A study of an implementation method for making a synchronised schedule of the mass transit network

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Abstract

In this paper we propose the concept of entropy of the timetable to evaluate the mass transit system. This concept simplifies the problem of the synchronisation of the timetable. Additionally, we show how to use the concept of the entropy for evaluation. We first describe the importance of the synchronised periodic railway timetable, and also the conditions for the synchronisation of the timetable. We next present a difficulty encountered when we synchronise the timetable completely everywhere in the transit network. This difficulty is attributed to the restriction of train headway in the synchronisation of the timetable. Then we must simplify the problem of synchronisation of the timetable by removing some constraints. Lastly, we indicate that using the concept of the entropy is useful for solving the complex problem.
Keywords: entropy, synchronisation of timetable, periodic timetable, evaluate mass transit system.

1 Introduction

In mass transit systems such as railway networks, there are a large amount of passengers, and OD-pair passengers travelling in a huge number of combinations. Even in the demand of the same OD-pair, it is common that there are various times when the passengers wish to start and end their travelling. In order to strictly manage all such demands for the travelling, a huge number of trains need to operate. However the capacity of the tracks is limited, and the number of trains required exceeds this capacity. In addition, the number of passengers on each train often becomes too small to operate a train efficiently. Vuchic [1] analysed

the topology of the railway network in the mega cities from the point of view of solving the capacity shortage of the infrastructure.

Actually, many mass transit systems are managed by a small number of service routes. The number of service routes is small when compared with the number of varieties for demand. By limiting the number of service routes, the passengers of various OD-pairs have to board trains together. Moreover, most of the passengers must select a scheduled train which is permitted by the passenger for travelling. That is, the passengers are limited in their behaviour for travelling by the timetable of trains.

However, the operator can manage the huge demand for travelling by limiting the number of service routes and trains, in the case that the capacity of the trains is large enough and the schedule of trains is appropriate. Furthermore, decreasing the number of trains leads to avoiding excess capacity on the tracks. It is ideal that many passengers can travel at the time they desire and have fewer detours caused by the provided transit service. Whereas, by limiting the service route, many passengers will have to change trains at some junctions to arrive at their destination. Therefore, it is important to decrease the waiting time at junctions as much as possible for passengers. This requirement causes an additional constraint for the timetabling problem. Adding this "connection constraint" causes the solution for timetabling problems to become more difficult to find. If a disruption occurs in the railway network with a completely synchronised timetable, the propagation of disruption becomes a serious problem [2].

In this paper, we consider the synchronisation of the timetable. First we will show some examples of the synchronisation of the timetable and the tight constraints for synchronisation. Secondly we will consider the case that the timetable is synchronised with little incompletion. Then we propose to use the concept of the entropy of the information theory to evaluate the level of synchronisation.

2 Synchronisation of timetable

In this section we describe the synchronisation of the timetable.

2.1 Definition

The synchronised timetable is the state in which the scheduled transfer time to the succeeding trains is not to long.

It is desirable to realise the synchronisation of the timetable at most of the junctions.

2.2 Importance of the synchronised timetable

When the timetable of the transit network is synchronised, we can reduce the stagnant time of the passengers who transfer trains at junctions. Reducing the stagnant time at the junctions means reducing the unnecessary time for the

passengers. Moreover the travelling time of the passengers is reduced. This is desirable for the passengers, and it is a great pleasure, not only the passengers but also for the operators of the transit systems. The reason for this is because the operators can manage the demands of passengers well, while using less rolling stock and less crew.

2.3 Example at a junction

We introduce the example of the timetable at Bern in Switzerland and we show that the timetable of "Inter City" and "Inter Ragio" at Bern is synchronised.

2.3.1 Synchronisation at Bern
The departure and arrival times at Bern station is as shown in Figure 1. The arrows towards the circumference from the interior of the circle show the arrival times of the trains. The arrows towards from the circumference of the circle to the outside show the departure times of the trains. The arrows with dashed tails describe the trains from/to east area (e.g. Zürich, Olten and Luzern). The arrows with solid tails describe the trains from/to south and west area (e.g. Spiez and Lausanne). All passenger transfer trains are described with different types of arrows.

We recognise that there are two groups of arrivals and departures. One group of arrows touches the circumference near the top the circle. The other one touches the circumference near the bottom the circle.

Within each group, the transfer time for trains within the same group is less than 20 minutes. Furthermore, the transfer time between the two groups is never more than 4 minutes. This shows that the passengers can transfer trains in the same group of this figure. Thus we can say that the timetable at Bern is synchronised.

2.4 A case of the cyclic route

We consider the synchronisation of the timetable of the cyclic route. And we assume that the period of the integrated timetable is h and the necessary time travelling round this circuit is r.

If equation (1) is satisfied, we can say the timetable is synchronised everywhere in the cyclic route.

$$\exists k \in \mathbb{N}^+, h \gg \delta \geq 0 \text{ s.t. } r + \delta = k \cdot h. \tag{1}$$

$\mathbb{N}^+$ describes the set of nonnegative integers. In addition, we can say that the timetable is synchronised enough when the transfer time at each junction is similar to or less than δ.

2.4.1 Example
Take the circuit of the railway network that consists, for example, of the links: Bern – Visp – Lausanne – Bern. When travelling on this circuit, we must transfer at Bern, Visp and Lausanne when we pass.
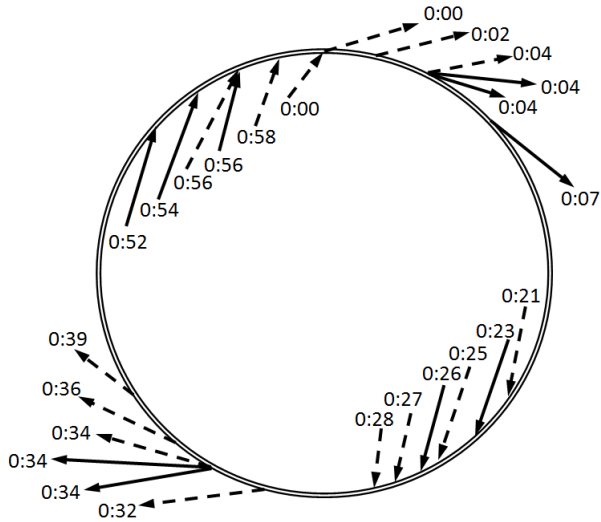
Figure 1: Configuration of connection at Bern.

There exists the trains operated by the timetable as in Table 1 every hour. The period of the clockwise route is $h_c = 60$ and the necessary time is $r_c = 229$. The period of the anticlockwise route is $h_{ac} = 60$ and the necessary time is $r_{ac} = 230$ similarly. The maximum transfer time on the clockwise route is 11 minutes, and on the anticlockwise route it is 10 minutes.

Table 1: Transfer timetable of circuit.

Clockwise			
Bern $\rightarrow$	Visp $\rightarrow$	Lausanne $\rightarrow$	Bern
0:07	1:02/1:06	2:40/2:50	3:56/(0:07)
Anti-clockwise			
Bern $\rightarrow$	Lausanne $\rightarrow$	Visp $\rightarrow$	Bern
0:04	1:10/1:20	2:53/2:57	3:54/(0:04)

Then δ and k satisfy the conditional expression (1) are as in Table 2. And the transfer time in each direction is equal to δ. Thus we can say that the timetable of this example is synchronised.

Table 2: Values: δ and k.

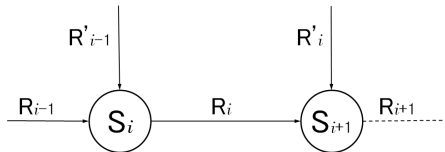| Direction	δ	k
Clockwise	11	4
Anti-clockwise	10	4

Figure 2: Example of a network.

2.5 A case of multiple merging

We consider the synchronisation of timetable when merging a route with another continuously.

2.5.1 Topology of network
We consider the configuration of railway network as in Figure 2. Two routes which are R_{i-1} and R'_{i-1}, come into S_i. And one route which is R_i, goes out from S_i.

2.5.2 Timetable
We assume the timetable is as follows.
- Trains running on R_i are operated by the periodic timetable T_i.
- Headway of R_i is h_i.
- Arrival time to S_i of a train running on R_{i-1} is a_{i-1}.
- Departure time from S_i of a train running on R_i is d_i.

Let τ_i be the set of d_i. Let $\hat{d}_i$ be the departure time of the train connecting from a_{i-1} at S_i, *i.e.*

$$\hat{d}_i = \arg\min_{d_i \in \tau_i}(d_i - a_{i-1}), \tag{2}$$

$$\hat{d'}_i = \arg\min_{d_i \in \tau_i}(d_i - a'_{i-1}). \tag{3}$$

Each predecessor train connects with a distinct successor train when the following conditions are satisfied,

$$0 \le \hat{d}_i - a_{i-1} \le h_{i-1}, \tag{4}$$

$$0 \le \hat{d'}_i - a_{i-1} \le h'_{i-1}. \tag{5}$$

From the definition of $\hat{d}_i$ and $\hat{d'}_i$, the following conditions are satisfied,

$$\hat{d}_i - h_i - a_{i-1} < 0 \Rightarrow \hat{d}_i - a_{i-1} < h_i, \tag{6}$$

$$\hat{d}'_i - h_i - a'_{i-1} < 0 \Rightarrow \hat{d}'_i - a'_{i-1} < h_i. \tag{7}$$

Furthermore the following conditions of headway are satisfied:

$$h_i \leq h_{i-1}, \tag{8}$$

$$h_i \leq h'_{i-1}. \tag{9}$$

Then we consider the headway as a sequence indexed by i, h_i as the monotonically decreasing sequence.

2.5.3 Conclusion of the case of multiple merging

It is necessary that the headway of the later successor is small enough to synchronise with the predecessor. In other words, when the synchronisations are iterated many times, the headway of the later successor becomes too small.

3 Entropy of timetable

It often happens that the perfect synchronisation of the timetable is difficult. It is especially difficult in the case when some routes arrive and depart by a different headway to each other at the junction.

We suppose the useful concept which is the entropy of the timetable. The concept of the entropy of the information theory is useful to evaluate the convenience of the timetable of transport networks [3]. Using the concept of the entropy, we can measure the level of synchronisation of the timetable.

3.1 Definition

3.1.1 Shannon's entropy of the information theory

Let X be a discrete random variable on a finite set $X = \{x_1, \ldots, x_n\}$, with probability distribution function $p(x) := Pr(X = x)$. The entropy $H(X)$ of X is defined as

$$H(X) = -\sum_{x \in X} p(x) \cdot \log p(x). \tag{10}$$

The convention $0 \log 0 = 0$ is adopted in the definition. The logarithm is usually taken to the base 2, in which case the entropy is measured in "bits".

3.1.2 Probability distribution function of the timetable

Let $\mathcal{T} = \{Tr_1, Tr_2, \ldots, Tr_n\}$ be the set of trains for travelling between the specific origin and the specific destination. Let b_i be the number of passengers boarding on the train Tr_i.

We define the probability distribution function p_i as

$$p_i = \frac{b_i}{\sum_{Tr_j \in \mathcal{T}} b_j}. \tag{11}$$

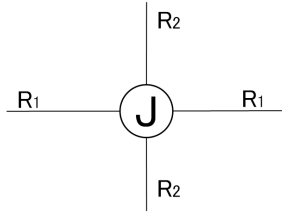If the passengers have to transfer the trains at the intermediate stations, Tr_i is a "train set" [3].

Figure 3: Example of the junction.

3.2 Properties of the entropy

As the passengers are equally distributed on each train, the entropy increases.

3.3 Examples

We consider the junction of the crossing two routes as shown in Figure 3.

3.3.1 Timetable
Each route is operated by the periodic timetable at regular intervals.

We denote f_i as the number of trains per hour of the route R_i. We consider the timetable of each frequency at J, that is shown in Table 3. This timetable expresses the arrival and departure of the jth train at the column (j) in the cases of some frequencies: 12, 10, 6, 5, 4, 3 and 2.

3.3.2 Passengers
We consider some cases in which we calculate the entropies. All passengers travel to J by the train running on the route R_1 and transfer to the train on the route R_2 at J.

We assume the behaviour of the passengers as follows:
  (1) Each passenger takes the train on the route R_1 by which the passenger is able to ensure the transfer time at J is 1 to 6 minutes.
  (2) All passengers come to the origin station according to a uniform distribution.
  (3) The number of passengers is large enough.
The above condition (1) says that "the limit of transfer time" is 6 minutes.

3.3.3 In the case of high frequency
We consider the case of $f_1 < f_2$. If the headway of the route R_2 exceeds the limit of transfer time, all trains on the route R_1 connect to the trains of R_2 in 6 minutes. Hence the entropy is

$$H = \log f_1. \tag{12}$$

Thus, we consider the case of $f_2 < 10$ in the following.

Table 3: Timetable of J.

f_i		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
$f_i = 12$	arr.	59	04	09	14	19	24	29	34	39	44	49	54
	dep.	00	05	10	15	20	25	30	35	40	45	50	55
$f_i = 10$	arr.	59	05	11	17	23	29	35	41	47	53	-	-
	dep.	00	06	12	18	24	30	36	42	48	54	-	-
$f_i = 6$	arr.	59	09	19	29	39	49	-	-	-	-	-	-
	dep.	00	10	20	30	40	50	-	-	-	-	-	-
$f_i = 5$	arr.	59	11	23	35	47	-	-	-	-	-	-	-
	dep.	00	12	24	36	48	-	-	-	-	-	-	-
$f_i = 4$	arr.	59	14	29	44	-	-	-	-	-	-	-	-
	dep.	00	15	30	45	-	-	-	-	-	-	-	-
$f_i = 3$	arr.	59	19	39	-	-	-	-	-	-	-	-	-
	dep.	00	20	40	-	-	-	-	-	-	-	-	-
$f_i = 2$	arr.	59	29	-	-	-	-	-	-	-	-	-	-
	dep.	00	30	-	-	-	-	-	-	-	-	-	-

3.3.4 The case: numbers of frequencies are relatively prime

We consider the pairs of relatively prime numbers that consist of the combination of 3, 4 and 5. The probabilities and the entropies of the example are shown in Table 4.

We describe the combined row of Table 4. As an example, "$k \to l$" of the first column means the transfer from R_1, whose frequency is k per hour, to R_2, whose frequency is l per hour. "$Tr_1^{(i)}$" in the row of "Predecessor" denotes the ith train of R_1. "$Tr_2^{(j)}$" in the row of "Successor" denotes the jth train of R_2. "-" in the row of "Successor" denotes that the $Tr_1^{(i)}$ above has no successor train of R_2 satisfies the passengers behaviour. The values in the row of p_i denote the probabilities of the train set $Tr_1^{(i)}$ and $Tr_2^{(j)}$. The value of the column "H" denotes the entropy of the timetable in the case of the transfer $k \to l$.

Table 4: Entropies (relatively prime).

$f_1 \rightarrow f_2$							H
5 → 4	Predecessor	$Tr_1^{(1)}$	$Tr_2^{(1)}$	$Tr_3^{(1)}$	$Tr_4^{(1)}$	$Tr_5^{(1)}$	0.722
	Successor	$\rightarrow Tr_1^{(2)}$	$\rightarrow Tr_2^{(2)}$	-	-	-	
	p_i	$\frac{4}{5}$	$\frac{1}{5}$	0	0	0	
5 → 3	Predecessor	$Tr_1^{(1)}$	$Tr_2^{(1)}$	$Tr_3^{(1)}$	$Tr_4^{(1)}$	$Tr_5^{(1)}$	0.971
	Successor	$\rightarrow Tr_1^{(2)}$	-	-	$\rightarrow Tr_3^{(2)}$	-	
	p_i	$\frac{2}{5}$	0	0	$\frac{3}{5}$	0	
4 → 5	Predecessor	$Tr_1^{(1)}$	$Tr_2^{(1)}$	$Tr_3^{(1)}$	$Tr_4^{(1)}$		0.811
	Successor	$\rightarrow Tr_1^{(2)}$	-	-	$\rightarrow Tr_5^{(2)}$		
	p_i	$\frac{1}{4}$	0	0	$\frac{3}{4}$		
4 → 3	Predecessor	$Tr_1^{(1)}$	$Tr_2^{(1)}$	$Tr_3^{(1)}$	$Tr_4^{(1)}$		0.811
	Successor	$\rightarrow Tr_1^{(2)}$	$\rightarrow Tr_2^{(2)}$	-	-		
	p_i	$\frac{3}{4}$	$\frac{1}{4}$	0	0		
3 → 5	Predecessor	$Tr_1^{(1)}$	$Tr_2^{(1)}$	$Tr_3^{(1)}$			0.918
	Successor	$\rightarrow Tr_1^{(2)}$	$\rightarrow Tr_3^{(2)}$	-			
	p_i	$\frac{2}{3}$	$\frac{1}{3}$	0			
3 → 4	Predecessor	$Tr_1^{(1)}$	$Tr_2^{(1)}$	$Tr_3^{(1)}$			0.918
	Successor	$\rightarrow Tr_1^{(2)}$	-	$\rightarrow Tr_4^{(2)}$			
	p_i	$\frac{1}{3}$	0	$\frac{2}{3}$			

3.3.5 The case: numbers of frequencies are not relatively prime.

We consider the case that the numbers of frequencies per hour are not relatively prime. Let f_c be the greatest common divisor of f_1 and f_2. The departure time of every (f_i/f_c)th train on the route R_i is the same as the departure time of a train on the other route. Therefore the probability of every (f_i/f_c)th train is not zero.

We consider the pairs of not relatively prime numbers that consist of the combination of 4, 6 and 10. The probabilities and the entropies of the example is shown in Table 5 and has the same notation as Table 4.

Table 5: Entropies (not relatively prime).

$f_1 \rightarrow f_2$							H
$10 \rightarrow 6$	Predecessor	$Tr_1^{(1)}$	$Tr_2^{(1)}$	$Tr_3^{(1)}$	$Tr_4^{(1)}$	$Tr_5^{(1)}$	1.995
	Successor	$\rightarrow Tr_1^{(2)}$	$\rightarrow Tr_2^{(2)}$	-	$\rightarrow Tr_3^{(2)}$	-	
	p_i	$\frac{2}{10}$	$\frac{1}{10}$	0	$\frac{2}{10}$	0	
	Predecessor	$Tr_6^{(1)}$	$Tr_7^{(1)}$	$Tr_8^{(1)}$	$Tr_9^{(1)}$	$Tr_{10}^{(1)}$	
	Successor	$\rightarrow Tr_4^{(2)}$	$\rightarrow Tr_5^{(2)}$	-	$\rightarrow Tr_6^{(2)}$	-	
	p_i	$\frac{2}{10}$	$\frac{1}{10}$	0	$\frac{2}{10}$	0	
$10 \rightarrow 4$	Predecessor	$Tr_1^{(1)}$	$Tr_2^{(1)}$	$Tr_3^{(1)}$	$Tr_4^{(1)}$	$Tr_5^{(1)}$	1.971
	Successor	$\rightarrow Tr_1^{(2)}$	-	$\rightarrow Tr_2^{(2)}$	-	-	
	p_i	$\frac{3}{10}$	0	$\frac{2}{10}$	0	0	
	Predecessor	$Tr_6^{(1)}$	$Tr_7^{(1)}$	$Tr_8^{(1)}$	$Tr_9^{(1)}$	$Tr_{10}^{(1)}$	
	Successor	$\rightarrow Tr_3^{(2)}$	-	$\rightarrow Tr_4^{(2)}$	-	-	
	p_i	$\frac{3}{10}$	0	$\frac{2}{10}$	0	0	
$6 \rightarrow 4$	Predecessor	$Tr_1^{(1)}$	$Tr_2^{(1)}$	$Tr_3^{(1)}$	$Tr_4^{(1)}$	$Tr_5^{(1)}$	1.918
	Successor	$\rightarrow Tr_1^{(2)}$	$\rightarrow Tr_2^{(2)}$	-	$\rightarrow Tr_3^{(2)}$	$\rightarrow Tr_4^{(2)}$	
	p_i	$\frac{2}{6}$	$\frac{1}{6}$	0	$\frac{2}{6}$	$\frac{1}{6}$	
	Predecessor	$Tr_6^{(1)}$					
	Successor	-					
	p_i	0					
$4 \rightarrow 6$	Predecessor	$Tr_1^{(1)}$	$Tr_2^{(1)}$	$Tr_3^{(1)}$	$Tr_4^{(1)}$		2.000
	Successor	$\rightarrow Tr_1^{(2)}$	$\rightarrow Tr_3^{(2)}$	$\rightarrow Tr_4^{(2)}$	$\rightarrow Tr_6^{(2)}$		
	p_i	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$		

3.4 Conclusion of the entropy

We considered the case of the travelling path with transfers at intermediate junctions. Then we obtained the following result. The entropy of the timetable

is inversely proportional to the amount of transfer time which the passengers need to travel by the timetable. As a consequence, the timetable in which the entropy is large, is synchronised well.

4 Conclusions

In this paper we describe the ways to implement the making of a synchronised timetable. One way of implementing it is the case of the cyclic route. The other is in the case with multiple merging.

However, sometimes we cannot make a timetable which is completely synchronised. In this case we substitute a timetable which is roughly synchronised. Therefore we proposed a method for evaluating the level of synchronisation in the timetable. The way used is the concept of the entropy of the information theory. We show that the concept of the entropy is one of the most useful methods to measure the level of synchronisation of the timetable.

In conclusion, the concept of the entropy is a useful and simple method to evaluate the quality of the synchronised timetable. Furthermore, calculating the entropy is the easiest way to evaluate the quality of the transit system.

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