

Water distribution network optimisation using HydroGen test instances

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Abstract

The water distribution network (WDN) optimisation problem is shown to be a NP-hard problem. Many (metaheuristic) techniques have already been developed in this research area. Despite the aforementioned scientific attention, only a few, high-quality benchmark networks are available for algorithm testing, which, in turn, hinders profound algorithm testing, sensitivity analysis and comparison of the developed techniques. This absence of high-quality benchmark networks motivated us to develop a tool to algorithmically generate close-to-reality virtual WDNs. The tool, called HydroGen, can generate WDNs of arbitrary size and varying characteristics in EPANET or GraphML format. HydroGen is used to generate an extensive library of realistic test networks on which (metaheuristic) methods for the optimisation of WDN design can be tested, allowing researchers in this area to run sensitivity analyses and to draw solid conclusions on the robustness and performance of their methods. An iterated local search technique is developed and tested on a set of Hydrogen-generated water distribution networks.

Keywords: HydroGen, water distribution network design optimisation, iterated local search.

1 Introduction

Much research has been done on the optimisation of water distribution network design (WDND) over the past thirty years. An overview of developed (metaheuristic) techniques for this challenging optimisation problem is given in [1]. One shortcoming in this research area, as stated in [1], is the lack of adequate testing of the developed optimisation methods. Profound testing is hindered by two factors: (1) the number of available test networks (the so called benchmark networks) is limited and (2) these available test networks do not resemble real



WDNs. This absence of high-quality test networks motivated us to develop HydroGen [2], a tool that algorithmically generates WDNs of arbitrary size and varying characteristics. A short overview of HydroGen is given in the following Section. In Section 3, a mathematical formulation of the WDND optimisation problem is given. In Section 4, an iterated local search (ILS) technique is presented and used for the optimisation of both existing benchmark networks and HydroGen-generated networks. Experimental results can be found in Section 5. The last Section concludes and provides thoughts for further research.

2 HydroGen

Some attempts to construct realistic, virtual WDNs can be found in the literature. The EXNET network [3], Micropolis [4] and Mesopolis [4] are three manually constructed water distribution networks. Modular Design System [5] and WaterNetGen [6] are tools that generate virtual networks algorithmically. A more detailed overview of these methods can be found in [2]. The developed WDN generation method, HydroGen, attempts to address the shortcomings of the aforementioned methods. The tool generates realistic WDNs of arbitrary size and characteristics algorithmically. The networks are available in EPANET input format, since this is the most frequently used hydraulic solver, and GraphML, an XML-based graph exchange file format. A database is available online (<http://antor.ua.ac.be/download/hydrogen-uw>).

The generation method is divided into six phases:

Generation of clusters. In a first step, random cluster centres are generated in a two dimensional plane. The water demand nodes are constructed in a circular layout around these cluster centres, with a uniform random polar angle and a radius for which holds: $P[R_{min} \leq radius \leq \alpha^d(R_{max} - R_{min})] = \alpha$, with α being the probability that a generated point will lay within a certain distance from the center and d an adjustable parameter. The generated water demand nodes are not equal to the drinking water needs of individual households, but to clustered demands of households, restaurants, hotels, etc.

Generation of tree structure. A minimum spanning tree, connecting all of the demand nodes in the plane, is drawn, using Prim's algorithm [7]. These connections represent the water distribution pipes. Every pipe has a begin-node, end-node, length, diameter and roughness coefficient. Begin-node, end-node and length are defined by the generation itself: nodes are assigned while drawing the spanning tree and the edge weights or pipe lengths are the corresponding Euclidean distances. Pipes implicitly contain shut-off valves with their initial status set on open.

Addition of reservoirs, tanks and pumps. For every reservoir or tank that has to be added to the WDN, a random demand node at the outside of the cluster is selected, and the water supply is connected to this demand node with a new pipe or pump. Reservoirs are infinite external sources of water to the network. Tanks have

a limited water storage capacity and operate within their minimum and maximum water levels.

Generation of loops with intra-cluster pipes. Although a tree structure could efficiently provide every demand node with sufficient drinking water, no real-life WDN is designed as a tree. Loops are added to increase water delivery reliability. Moreover, huge pressure changes are avoided by an interwoven net. The user can predefine a cluster type for each cluster. Each cluster type corresponds with a specific meshedness coefficient M ($M =$ number of actual loops in comparison to the maximum number of loops), which in turn is related to the number of intra-cluster pipes that is added. Depending on the cluster type, some clusters will have more loops than others, which can be seen in Figure 1. Duplication of pipes and the crossing of pipes are avoided by a preliminary check. Moreover, in reality, the number of neighbouring pipes (or the degree of that node) is limited to four, which is also taken into account in the generation procedure. There are three pre-defined cluster types:

1. rural area ($M = 0$): no intra-cluster pipes, retaining minimum spanning tree configuration
2. very densely populated area ($0 \ll M$): generation of a complete planar graph (via triangulation), followed by adjustments so that the degrees of the nodes are ≤ 4 .
3. urban area ($M = 0.16$): generation of loops, avoiding duplication and crossing of pipes and maintaining degrees of the nodes ≤ 4 .

Three example networks, with an equal number of nodes but a different cluster type are shown in Figure 1.

Generation of inter-cluster pipes. Additionally, inter-cluster pipes can be added to the WDN. The user can specify the number of inter-cluster pipes that are added randomly, connecting different clusters. In contrast to the intra-cluster pipes, inter-cluster pipes can cross other pipes.

Assignment of base load and demand patterns. In a final step, both the demand pattern and the base load must be specified for every demand node. Five different user categories with corresponding base loads [8] and demand patterns are defined:

- residential (households): $0.01 \text{ m}^3/\text{h}$
- industry (chemical industry, production plants, factories, ...): $0.17 \text{ m}^3/\text{h}$
- commercial (restaurants, cafes, hotels, ...): $0.20 \text{ m}^3/\text{h}$
- public services (hospitals, universities, ...): $0.48 \text{ m}^3/\text{h}$
- energy (refinery, electricity and gas production): $121.68 \text{ m}^3/\text{h}$

The water demand of residential users, commercial users and public services is assigned to the pipelines and subsequently clustered to the connected demand nodes, based on the average number of customers per kilometre of pipeline in Flanders (Belgium), which is 43 customers per kilometre of pipeline for residential users [9], 20 commercial users per kilometre and 5 public services buildings per kilometre. The water demand of industrial users and energy plants is assigned to the nodes directly.

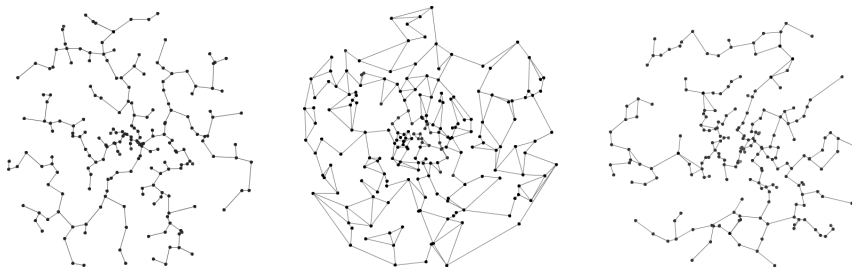


Figure 1: HydroGen examples of: rural area – type 1 (left); very densely populated area = type 2 (center); urban area = type 3 (right).

An abundant number of realistic network settings can be built by adjusting HydroGen's parameters. In [2], it is shown that HydroGen networks show high resemblance to real networks by using a graph-theoretical analysis. This high resemblance is in sharp contrast to the non-realistic benchmark networks and previously developed techniques.

3 WDND problem formulation

In this section, we discuss the basic WDND optimisation problem. The aim of this problem is to determine the optimal diameter and material for each pipe in a given network layout. The basic problem is a simplification of reality in that demand for water is considered to be static (or single period), no pumps are assumed to be present in the network (for this reason, these networks are called “gravity-fed”) and there is only one objective (cost minimisation).

The objective of this basic WDND optimisation problem is to minimise the total investment cost of the network design. The cost of an individual pipe depends on the type t that is chosen for this pipe from a list of commercially available types T . The type of a pipe determines both its diameter and the material of which it is made, which in turn determine its hydraulic properties. If the cost per meter of a pipe p of type t is represented by IC_t and the length of pipe p is represented as L_p , the objective function of the single period, gravity-fed WDND optimisation problem can be written as:

$$\text{minimize } \sum_{p \in P} \sum_{t \in T} L_p IC_t x_{p,t} \quad x_{p,t} \in \{0, 1\} \quad (1)$$

where $x_{p,t}$ is a binary decision variable that determines whether pipe p is of type t ($x_{p,t} = 1$) or not ($x_{p,t} = 0$).

The objective function is conditioned by physical mass and energy conservation laws, and by minimum head requirements in the demand nodes.

The *mass conservation law* must be satisfied for each node $n \in N$. This law states that the volume of water flowing into a node in the network per unit of time

must be equal to the volume of water flowing out of this node. Let Q_{in} represent the water flowing from node i to node j , and let S_n be the supply and D_n the demand of node n (all expressed in m^3s^{-1}) then the following should hold:

$$\sum_{i \in N/n} Q_{in} - \sum_{j \in N/n} Q_{nj} = D_n - S_n \quad \forall n \in N \quad (2)$$

Furthermore, for each closed loop $l \in L$, the *energy conservation law* must be satisfied. This law states that the sum of pressure drops in a closed loop is zero. Pressure drops (also called head losses) in piping systems are caused by wall shear in pipes and friction caused by piping components such as junctions, valves, and bends. In the basic WDND optimisation problem, only the first type of friction losses (in the pipes) are taken into account. For the closed loop l , the energy conservation law can therefore be stated as:

$$\sum_{p \in l} \Delta H_p = \sum_{p \in l} \frac{10.6668 y_p Q_p^{1.852}}{\sum_{t \in T} (x_{p,t} C_t^{1.852} D_t^{4.871})} = 0 \quad \forall l \in L \quad y_p \in \{-1, 1\} \quad (3)$$

In this equation, head losses in the pipes of the network are approximated using Hazen–Williams equations, with the parameters set to the values used by EPANET2.0, a frequently used hydraulic solver. y_p is the sign of Q_p , which is the water flow rate through pipe p (in m^3s^{-1}), L_p is the pipe length (in m), C_t is the Hazen–Williams roughness coefficient of pipe type t (unitless), D_t is the diameter of pipe type t (in m). Parameters D_t and C_t are determined by the type of a pipe and are assumed given for each available type.

Finally, *minimum pressure head requirements* exist for every (demand) node $n \in N$. Let H_n be the pressure head in node n (in m) and H_n^{min} the minimum pressure head in node n (in m). This constraint therefore can be represented as:

$$H_n \geq H_n^{min} \quad \forall n \in N \quad (4)$$

Several variants of the basic WDND optimisation problem have been proposed. These extensions are more realistic in that the water demand is dynamic (changes during the course of the day), that tanks (with finite capacity) are used in addition to reservoirs for the water supply and that pumps are added to the formerly exclusively gravity-fed networks. Another extension is the formulation of the above single-objective optimisation problem as a multi-objective (where other objectives such as reliability could be added to the cost minimisation objective). In Section 2 can be seen that HydroGen is not only able to generate test networks for the basic WDND optimisation problem, but also for the more complex extensions containing demand patterns, tanks and pumps.

4 Iterated local search heuristic

The WDND problem is shown to be NP-hard [10], therefore, metaheuristic techniques are applied to find satisfying solutions in reasonable times. An iterated



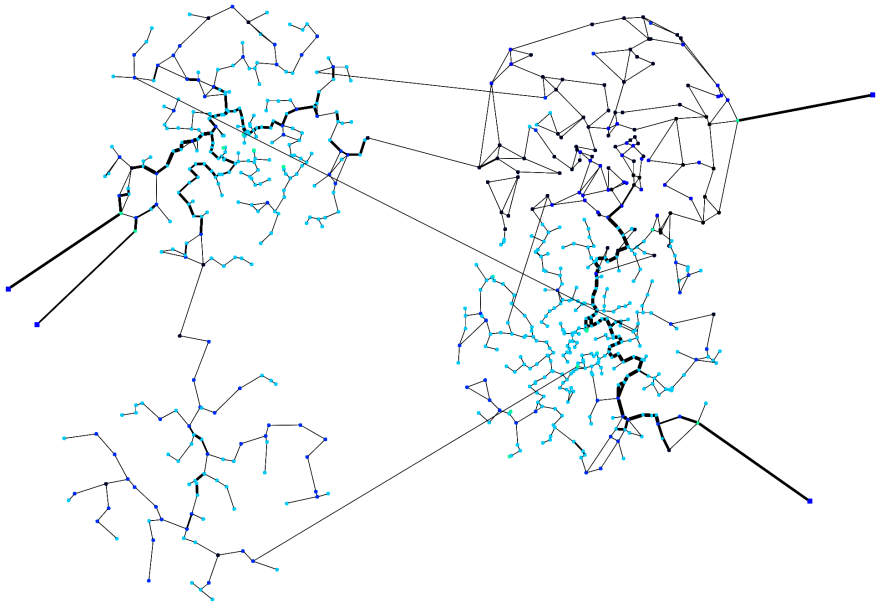


Figure 2: Optimised HydroGen network of 4 clusters: (1) clustertype 1–75 demand nodes, (2) clustertype 2–120 demand nodes, (3) clustertype 3–230 demand nodes, (4) clustertype 3–310 demand nodes.

local search (ILS) algorithm has been developed. ILS iteratively applies a large random change (perturbation) to the current solution, on which the local search algorithm is applied afterwards.

In this work, the basic version of the WDND problem is studied: single-period, gravity-fed, single-objective network design optimisation. ILS iteratively applies a large random change (perturbation) to the current solution, on which the local search algorithm is applied afterwards:

Sort. In a preliminary step, the set of pipes is sorted according to decreasing pipe length and the discrete set of available pipe types is sorted according to decreasing diameter.

Initial solution. In a first step, an initial feasible solution is generated. This solution is the configuration where each pipe is assigned the biggest diameter out of the set of available pipe types, in order to guarantee feasibility.

Local search. The local search algorithm iteratively moves to neighbour solutions by applying a move that tries to decrease the diameter of every pipe with one size. A first improving strategy (adjusting the current solution as soon as a feasible, lower cost solution is encountered) is applied until a local optimum is reached. If

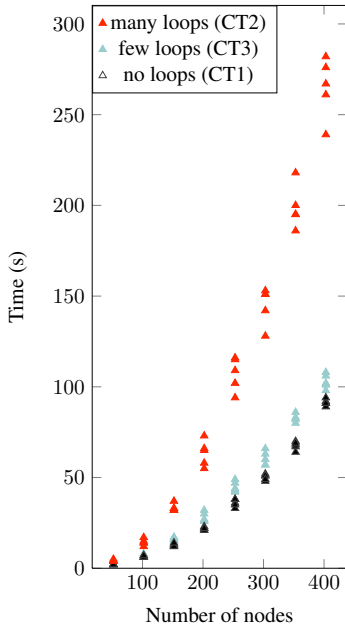


Figure 3: Calculation times of ILS in function of the number of nodes.

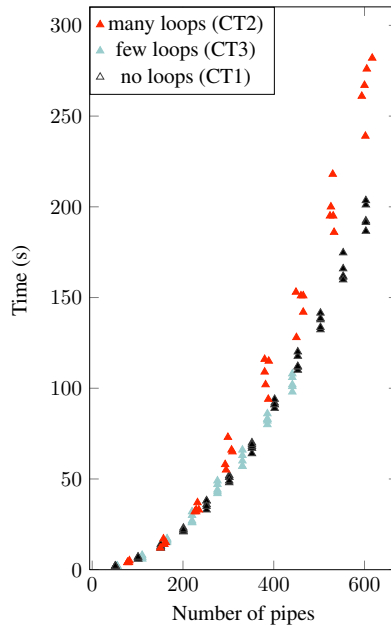


Figure 4: Calculation times of ILS in function of the number of pipes.

this solution is better than the incumbent solution, this new solution replaces the incumbent one as current local optimum.

Perturbation. A perturbation is applied on the current solution to escape local optima. In this perturbation step, for a percentage of randomly selected pipes, diameters are increased with one size. This perturbed solution is the input for the local search algorithm.

Termination criterion. For the current experimental setup, the heuristic stops after 100 iterations.

5 Experimental results

The heuristic was tested on the available benchmark networks (New York City Tunnels [11], Hanoi network [12] and Two loop network [13]) and on HydroGen test instances. ILS finds optimal solutions for the available benchmark networks, which confirms the not-too-challenging nature of those networks, as stated in the Introduction.

The HydroGen instances and the set of available pipe types with corresponding costs can be found at the ANT-OR website (<http://antor.ua.ac.be/>



Table 1: Results ILS on HydroGen test instances with p = number of pipes, l = number of loops, n = number of nodes. Time is in seconds.

Name	p	l	n	Cost (avg)	Time	Name	p	l	n	Cost (avg)	Time
50-1-1	51	0	52	82,072	2	50-3-1	55	4	52	64,165	2
50-1-2	51	0	52	93,088	2	50-3-2	55	4	52	106,161	2
50-1-3	51	0	52	61,020	2	50-3-3	55	4	52	57,694	2
50-1-4	51	0	52	74,403	2	50-3-4	55	4	52	93,675	2
50-1-5	51	0	52	72,030	2	50-3-5	55	4	52	82,623	2
100-1-1	101	0	102	116,101	6	100-3-1	110	9	102	198,003	7
100-1-2	101	0	102	89,126	6	100-3-2	110	9	102	98,772	8
100-1-3	101	0	102	129,827	6	100-3-3	110	9	102	102,945	7
100-1-4	101	0	102	108,089	7	100-3-4	110	9	102	218,055	6
100-1-5	101	0	102	119,407	7	100-3-5	110	9	102	119,562	6
150-1-1	151	0	152	124,161	13	150-3-1	165	14	152	206,937	17
150-1-2	151	0	152	217,761	13	150-3-2	165	14	152	208,064	17
150-1-3	151	0	152	182,722	12	150-3-3	165	14	152	193,410	16
150-1-4	151	0	152	119,896	12	150-3-4	165	14	152	182,723	16
150-1-5	151	0	152	128,207	14	150-3-5	165	14	152	195,338	15
200-1-1	201	0	202	188,806	22	200-3-1	220	19	202	224,773	26
200-1-2	201	0	202	175,398	23	200-3-2	220	19	202	246,827	26
200-1-3	201	0	202	157,010	21	200-3-3	220	19	202	212,997	30
200-1-4	201	0	202	224,879	21	200-3-4	220	19	202	259,114	32
200-1-5	201	0	202	147,733	22	200-3-5	220	19	202	249,628	27
250-1-1	252	0	253	182,099	36	250-3-1	276	24	253	222,686	47
250-1-2	252	0	253	192,563	38	250-3-2	276	24	253	237,939	42
250-1-3	252	0	253	175,797	38	250-3-3	276	24	253	220,729	43
250-1-4	252	0	253	177,041	33	250-3-4	276	24	253	207,337	49
250-1-5	252	0	253	196,086	35	250-3-5	276	24	253	236,656	44
300-1-1	302	0	303	199,627	48	300-3-1	331	29	303	293,342	60
300-1-2	302	0	303	174,222	51	300-3-2	331	29	303	249,475	57
300-1-3	302	0	303	157,977	52	300-3-3	331	29	303	221,862	63
300-1-4	302	0	303	185,115	51	300-3-4	331	29	303	211,990	57
300-1-5	302	0	303	202,527	49	300-3-5	331	29	303	275,222	66
350-1-1	352	0	353	249,320	70	350-3-1	386	34	353	251,396	82
350-1-2	352	0	353	166,115	64	350-3-2	386	34	353	311,434	83
350-1-3	352	0	353	201,251	67	350-3-3	386	34	353	212,278	80
350-1-4	352	0	353	180,788	68	350-3-4	386	34	353	242,706	83
350-1-5	352	0	353	249,730	69	350-3-5	386	34	353	258,463	86
400-1-1	402	0	403	218,097	89	400-3-1	441	39	403	297,728	101
400-1-2	402	0	403	241,361	92	400-3-2	441	39	403	320,679	108
400-1-3	402	0	403	199,615	91	400-3-3	441	39	403	288,329	98
400-1-4	402	0	403	215,811	91	400-3-4	441	39	403	234,989	102
400-1-5	402	0	403	184,748	94	400-3-5	441	39	403	217,215	106
450-1-1	453	0	454	354,389	110	450-3-1	497	44	454	278,123	143
450-1-2	453	0	454	207,621	112	450-3-2	497	44	454	256,779	141
450-1-3	453	0	454	207,597	118	450-3-3	497	44	454	383,187	136
450-1-4	453	0	454	294,367	120	450-3-4	497	44	454	342,294	128
450-1-5	453	0	454	243,030	112	450-3-5	497	44	454	211,830	133
500-1-1	503	0	504	253,956	138	500-3-1	552	49	504	240,588	161
500-1-2	503	0	504	268,311	132	500-3-2	552	49	504	259,373	173
500-1-3	503	0	504	224,787	134	500-3-3	552	49	504	255,199	177
500-1-4	503	0	504	298,962	142	500-3-4	552	49	504	249,443	169
500-1-5	503	0	504	245,501	139	500-3-5	552	49	504	347,346	153

Table 1: Continued.

Name	p	l	n	Cost (avg)	Time	Name	p	l	n	Cost (avg)	Time
550-1-1	553	0	554	281,400	175	550-3-1	607	54	554	341,069	192
550-1-2	553	0	554	240,978	166	550-3-2	607	54	554	322,642	211
550-1-3	553	0	554	214,410	161	550-3-3	607	54	554	313,738	202
550-1-4	553	0	554	232,131	162	550-3-4	607	54	554	336,675	196
550-1-5	553	0	554	250,098	160	550-3-5	607	54	554	307,190	200
600-1-1	603	0	604	295,176	187						
600-1-2	603	0	604	310,072	192						
600-1-3	603	0	604	223,043	192						
600-1-4	603	0	604	248,105	201						
600-1-5	603	0	604	203,011	204						
50-2-1	78	27	52	517,296	4	250-2-1	382	130	253	9,834,840	100
50-2-2	78	27	52	523,516	4	250-2-2	389	137	253	8,547,468	115
50-2-3	81	30	52	1,247,833	4	250-2-3	380	128	253	10,097,439	109
50-2-4	78	27	52	2,020,499	4	250-2-4	388	136	253	7,631,993	94
50-2-5	82	31	52	410,842	5	250-2-5	380	128	253	13,901,087	116
100-2-1	157	56	102	2,536,095	17	300-2-1	449	147	303	15,290,843	153
100-2-2	150	49	102	1,928,193	12	300-2-2	465	163	303	11,079,920	151
100-2-3	162	61	102	1,793,187	15	300-2-3	465	163	303	11,811,356	142
100-2-4	159	58	102	3,481,872	14	300-2-4	460	158	303	12,820,643	151
100-2-5	158	57	102	2,684,722	14	300-2-5	450	148	303	8,924,209	128
150-2-1	229	78	152	4,647,135	33	350-2-1	531	179	353	16,240,748	195
150-2-2	226	75	152	4,814,073	32	350-2-2	530	178	353	19,977,708	218
150-2-3	229	78	152	3,672,382	32	350-2-3	533	181	353	11,680,479	186
150-2-4	235	84	152	4,486,011	33	350-2-4	524	172	353	13,018,354	195
150-2-5	232	81	152	4,553,862	37	350-2-5	526	174	353	13,025,610	200
200-2-1	309	108	202	8,458,857	65	400-2-1	617	215	403	18,863,235	282
200-2-2	299	98	202	8,074,587	73	400-2-2	602	200	403	15,736,287	239
200-2-3	295	94	202	4,179,127	55	400-2-3	600	198	403	22,427,996	267
200-2-4	307	106	202	5,373,698	66	400-2-4	594	192	403	14,343,928	261
200-2-5	293	92	202	6,141,137	58	400-2-5	605	203	403	21,645,310	276

download/hydrogen-uw). A minimal pressure of 30 *m* is required in every node. For this experimental set-up, the termination criterion was set at 100 iterations. The perturbation rate is set at 60 percent. Every network was run 10 times, reported data are averages of the results of 10 runs. EPANET version 2.0 was used as hydraulic solver and experiments were done using a personal computer Intel Core i7 with 2.70 GHz processor and 3 GB Ram.

The ILS algorithm is used to optimise several HydroGen instances. In Table 1, results are presented. Since no other techniques have been applied on these networks yet, comparison is difficult. From Figures 3 and 4, it is clear that, within a certain cluster type, running times increase when the number of nodes (top) or number of pipes increases. Moreover, for a given number of nodes, running times also increase when the networks become more clustered.

6 Conclusions and further work

It has been shown that many (metaheuristic) techniques for WDND optimisation are poorly tested, which makes it difficult to draw solid conclusions on their performance. This lack of rigorous analyses is mainly caused by the absence of high-quality test networks, which motivated us to develop a tool to algorithmically generate realistic, artificial WDNs. The developed tool, HydroGen, generates WDNs of arbitrary size and varying characteristics in EPANET and GraphML format. The use of cluster types; the possibility to add dynamic demand profiles, tanks and pumps and parameter fine-tuning enables HydroGen to generate close-to-reality WDNs. HydroGen is used to generate an extensive library of realistic test networks, which is available via <http://antor.ua.ac.be/download/hydrogen-uw>.

An iterated local search metaheuristic has been developed and tested on both benchmark as HydroGen test instances. The heuristic finds optimal results for the benchmark networks, which confirms the not-too-challenging nature of these networks. Results on the HydroGen instances are presented, which will hopefully stimulate researchers in this area to test and compare their techniques on these instances and will allow researcher to draw more solid conclusions on the robustness and performance of their methods.

As stated before, the optimisation of the static, single-period problem could be extended to a multi-period problem with hourly water demand patterns. Pumps could be added to the gravity-fed network setting. Moreover, the single-objective problem could be extended into a multi-objective optimisation problem, where other objectives such as maximisation of network reliability could be taken into account on top of the current cost minimisation goal.

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