for the smooth turbulent flows and when

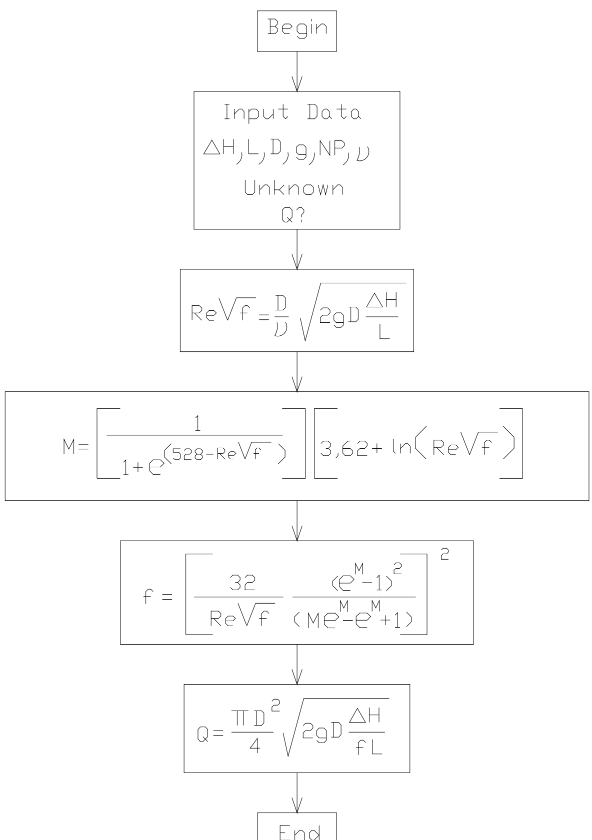

Figure 2: Block diagram for problem 2.

"$Ref^{1/2}$<528", M is calculated for the laminar flows. It is important to notice that "528" is the $Ref^{1/2}$ value for the critical point and this value is written in the formula to calculate M. The formula to calculate the laminar, the critical point and smooth turbulent flows was developed through mathematical operations on the friction factor formula for these flow regimes in fig. 1.

3.3 Block diagram for problem 3

The third formula is used to calculate the diameters of pipes given its discharges, lengths, head losses, the kinematic viscosity of the fluid that flows in the pipes and the gravity acceleration.

It is important to notice that "$Ref^{1/5}$" is defined in fig. 3. When "$Ref^{1/5}$=1498", M is calculated for the flow in the critical point, when "$Ref^{1/5}$>1498", M is calculated for the smooth turbulent flows and when "$Ref^{1/5}$<1498", M is calculated for the laminar flows. It is important to observe that "1498" is the

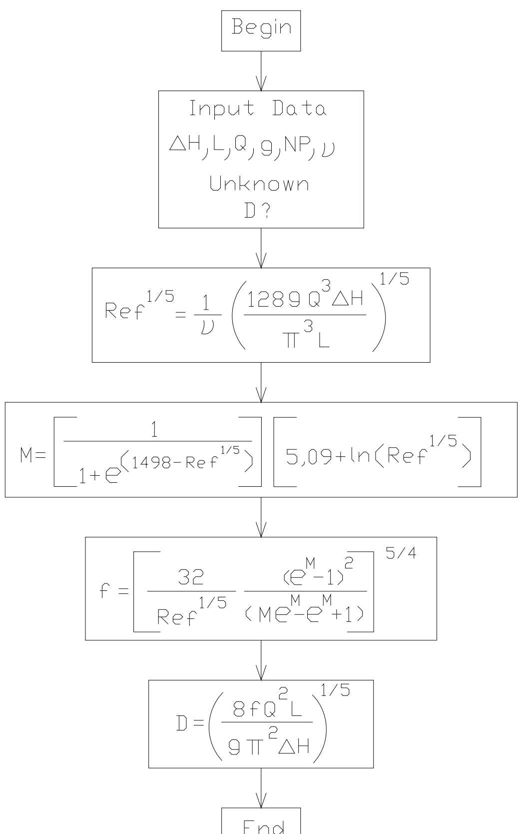

Figure 3: Block diagram for problem 3.

$Ref^{1/5}$ value for the critical point and this value is written in the formula to calculate M. The formula to calculate the laminar, the critical point and smooth turbulent flows was developed through mathematical operations on the friction factor formula for these flow regimes in fig. 1.

3.4 Block diagram for problem 4

The fourth formula is used to calculate the diameters of pipes given its lengths, head losses, the kinematic viscosity of the fluid that flows in the pipes, the fluid velocities in the pipes and the gravity acceleration.

It is important to regard that "$(Re/f)^{1/2}$" is defined in fig. 4. When "$(Re/f)^{1/2}=311$", M is calculated for the flow in the critical point, when "$(Re/f)^{1/2}>311$", M is calculated for the smooth turbulent flows and when "$(Re/f)^{1/2}<311$", M is calculated for the laminar flows. It is important to observe that "311" is the $(Re/f)^{1/2}$ value for the critical point and this value is written in

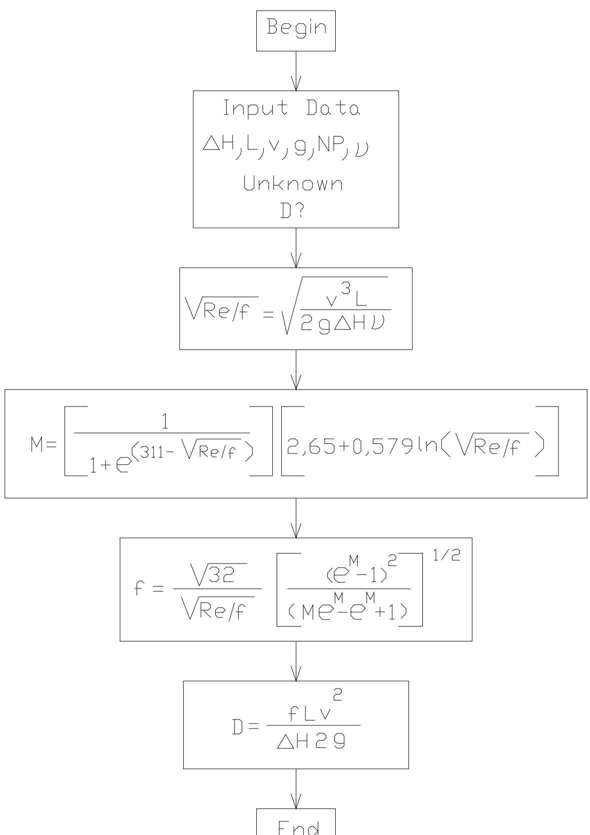

Figure 4: Block diagram for problem 4.

the formula to calculate M. The formula to calculate the laminar, the critical point and smooth turbulent flows was developed through mathematical operations on the friction factor formula for these flow regimes in fig. 1.

4 Conclusions

It is concluded the developed formulae to calculate the friction factors have the potential to be used in real hydraulic networks provided that that the fluid flow regime fits either in the laminar or in the critical point or in the smooth turbulent regimes. Another conclusion is that the developed formulae to calculate the friction factors can be used to calculate the discharges, head losses and diameters of pipes for the steady state and for the extended period simulation provided that the necessary input data is yielded. It is also concluded that the four presented formulae to calculate the friction factors are correct. First, because based on the researched literature, it is concluded that the formula to calculate the friction factors for problem 1 is correct. Second, because the three other formulae were developed based on the first formula. Another reason to conclude that the four formulae are correct it is because the problems these new formulae solve are the same problems presented in Diniz and Souza [2] solved with other formulae to calculate the friction factors as already mentioned. These authors have already applied the four problems to a real hydraulic network and the results were very reliable. It is also concluded that the developed formulae made easier the calculation of the friction factors for the three already mentioned flow regimes, because it is necessary only one formula (for each problem) to calculate the friction factor for the flow regime.

5 Recommendations

As surely noticed, it is missing the formulae to calculate the friction factors for the transition zone end for the wholly rough turbulent flow. So it is recommended to develop these formulae to publish them in a future work. Another recommendation is to test the developed formulae to calculate the friction factors for the laminar, the critical point and smooth turbulent flows at least theoretically, because in practice it is difficult to find a real situation where the fluid flow regime fits either in the laminar or in the critical point or in the smooth turbulent regimes.

6 Symbol list

{ΔH}	head loss of the pipes in any given hydraulic system (m)
{Q}	discharges in pipes in any given hydraulic system (m^3/s)
{D}	pipe diameters in any given hydraulic system (m)
{NP}	number of pipes in any given hydraulic system
{g}	gravity acceleration (m/s^2)
{υ}	kinematic viscosity of the fluid in any given hydraulic system (m^2/s)

{L} pipe lengths in any given hydraulic system (m)
{v} fluid velocity in pipes in any given hydraulic system (m/s)
{e} the natural logarithm base equal to 2,71828…

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