

# The standing wave effect of a spiral spring in steel-spring floating-slab track on the noise in an urban transit carriage

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## Abstract

Steel-spring floating-slab (SSFS) has been widely used in track structure because of its effective vibration isolation. The energy transferred to the foundation is significantly reduced. Usually, the frequencies are low in the course of analyzing the vibration of a floating slab. When considering the vibration isolation effect of SSFS, the mass of spiral spring and its detailed structure in the supports is usually ignored. This assumption will cause errors in the high frequency vibration characteristics. In fact, there is certain mass in the supports of floating slab. In addition, when the diameter of spring is equal to integer times of a half-elastic wave in it, the wave effect can be produced in the supports. At this time, the standing wave effect in the spiral springs should not be ignored. Based on the linear assumption and using the four-end parameter analysis method, we get the range of frequency that is caused by the wave effect. Three groups of noise in a carriage of an actual running vehicle are measured. In addition, it is explained why the noise is higher when the vehicle is running over an SSFS zone, compared with the common track. Some suggestions are put forward that will reduce the low frequency noise in the urban transit carriage.

*Keywords: steel-spring floating-slab (SSFS), spiral spring, standing wave, noise.*

## 1 Introduction

As people have the higher requirements on the influence of rail transit on the environment, reducing vibration and noise of transit system becomes very important. Steel-spring floating-slab (SSFS) due to its effective vibration isolation effect, has been widely used in the track structure. The main goal of



SSFS system is reducing the transmission of wheel-rail vibration to the foundation. The main energy of vehicle and track vibration is concentrated within the band that will make the vehicle and track to vibrate at low frequencies, so in the standard of GB10070-88 [1] and JGJ/T 170-2009 [2], the range of frequency is 0–80 Hz and 0–200 Hz. The vibration energy at high frequencies will excite the panel of vehicle and track to vibrate. In addition, this kind of vibration will radiate noise. Therefore, the research of vibration and noise must be extended to a higher frequency range [3]. In the analysis of wheel-rail vibration, the mass of spiral spring in SSFS is often neglected. This assumption will cause errors when analyzing the vibration characteristics of wheel-rail system at high frequencies. At this time, we have to consider the standing wave effect of spiral spring in the floating slab supports.

## 2 Noise tests in the interior of the carriage

Structure-borne vibrations due to railway traffic have become important environment issues, which are particularly critical when new rail infrastructure is introduced in an existing urban environment [4]. Low frequency vibrations have become a special concern [5–7]. After a large number of SSFS track structures are used on site, engineers meet some new problems, such as the vibration of wheel and rail become violently, which exacerbates the wheel flange wear too. Further more noise in the range of low frequencies occurs in the interior of vehicle while the train is running over such zone. Three different groups of noise data in the carriage were measured when running over SSFS, high damping bearing floating slab track (FST) and common track area (Figure 1 and Figure 3) respectively. The tested vehicle is for Shanghai metro line which is a full aluminium metro vehicle. Vehicle parameters are shown as in Table 1. Measured points were located in the centre of vehicle and the position is 1.6 metres above the floor. There are 35 supports in each SSFS. The SSFS track design drawing is as Figure 4. And the detailed parameters are in Table 2 and Table 3.

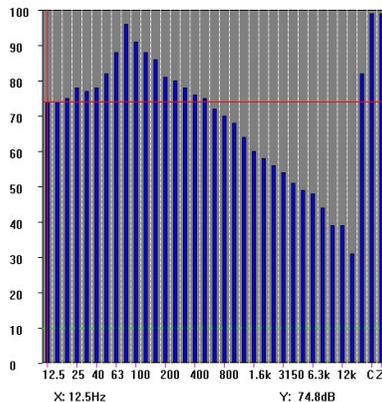


Figure 1: Noise frequency domain (SSFS area).

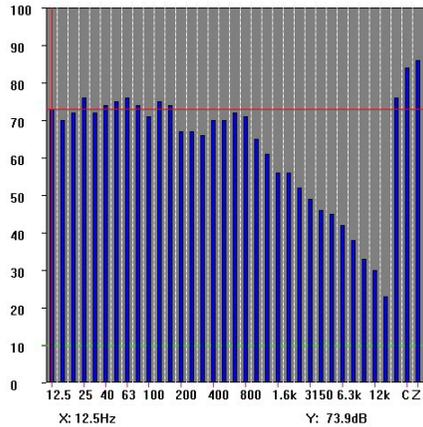


Figure 2: Noise frequency domain (common slab area).

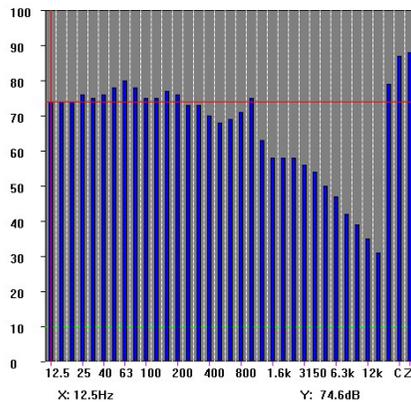


Figure 3: Noise frequency domain (High damping bearing FST area).

The results from Figure 1 shows: noise over 75 dB in the carriage occurs in the range of 25 Hz to 400 Hz when the train is crossing over SSFS area. However, there is no such value when running on common slab and high damping bearing FST (Figure 2). It can be seen that between 25 Hz to 200 Hz, there are vibration above 80 dB when crossing SSFS. The peak value of SSFS is bigger about 20 dB than that of common slab track.

Because there is the noise at low frequencies when the train is running over SSFS area, we firstly checked whether it was caused by the track slab vibration. We compared the vibration of different types of track. It is found that in the range of 0–200 Hz the vibration of SSFS track is larger than that of common track

(Figure 5 and Figure 6). The peak of SSFS vibration acceleration degree is concentrated between 50–80 Hz. Although floating slab has lower and higher

orders' vibration modes, the low order natural vibration modes play a key role in the vibration isolation effect.

As the peak of floating slab vibration is 50–80 Hz and interior noise peak is 63–200 Hz, there are some differences between each other. Then we tested the interior noise when the train was running over high damping bearing floating slab area from common slab track. There is also no obvious peak noise when the vehicle is running over the high damping spring floating slab (Figure 3). Because the vehicle is same in the test, the difference is running over different tracks. The length, the width and the height of the track are also same. The supports of SSFS are different from those of the high damping spring floating slab. From the test data, we can get the results that the vehicle interior noise is caused by the different bears of floating slab. In the SSFS system, we often ignored the mass of spiral spring in the supports when analyzing the isolate effect. After compare with the test data, especially in the range of 400 Hz, we consider the spiral springs contribute to the vehicle interior noise.

Table 1: Vehicle parameters.

Parameters	Value
mass of vehicle ( <i>kg</i> ) ( <i>kg</i> )	50160
mass of body frame ( <i>kg</i> )	4618
mass of wheel set ( <i>kg</i> )	1510
nod inertia of vehicle ( <i>kg m<sup>2</sup></i> )	$1.89725 \times 106$
nod inertia of body frame ( <i>kg m<sup>2</sup></i> )	3940
first suspension stiffness ( <i>kN/mm</i> )	2.9
first suspension dampness ( <i>N s/m</i> )	0
second suspension stiffness ( <i>kN/mm</i> )	0.48
second suspension dampness ( <i>N s/m</i> )	10000
Distance between bogie centres ( <i>m</i> )	15.70
Distance between axles ( <i>m</i> )	2.50
Radius of wheel ( <i>m</i> )	0.42
Axle load ( <i>kg</i> )	16270

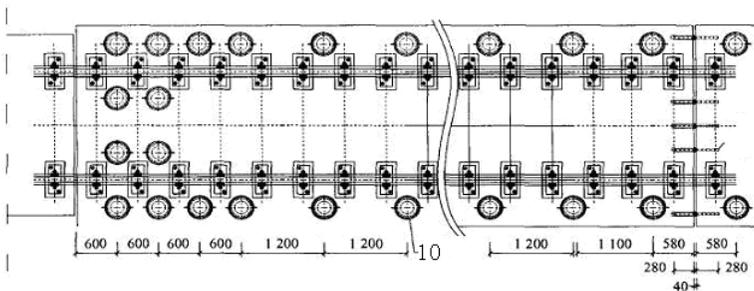


Figure 4: Design drawing of SSFS.

Table 2: Vehicle parameters.

parameters		reference value
Rail	Type of rail ( <i>kg/m</i> )	60
	cross-section area ( <i>cm<sup>2</sup></i> )	77.45
	elastic modulus ( <i>GPa</i> )	210
fastener	stiffness ( <i>kN/mm</i> )	40
	damping ( <i>N s/m</i> )	$7.5e^4$
SSFS	Length ( <i>m</i> )	25
	Width ( <i>m</i> )	2.5
	Density ( <i>kg/m<sup>3</sup></i> )	2775
Bearing of SSFS	stiffness ( <i>kN/mm</i> )	10
	damping ( <i>N s/m</i> )	0 or $7.5e^4$

Table 3: The natural frequencies of SSFS (*Hz*).

Parameters	Value	Parameters	Value
1st order	10.105	6th order	41.800
2nd order	12.351	7th order	52.078
3rd order	17.544	8th order	63.383
4 <sup>th</sup> order	24.438	9 <sup>th</sup> order	75.697
5 <sup>th</sup> order	32.571	10 <sup>th</sup> order	76.421

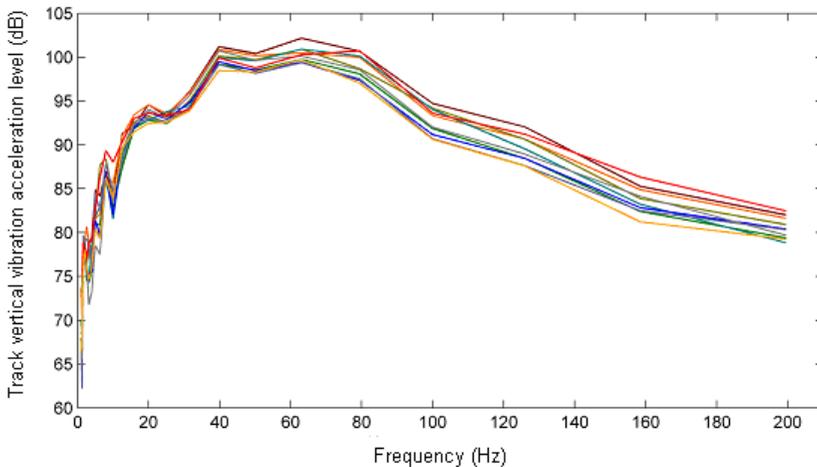


Figure 5: Track vertical vibration acceleration level (SSFS).

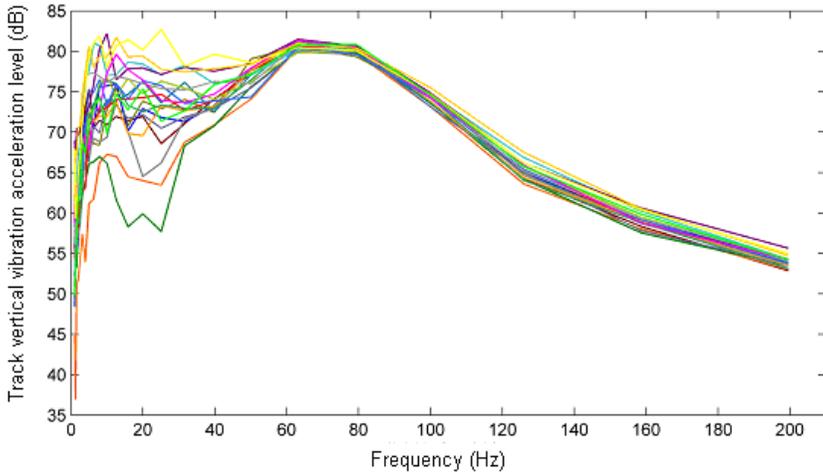


Figure 6: Track vertical vibration acceleration level (common slab track).

### 3 Using the four-end parameter method to analyze the standing wave effect of spiral spring

From the test results we can be certain that the noise in the carriage is caused by the spiral spring, in the following content we analyze its effect. In the course of studying the vibration isolation effect of the floating slab, fasteners are taken as input ends and steel spring bottom are as the output of the system. We use four-end parameter analysis method to analyze. This is shown in Figure 7.

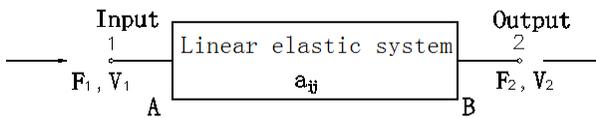


Figure 7: Four-end parameter method.

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} F_2 \\ V_2 \end{bmatrix} \tag{1}$$

where  $F_1, V_1$  and  $F_2, V_2$  stand for load and velocity of input and output end respectively.  $a_{11}, a_{12}, a_{21}, a_{22}$  are the four-end parameters of this system [7].

If the mass of spring is considered in the spiral spring system (shown in Figure 8):

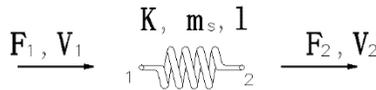


Figure 8: The four- end parameter of spiral spring.

Because of the differential movement the equation of the spring is:

$$\frac{\partial^2 \xi}{\partial x^2} - \frac{1}{C^2} \cdot \frac{\partial^2 \xi}{\partial t^2} = 0 \tag{2}$$

in which:

$\xi(x, t)$  ----Longitudinal displacement of spring in the position x at time t

C -----Wave propagation velocity of the spring;

$$C = l \cdot \sqrt{\frac{K}{m_s}}$$

l -----Length of the spring;

K ----- Stiffness of the spring;

$$K = \frac{Gd^4}{8nD^3}$$

$m_s$  -----Mass of the spring;

G ----- Shear modulus of the spring;

D -----Diameter of the spring;

d -----Diameter of the spring wire.

The four-end spiral spring parameters when considering the mass of the spring is:

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} \cos\left(\sqrt{\frac{m_s}{K}} \cdot \omega\right) & j\sqrt{Km_s} \cdot \sin\left(\sqrt{\frac{m_s}{K}} \cdot \omega\right) \\ \frac{-1}{j\sqrt{K \cdot m_s}} \cdot \sin\left(\sqrt{\frac{m_s}{K}} \cdot \omega\right) & \cos\left(\sqrt{\frac{m_s}{K}} \cdot \omega\right) \end{bmatrix} \begin{bmatrix} F_2 \\ V_2 \end{bmatrix} \tag{3}$$

The transmission rate when considering the spring wave propagation effect is shown as Figure 9. The mass of structure which needs to be isolated from vibration is  $m$ .

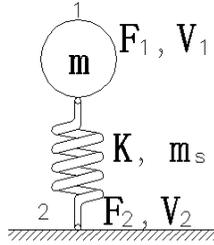


Figure 9: The series system with mass spiral spring.

The Four-end parameters equation is:

$$\begin{bmatrix} F_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} 1 & j\omega m \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\left(\sqrt{\frac{m_s}{K}} \cdot \omega\right) j\sqrt{K m_s} \cdot \sin\left(\sqrt{\frac{m_s}{K}} \cdot \omega\right) \\ -1 \\ \frac{-1}{j\sqrt{K \cdot m_s}} \cdot \sin\left(\sqrt{\frac{m_s}{K}} \cdot \omega\right) \cos\left(\sqrt{\frac{m_s}{K}} \cdot \omega\right) \end{bmatrix} \begin{bmatrix} F_2 \\ V_2 \end{bmatrix} \quad (4)$$

To the supports of SSFS, there is  $V_2 = 0$ . Then

$$F_1 = \left[ \cos\left(\sqrt{\frac{m_s}{K}} \omega\right) - \frac{\omega m}{\sqrt{K \cdot m_s}} \cdot \sin\left(\sqrt{\frac{m_s}{K}} \cdot \omega\right) \right] \cdot F_2 \quad (5)$$

$$T_A = \frac{F_2}{F_1} = \frac{1}{\cos\left(\sqrt{\mu} \cdot \frac{\omega}{\omega_n}\right) - \frac{1}{\sqrt{\mu}} \cdot \left(\frac{\omega}{\omega_n}\right) \cdot \sin\left(\sqrt{\mu} \cdot \frac{\omega}{\omega_n}\right)} \quad (6)$$

in which,

$$\mu = \frac{m_s}{m}, \quad \omega_n = \sqrt{\frac{K}{m}}$$

To the first resonance frequency, according to  $\mu \ll 1$ , it can be calculated that:

$$f_n = \frac{1}{2\pi} \sqrt{\frac{K}{m}} \quad (7)$$

To the later series of resonant frequencies, it is almost consistent with the pulse frequency of the spiral spring:

$$f_s = \frac{i}{2\pi} \sqrt{\frac{K}{m_s}} \quad (i = 1, 2, 3, \dots) \quad (8)$$

According isolation principle, when  $\mu = 0$ , the sloped of the transmission curve of high frequencies is:

$$\frac{d(T_A)}{d\left(\frac{\omega}{\omega_n}\right)} = -\frac{1}{\left(\frac{\omega}{\omega_n}\right)^2} \quad (9)$$

After the fluctuation occurs, the slope of the tangent is:

$$\frac{d(T_A)}{d\left(\frac{\omega}{\omega_n}\right)} = -\frac{1}{\left(\frac{\omega}{\omega_n}\right)} \quad (10)$$

Due to the spiral spring standing wave effect, the vibration isolation characteristics of the system will decline in the range of high frequencies. There are equally spaced frequency peaks. Because in the actual SSFS, there is certain damping while it is low, the peak of the first resonant vibration is not infinite. The later resonance frequencies are constant with those of the pulse vibration of spiral. The standing wave effect of the spring has great influence on the vibration characteristics of the system.

Figure 8 shows the transmissibility under different damping system. Before the frequency when appear the standing wave, transfer characteristics of vibration isolation system matches with the concentration parameters system. As frequency increases, peaks appear at the frequency  $\omega_j$ .

$$\omega_j = \omega_0 j \pi \left( \frac{M}{m_s} \right)^{\frac{1}{2}} \quad (j = 1, 2, 3, \dots) \tag{11}$$

Here  $\omega_0 = \sqrt{K / m}$ .

So, to reduce such unexpected effect, we can adopt suitable damping supports. In Figure 10 the parameter  $n$  is the damping ratio. From the result we can see that due to the standing wave effect, the vibration transmission rate is raised in the region of higher frequency. At the same time, the vibration isolation effect becomes poor. Furthermore at some frequencies there are peaks. As the increase of the damping ratio of supports, these value of peaks become smaller, and the curve of transmissibility become gradually smoothing. The frequencies of peaks concentrate from 50 Hz to 200 Hz. It is consistent with the frequencies of noise in the carriage.

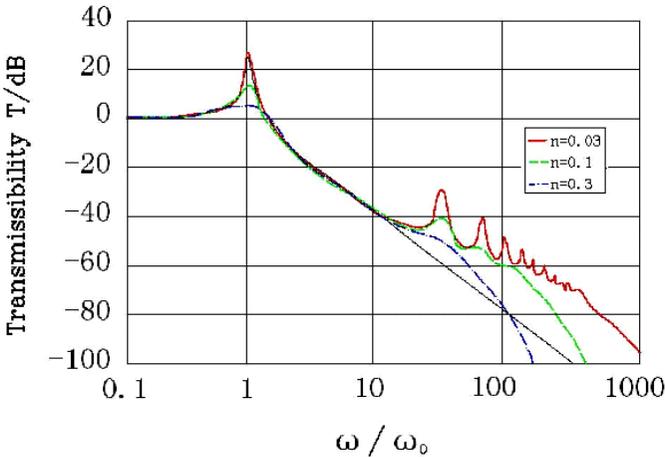


Figure 10: Transmissibility theoretical curve ( $m/m_s = 100$ ).

#### 4 The source and transfer path of noise at low frequency in carriage

From the test on site and the theoretical analysis, we get the result that the additional noise at the frequency of 50–400 Hz is caused by the spiral spring in SSFS. In general, the sources of noise in the carriage at low frequency are various. Theoretically, the sources are mainly divided into such three ways as; structural sound transmission, air sound transmission and standing wave.

In the train-track system, when trains are running over SSFS, the floating slabs are excited by the impacts of wheel. A part of the vibration energy is

dissipated by the floating slab, a part is radiated directly as sound into the carriage, and the remainder is transmitted through the wheels and bogies into the carriage structure.

On one hand, the radiated sound energy excites the backplane and walls of carriage. Then the vibrations in the backplane and walls radiate sound. On the other hand, the vibration in wheels and bogies travel to the carriage and radiate sound. The paths of structure-borne noise can be identified by waves [8].

All the components of SSFS vibrate at the same time, including the steel spring supports. The main load bearing components are the spiral springs in SSFS. Standing waves caused by steel spring travel into the carriage by the air from the gap of vehicle structure. In addition the vibration transmits to the vehicle's floor through wheels and a bogie, which causes the floor vibrate, and then produces the second noise. The range of low frequency noise (25 Hz to 400 Hz) covers the above resource noise, which are in accord with the frequencies of wave effect of steel spiral spring.

## 5 Suggestions

Vibration isolation is good and necessary. However if we only focus on this point, some other problems are probably ignored. And these problems are one part of vibration and noise of vehicle and track system. When design the floating slab, it is better if the natural frequency is away from stimulating frequency and that of vehicle.

To reduce the noise level in the carriage by requirement, it can work if taking account each path that the sound energy produces and transmits. Furthermore some suggestions are put forward for reference:

- 1) Reduce energy input to the floating slab.  
Because in SSFS track system, fasteners are taken as the input end, if we use some device to lower the energy transferred to the fasteners (such as using energy absorbers on rail), the energy in SSFS will be reduce.
- 2) Increase energy dissipation in the slab.  
Using damping spiral spring supports will increase energy assumption. As the damping ratio of the steel spring FST is only greater than 0.05, in the process of impact between wheel and rail, vibration energy can not be effectively consumed. On the contrary it aggravates the vibration of floating slab and the vehicle, and results in noise in carriage increase. Considering the viscoelastic damping materials can reduce the standing waves in the supports, it can be developed in FST construction.
- 3) Reduce the structure-borne noise along its propagation path to the carriage.  
From the analysis the cause of low frequencies noise, some components of vehicle will produce noise. If there are some vibration isolation measures on the path of vibration propagation, the noise in the carriage will also be reduced.

## Acknowledgement

The authors wish to acknowledge the support and motivation provided by the Fundamental Research Funds for the Central Universities (Tongji University).

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