A comparison of vehicle routing approaches with link costs variability: an application for a city logistic plan

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Abstract

In this paper the comparisons of some solution approaches for vehicle routing are reported. The scenarios of vehicle routing with static travel costs, pseudo-dynamic travel costs, dynamic travel costs are considered. Some characteristics of link costs are discussed. For each scenario the changes in vehicle routing (i.e. path search) are highlighted; a test application for freight distribution in city logistic is done. The goal is to verify how the cost variation in the time influences the route optimization.
Keywords: link cost functions, genetic algorithm, optimization models.

1 Introduction

In this paper the comparison of some solution approaches for vehicle routing are reported. Three scenarios in the vehicle routing problem are considered, related to the travel costs: i) static travel costs, ii) pseudo-dynamic travel costs, iii) dynamic travel costs. The goal is to verify how the cost variation in the time influence the route optimization. In first case, the costs are established in advance, and they are the same during the simulation period. In second case, the simulation period is split in time intervals and for each interval the travel costs are supposed constant, depending on the interval. In third case, the travel costs are calculated dynamically instant by instant.

To frame the vehicle routing problem, below is reported a literature review. Dantzig and Ramser did the vehicle routing problem in 1959 considering a truck

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fleet [1]. Since then, the problem has been formulated considering various aspects: time windows, delivery and pick-up, variations in the demand, variations in the travel cost and so on. Below, here are some papers that explore the problem: nevertheless, the list can be not complete, because the papers in this field are very numerous. The vehicle routing with time windows imposes explicit constraints in the time interval in which it is possible done the delivery and/or pick-up operations [2–5]. The vehicle routing with simultaneously delivery and pick-up involves the distribution and the collection of freight to/from users [6–8]. The vehicle routing with variations in the demand concerns the insertion/removal of users in the vehicle route [9–11]. The vehicle routing with variations in travel cost considers the travel cost modification in the time, sometimes with the probability in the cost variability [12–16]. Another question concerns the solution approaches in the vehicle routing: generally, due the computational difficulties, heuristic procedures are applied [17–20]; however, for some problem classes, exacts procedures are proposed [2, 21–23].

In this paper, a comparison between the static, pseudo-dynamic and dynamic approach is reported. In Figure 1 the three cases are reported: in static approach the cost are defined first and not change; in pseudo-dynamic the cost are also defined first but depend on a time slices; in dynamic case it is necessary to evaluate the costs instant by instant inside an optimization procedure.

In Figure 1(a), the static approach is reported: the Static Path Search (SPS) procedure have as input the supply and the users; it gives as output a shortest path cost matrix (a matrix element is the shortest path cost between each pair of users). The shortest path cost matrix is used, as usually, to evaluate the route cost in the VRP. In Figure 1(b), the pseudo-dynamic approach is reported: in this case, some time slices are defined. In each time slice, a modification in the supply is possible. A Pseudo Static Path Search (PSPS) is applied to find the shortest path: this involve applying a SPS for each time slice considered; in a global optimization the cost is fixed in each time slice but it is different from a time slice to another. The output are some path cost matrices (a matrix for each time slice), used as input for the VRP. In the route cost evaluation, it is necessary to consider, for each user, the time slice in which is included the departure time (as explained in Section 3.2). In Figure 1(c), the dynamic approach is reported: in this case, the route cost should be evaluated with the route construction inside the routing problem, because the cost function is a continue function respect the clock time (as explained in Section 3.3) in each departure time instant. This approach is useful, for example, in urban freight distribution [24] with great distance route, reducing the travel time that continuously changes during the route.

The paper is structured as follows. In Section 2 the link cost functions and the procedure are examined, considering static (2.1, 2.4), pseudo-dynamic (2.2, 2.5) and dynamic (2.3, 2.6) cost functions. In Section 3 an application and a comparison for freight distribution in city logistic is made; the comparison among the results obtained with the three approaches are reported. Finally, the conclusions and the possible developments are reported.

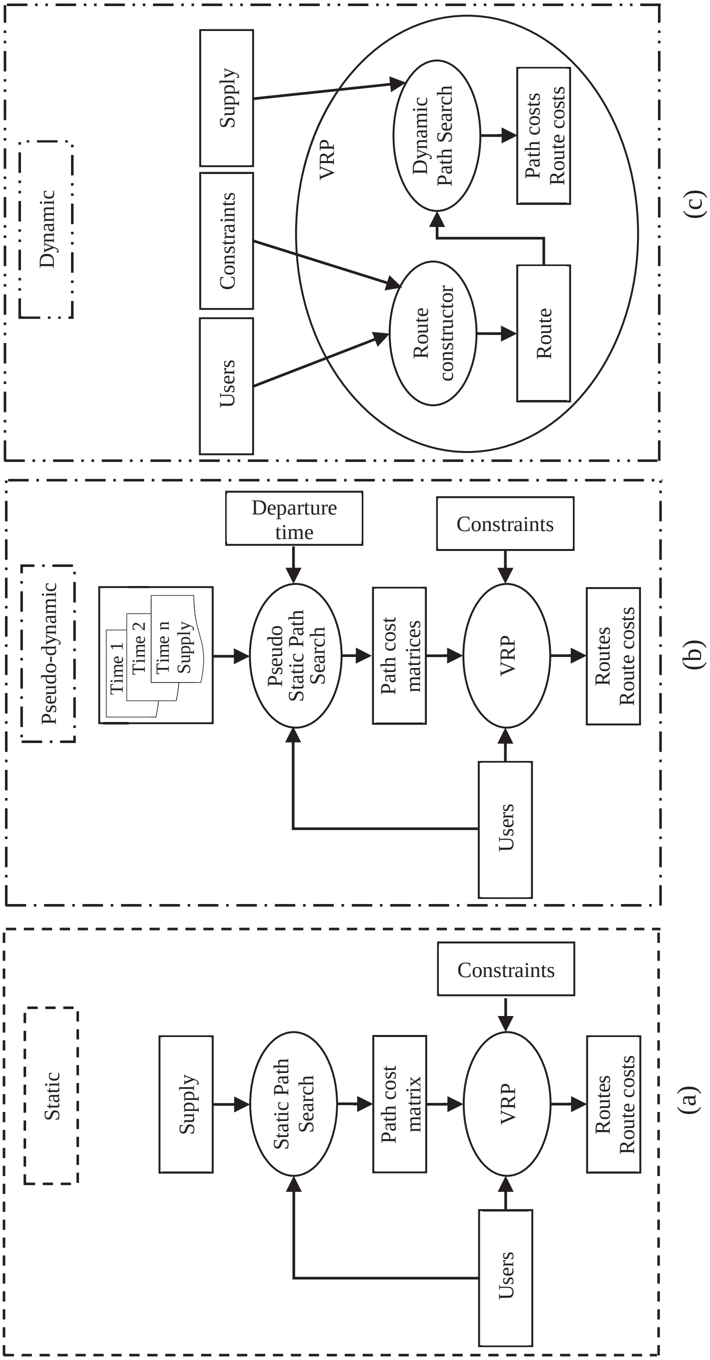

Figure 1: Vehicle routing comparison.

2 Link cost functions and vehicle routing procedures

In this section, the link cost functions are introduced. The link cost, generally, depends on physicals (i.e. length, width, lanes number) and functional characteristics (i.e. speed, final node regulation) of the link [25] and summarize the link performances [26, 27]. Because several vehicles use the same link, link cost is also flow dependent.

In some cases, in the link cost function is considered also the wait cost at the junction [28, 29]. Moreover, the link cost function is used in the set formation level [15] to define the set of paths perceived by the drivers.

Three cases are considered: static, pseudo-dynamic and dynamic. In the first case, the costs not depend on the clock time: each link (and each path) have a fixed cost in the time, but can be dependent on the flow. In the second case, some slice times are defined. The costs do not depend on the time within the same time slice. The cost is different for each time slice in relation to an average flow estimated. In the third case, the costs depend on the time, being the link cost a function of the departure time from each node.

In relation to the solution procedure, some approaches applicable to find the solution in static, pseudo-dynamic and dynamic approach are reported. In general, to optimize the vehicle route a genetic algorithm is applied [5, 23]; the variations are in the methodology used to evaluate the path cost. As in Figure 1, in the static and in the pseudo-dynamic approaches, the shortest path problem can be solved before and the vehicle routing after. In the dynamic approach, the vehicle routing problem need, to find the route, to solve a shortest path problem for each pair of user in the route with an optimization approach.

2.1 Static cost functions

In the static case the shortest path cost can be found using the Belmann inequality. Operatively, it is possible to found a cost matrix with entry the path cost between the nodes that can be reached. Various cost functions can be specified, considering the network characteristics and their conditions (like congestion). Generally, in this cases, the link cost is a function of link flows:
$c = c(\mathbf{f})$

where $\mathbf{f}$ is the flow vector with entry f_i the flow on link i.

Figure 2 reports the qualitative trend of a static cost function, note that the cost increase with the flows, regardless of the time [25].

2.2 Pseudo-dynamic cost functions

In the pseudo-dynamic approach the time (for example a day) is split in some intervals τ and for each interval the link cost is evaluated. In this approach, the link cost functions refer to an instant t_τ of the interval.

In this case, the link cost is a function of link flows and time slice (in each time slice the cost function is fixed and depend on the flow $\mathbf{f}_\tau$):

$$\mathbf{c}_\tau = \mathbf{c}(\mathbf{f}_\tau, t_\tau)$$

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where

- $\mathbf{f}_\tau$ is the flow vector with entry $f_{\tau,i}$ the flow on link i in time slice τ supposed stationary;
- t_τ is a predefined instant representative of the generic time slice τ.

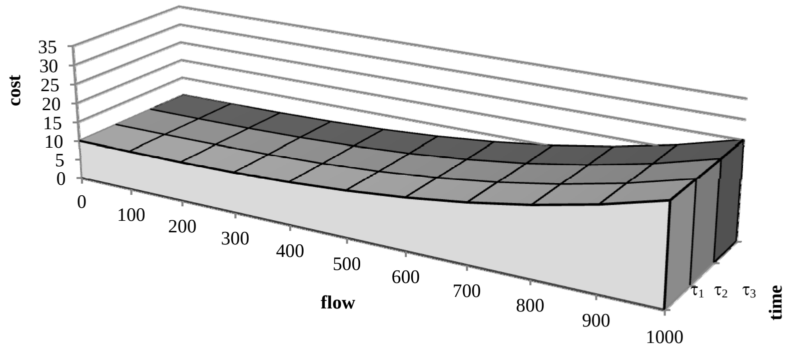

Figure 2: Static approach: qualitative cost function trend.

Figure 3 reports the qualitative trend of a pseudo dynamic cost function, note that: the cost increase with the flows; for the same flow value the cost can change changing the time slice. For example, considering the time slice τ_2 the link cost corresponding to a flow of 900 vehicle/h is about 21 seconds: this same cost is 18 second and 26 second in first and third time slice respectively.

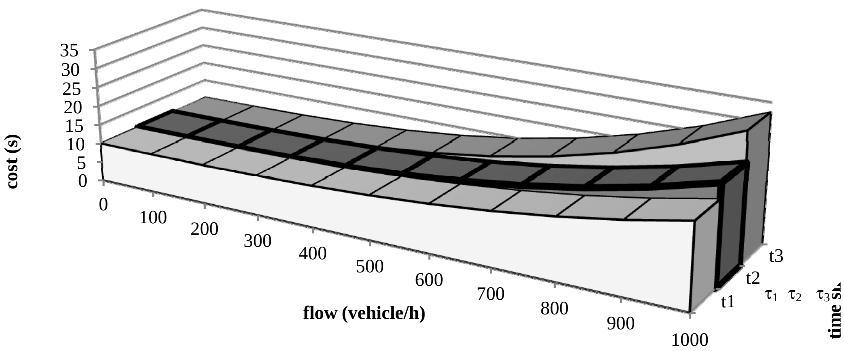

Figure 3: Pseudo dynamic approach: qualitative cost function trend.

2.3 Dynamic cost functions

In the dynamic approach the costs are assumed as a function of the clock time t, depending on the arrival time at node. Various cost functions can be defined to capture the cost variability in the time [16, 31, 32]; the aim is to reproduce the real conditions in the network instant by instant (this can be useful in planning and simulation [33, 34]).

In this case, the link cost is a function of link flows and time:

$\mathbf{c}_t = \mathbf{c}(\mathbf{f}_t, t)$

where

- $\mathbf{f}_t$ is the time dependent flow vector with entry f_i the flow on link i;
- t is the time.

Defined the link cost functions, the path costs can be found by a generalization of Bellman inequality. Figure 4 reports the qualitative trend of a dynamic cost function, note that: the cost increase with the flows; for the same flow value the cost can change changing the clock time; fixed the clock time the cost change with the flow. For example, in Figure 4 the broken line represent a trend of the cost with the flow fixing the clock time while the dotted line represent the cost fixing the flow varying the time.

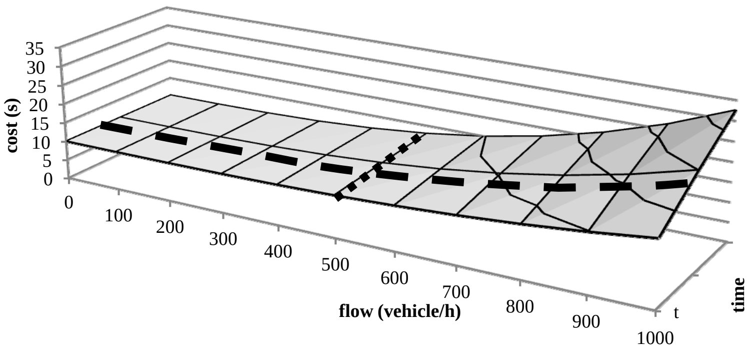

Figure 4: Dynamic approach: qualitative cost function trend.

Note that the path cost can be specified case by case for static (2.1), pseudo-dynamic (2.2) and dynamic (2.3) approach (for the symbols definition, see sections 2.4, 2.5, 2.6):

$$g_{k(u,w)} = \sum_i \sum_j \delta_{ij,\, k(u,\, w)} \cdot c_{ij}(\mathbf{f}) \tag{2.1}$$

$$g_{k(u,w)\tau} = \sum_i \sum_j \delta_{ij,\, k(u,\, w)\ \tau} \cdot c_{ij}(\mathbf{f}_\tau, t_\tau) \tag{2.2}$$

$$g_{k(u,w)(t_u,\, \ldots,\, t_{B(w)})} = \sum_i \sum_j \delta_{ij,\, k(u,\, w)\ (t_u,\, \ldots,\, t_{B(w)})} \cdot c_{ij}(\mathbf{f}_t, t) \tag{2.3}$$

where

- c_{ij} the link cost obtained from the vector cost $\mathbf{c}$ or $\mathbf{c}_t$ or $\mathbf{c}_\tau$;
- $\delta_{ij,\, k(..)}$ variable equal to 1 if link ij is used by path $k(..)$, zero otherwise.

Case 2.1 is not influenced by time, case 2.2 depends on the time slice into which the departure time from the node, case 2.3 depends on the instant in which the vehicle start from each initial node of the links.

2.4 Static procedure

In the static approach, the link cost and the path do not depend on the time. In this case, it is possible to evaluate the path cost between each pair of users defining a path cost matrix **G** that is used as input for the vehicle routing to calculate the route cost. Given:

- **G** path cost matrix, with entry $g_{k(u,\,w)}$;
- R a generic route;
- ζ_R the route cost (objective function to consider in the VRP);
- $x_{k(..)v}$ the decision variables of the VRP relative to the path k and vehicle v.

Figure 5 depicts the procedure.

$\mathbf{f}$:= link flow vector;
$\mathbf{c}(\mathbf{f})$:= link cost vector;
$\forall$ client pair (u, w)
  $k(u, w)$:= shortest path between the two users;
  $g_{k(u,\,w)} = \Sigma_{ij}\delta_{ij,\,k(u,\,w)} \cdot c_{ij}(\mathbf{f})$, shortest path cost;
  $\mathbf{G} = [g_{k(u,\,w)}]_{|T|\times|T|}$, shortest path cost matrix;
$\forall$ route R
  $\zeta_R = \Sigma_i\Sigma_j g_{k(u,\,w)} \cdot x_{k(u,\,w)v}$, route cost;

Figure 5: Path cost in static approach.

2.5 Pseudo dynamic procedure

In pseudo dynamic approach, the link and the path cost depend on the time slice in which the cost is evaluated. In this case, it is possible to evaluate a shortest path cost matrix $\mathbf{G}_\tau$ for each time slice τ. Respect the previous case the difference is that the path is defined in each departure time interval τ.
Figure 6 depicts the procedure.

t_τ := predefined instant representative of the generic time slice τ;
$\mathbf{f}_\tau$:= link flow vector in time slice τ;
$\mathbf{c}_\tau = \mathbf{c}(\mathbf{f}_\tau, t_\tau)$:= link cost vector in time slice τ;
$\forall$ client pair (u, w)
  $\forall$ time slice τ
    $k(u, w)_\tau$:= shortest path between the two users in time slice τ;
    $g_{k(u,\,w)\tau} = \Sigma_{ij}\delta_{ij,\,k(u,\,w)} \cdot c_{ij}(\mathbf{f}_\tau, t_\tau)$, shortest path cost;
    $\mathbf{G}_\tau = [g_{k(u,\,w)}]_{\tau,\,|T|\times|T|}$, shortest path cost matrix in time slice τ;
$\forall$ route R
  $\zeta_R = \Sigma_u\Sigma_w\, g_{k(u,\,w)\tau} \cdot x_{k(u,\,w)v}$, route cost.

Figure 6: Path cost in pseudo-dynamic approach.

2.6 Dynamic procedure

In dynamic approach, the procedure can be view as a two levels procedure: at the first level is built the route configuration (clients' sequence), at the second the cost is evaluated considering the dynamic shortest path cost between a client and the next. The cost are assumed dependent on departure time from intermediate nodes.

Given FS_i the forward star of node i, Figure 7 depicts the procedure.

In this case, a clarification is required regarding the time dependence of the link cost and of the path cost. Because the path cost depend on time (arrival or departure at/from a node) is not possible built one (or more) shortest path cost matrix just as in the static and pseudo-dynamic case. Supposing to start from a client u at time t_u, get $g_{k(u,\ i)}$ the cost to arrive at a generic node i, at time t_i, belonging to the shortest path tree with root in u. At node i, two options are available [16]: wait or not wait at the node. Assuming, for simplicity sake, that the wait is not possible, the arrival time and the departure time are coincident.

Given client pair (u, w)
t_u := departure time from u
$\mathbf{f}_t$:= time dependent link flow vector;
$\mathbf{c}_t = \mathbf{c}(\mathbf{f}_t, t)$:= time dependent link cost vector;
$B(w)$:= node belonging to the backward star of w and to the minimum path starting at instant t_u from u;
$k(u, w)\ (t_u,\ldots, t_{B(w)})$:= minimum path between u and w starting at instant t_u from u to w ;
Generation of the minimum path from u to w with time-generalized Bellman inequality.
  i := generic node;
  $t_i = t_u + g_{k(u,\ i)}(t_u,\ \ldots,\ t_{B(i)})$
  $\forall$ node $j \in FS_i$
    $g_{k(u,\ j)}\ (t_u,\ \ldots,\ t_{B(i)},\ t_i) \le t_i + c_{ij}\ (\mathbf{f}_t,\ t_i)$
$g_{k(u,\ w)}\ (t_u,\ldots,\ t_{B(w)}) = \sum_{ij}\ \delta_{ij,\ k(u,\ w)\ (t_u,\ldots,\ t_{B(w)})} \cdot c_{ij}\ (\mathbf{f}_t,\ t_i)$
$\zeta_R = \sum_u \sum_w g_{k(u,\ w)}\ (t_u,\ldots,\ t_{B(w)}) \cdot x_{k(u,w)(\ t_u,\ldots,\ t_{B(w)})v}$, route cost.

Figure 7: Path cost in dynamic approach.

3 Application

A test application is made on a real urban test network with 550 links and 150 nodes. The users to reach are 24 (coded from 2 to 25, 1 being the depot). In the dynamic approach, the cost function employed is a sinusoidal function specified and calibrated from real observed data in [16], as in equation 3.1:

$$c(t) = a + b \cdot sin(2\pi \cdot t/P + \phi) + t \tag{3.1}$$

where

- a, b, are parameters;
- P is the period;
- ϕ is the phase difference.

Note that for simplicity's sake, in relation to this application, the cost function is specified for non-congested system but it is time dependent.

The algorithm used to build the routes is a genetic algorithm [5], [20] that in the three cases (the static, pseudo-dynamic and dynamic approaches are analysed in the case of freight distribution in city logistic) starts with the same parameters.

In static case, the shortest path matrix cost is unique and not depend on the time. The costs, as explained in Section 2.1, can depend on the flow. The link cost function used in this case is $c = c(\mathbf{f})$.

Four routes compose the solution; the total cost is 6583 seconds (Table 1).

Table 1: Routes in static approach.

	sequence										cost (s)
Route1	1	8	10	14	11	7				1	1385
Route 2	1	12	19	20	23	21	16			1	1713
Route 3	1	9	4	3	2	6				1	1578
Route 4	1	13	15	22	25	24	18	17	5	1	1907
Total											**6583**

In the pseudo-dynamic approach, it is assumed that the study time is split into ten intervals; for each interval, a shortest path matrix cost is defined (note that the matrix cost in the first time slice is the same as that for static approach). The link cost function used in this case is $c_\tau = c(f_\tau, t_\tau)$.

Four routes compose the solution; the total cost is 5911 seconds (Table 2).

Table 2: Routes in pseudo-dynamic approach.

	sequence											cost (s)
Route1	1	18	17	22	24	20	9	11	5		1	1831
Route 2	1	13	14								1	1011
Route 3	1	12	15	16	21	23	19	25	10	7	1	2097
Route 4	1	2	6	8	4	3					1	972
Total												**5911**

In the dynamic approach, the path search should be made after the route construction, to take into account the departure time for each node in the paths connecting the users. The link cost function used in this case is $\mathbf{c}_t = \mathbf{c}(\mathbf{f}_t, t)$.

Four routes compose the solution; the total cost is 4727 seconds (Table 3).

The algorithm used to find the routes is a genetic algorithm, with allows solving the routing problem in acceptable computing time, although it does not ensure that the solution is the optimal one [5]. Starting from the same initial algorithm parameters (population, crossover and mutation rate) in this experiment, the pseudo-dynamic and the dynamic approach allow a gain, respect to the static approach, of 10.20% and 28.20% respectively.

Table 3: Routes in dynamic approach.

	sequence											cost (s)
Route1	1	19	20	25	24	23	22	18	17	5	1	1600
Route 2	1	8	21	16	15	14	11	10			1	1317
Route 3	1	6	13	12	9	7					1	1114
Route 4	1	3	4	2							1	696
Total												**4727**

4 Conclusions

In this paper, three scenarios for the vehicle routing problem are considered: static, pseudo-dynamic and dynamic. For each scenario, the cost functions are analysed, highlighting the differences in the cost formulation. Moreover, the algorithms to solve the vehicle routing problem in static, pseudo-dynamic and dynamic context are implemented. A test application is performed to evaluate the procedures and compare the solutions. The scenario with dynamic cost functions seems is the best scenario if compared with static and pseudo-dynamic approach. This can be because the continuous cost functions are able to simulate better the reality.

The comparison among the three scenarios highlights that implementing the vehicle routing with time dependent function (this, in practical applications, can be imply the real time information at the drivers) can have a significant reduction in the route time. The cost reduction can be imputes to a link cost evaluation closer to reality that implies, also, a changing in the order to visit the users.

Future developments can be realised considerint also the technological resources able to evaluate in real time the link cost for a rate of network link and forecast the cost for the other links.

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