Estimation of time-dependent O–D demand and route choice from link flows

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Abstract

Given time-dependent link traffic flow of a network, we propose a Maximum Likelihood Estimation (MLE) approach to estimate the parameters of dynamic route choice and time-dependent O–D demand simultaneously with consideration of dependencies among paths and links with repeated sampling, as well as some flow propagation information by utilizing time-dependent traffic data and any historical traffic information. By presenting approximate joint probability distribution functions of the temporal link traffic flows on a network, the full likelihood functions for estimating dynamic O–D and route choice parameters are derived with investigation of parameter identifiability. Although the methods can be applied using link flow data alone, incorporation of prior information into the inferential process is also discussed. The impact of sample size and the sensitivity to the number of O–D pairs and measurement errors have also been studied. The numerical example demonstrated that estimation of O–D matrix and dynamic route choices, as well as some flow propagation information, is feasible based on repeated-sampled link flow data alone.
Keywords: dynamic O–D demand, dynamic route choice, maximum likelihood estimation (MLE), flow propagation, DTA, repeated sampling.

1 Introduction

Dynamic Traffic Assignment (DTA) is a process of assigning time-dependent origin-destination (O–D) trips through a transportation network. Both analytical and simulation-based DTA models can adopt Dynamic User Optimal (DUO) or Stochastic Dynamic User Optimal (SDUO) principles to estimate travelers' route choice and provide a close-reality solution of time-dependent traffic conditions for evaluation and management. Both models require time-dependent O–D

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demand tables, and the flow propagation is assumed to use time-dependent shortest path algorithm and empirical travel time functions or models, including BPR function, Greenshield's function, car-following model, cell transmission model, platoon dispersion, etc. How should one evaluate the estimation and prediction of route choice resulting from either analytical or simulation DTA models? How should one obtain and evaluate the network-wide dynamic O–D demand matrices that essentially drive dynamic traffic assignment models? Currently, satisfactory answers to these questions are big challenges to transportation researchers. Fortunately, with the help of advanced technologies, such as loop detectors, video cameras, radar, GPS trackers, and probe vehicles, time-dependent link/path traffic information in certain areas is available, which enables the estimation of time-dependent O–D matrices. Given traffic flows on the links of a network during one or more time periods and any relevant prior information, estimating the intensity of traffic flow between an origin and a destination, dynamic route choice, and flow propagation is the objective of our study. The majority of early papers in solving static and dynamic origin-destination (O–D) matrix estimation problem attempted to find the 'most plausible' O–D matrix that was entirely consistent with the set of observed link flows. The plausibility of a given O–D matrix was judged by its similarity to some target or prior estimate of the O–D matrix. While this type of approach is heuristically reasonable, it is not well founded from a statistical point of view. The observed link flows are by nature stochastic, and the travel demand and cost between any particular O–D pair vary from day to day. The elements of the O–D matrix that we are attempting to estimate can legitimately be interpreted as mean O–D flows. There is no reason why these should precisely be consistent with a particular set of link data [1]. In static O–D matrix estimation, the relationship between the link flows and the O–D flows is specified in the form of a link-path incidence matrix. The disadvantage of such static models is that they do not capture the dynamic characteristics of traffic flow, and they assume that the observed link flows represent a steady-state situation that persists over a block of time. Statistical methods for static OD matrix estimation, such as maximum likelihood estimation, are demonstrated by Davis and Nihan [2]. In dynamic O–D matrix estimation, Kalman filter technology employed by some researchers (Cremer and Keller [3], Nihan and Davis [4]) is restricted to intersections /junctions or small segments of network corridors where the travel time between each O–D pair is roughly constant. Bell [5] and Wu and Chang [6] relaxed the constant travel time limitation by integration of platoon dispersion factors, that is, travel times can be variable for a few time steps. Ashok and Ben-Akiva [7] assumed knowledge of an assignment matrix or mapping that defines the temporal and spatial relationships between the link flows and O–D volumes, which is error-prone and a difficult task in reality. Sherali and Park [8] employed dynamic traffic assignment technology to obtain flow propagation information. Statistical optimality can be guaranteed by employing the maximum likelihood estimation method. He and Ran [9] have proposed to use this technique for time-dependent route choice estimation, but many issues remain to be resolved. For example, traffic counts on separate links or on different paths may be highly correlated. Thus, ignoring the inter-link or inter-path dependency will generally lead to bias in the O–D matrix and route choice estimation procedure. In this paper, we present a new methodology to estimate the parameters of dynamic route choice and time-dependent O–D demand simultaneously with consideration of dependencies among paths and links with repeated sampling, as well as some flow propagation information by utilizing time-dependent traffic data and any historical traffic information. Under the standard assumption of Poisson distributed number of O–D trips, we derive the joint distribution of the temporal link flows with consideration of variation in dynamic route choices. This function is relatively intractable, but we are able to obtain useful approximations to it based upon the multivariate normal distribution. Using these approximate likelihoods, we can obtain consistent estimates of the elements of the O–D matrix and the route choice probabilities. The estimation results enable us to perform related statistical tests and travel time evaluation. Our work demonstrates that estimation of O–D matrix and dynamic route choices, as well as some flow propagation information, is feasible based on repeated-sampled link flow data alone. Nevertheless, if prior information about the O–D matrix, route choice probabilities and flow propagation does exist, it may be combined with the likelihood functions in the standard Bayesian fashion. We also consider the study in which the link traffic data are measured with error.

2 Model formulation

Consider a transportation network with n nodes and m directed links. Suppose that we observe the traffic system over N homogeneous, independent time periods. For example, these measurement periods could be 10:00am–12:00pm or 2:00pm–4:00pm on a sequence of Tuesdays. N is basically the number of repeated samples. The length of each measurement period should be sufficiently large so that a typical journey can be finished during that period. Since we are studying dynamic traffic condition, we discretise time and use our sampling interval h as time unit. The maximum number of sampling intervals in one measurement period is T, so the sampling intervals are [1, 2, …, h, …, T].

Start with standard assumption of $q^{rs}(t)$, the total O–D demand from origin r to destination s during departure time t, as a standard Poisson random variable with the following expectation and variance:

$$E(q^{rs}(t)) = \lambda^{rs}(t) \tag{1}$$

$$Var(q^{rs}(t)) = \lambda^{rs}(t) \tag{2}$$

Let route flow $f_k^{rs}(t)$ be the part of traffic $q^{rs}(t)$ that takes path k (one of the K paths between r and s) during departure time t, and is multinomially distributed with the following probability distribution function:

$$\mathrm{P}\{F_k^{rs}(t) = f_k^{rs}(t), \forall k\} = q^{rs}(t)\prod_{k=1}^{K}\frac{[p_k^{rs}(t)]^{f_k^{rs}(t)}}{f_k^{rs}(t)!} \tag{3}$$

and

$$\sum_{k=1}^{K} f_k^{rs}(t) = q^{rs}(t) \tag{4}$$

with mean and variance-covariance as follows:

$$E(f_k^{rs}(t)) = q^{rs}(t) p_k^{rs}(t) \tag{5}$$

$$Var(f_k^{rs}(t)) = q^{rs}(t) p_k^{rs}(t)(1 - p_k^{rs}(t)) \tag{6}$$

$$\begin{aligned} Cov(f_k^{rs}(t), f_{k'}^{r's'}(t')) &= 0, \quad \text{if (rs)} \neq \text{(r's') or t} \neq \text{t'}; \\ &= -q^{rs}(t) p_k^{rs}(t) p_{k'}^{rs}(t), \quad \text{if (rs) = (r's'), t = t', and k} \neq \text{k'}. \end{aligned} \tag{7}$$

where $p_k^{rs}(t)$ is the probability of traffic $q^{rs}(t)$ that takes path k (one of the K paths between r and s) departing at time t.

Utilizing the properties of Poisson and Multinomial distributions, the mean and variance of unconditional path flows F can be determined as follows:

$$E(f_k^{rs}(t)) = \lambda^{rs}(t) p_k^{rs}(t) \tag{8}$$

$$Var(f_k^{rs}(t)) = \lambda^{rs}(t) p_k^{rs}(t) \tag{9}$$

$$Cov(f_k^{rs}(t), f_{k'}^{r's'}(t')) = 0 \tag{10}$$

where $p_k^{rs}(t)$ – probability (proportion) of travelers choosing route k from r to s departing at time t. k = 1, 2, …, K (K is the number of routes from r to s).

Define link flows X as linear combinations of path flows F:

$$X = \Delta \bullet F \tag{11}$$

where Δ is the link-route incidence matrix.

Then, link flows X are random variables with mean:

$$E(X) = \Delta \bullet E(F) \tag{12}$$

and variance-covariance matrix, Σ_X :

$$\Sigma_X = \Delta \Sigma_F \Delta^T \tag{13}$$

The link-route incidence matrix is as follows:

$$\Delta = (\delta_{k,h}^{a,t})_{k,h,a,t} \tag{14}$$

where a – link, k – route, h – time interval [1, …, T], t – departure time.

The link flows on link a for sampling interval h are combinations of route flows:

$$x_a(h) = \sum_{k,t \leq h} \delta_{k,h}^{a,t} f_k(t) \tag{15}$$

To obtain the estimation of dynamic O–D and route choice parameters, it is needed to maximize the likelihood function of link flow X. However, maximization of the likelihood function

$$L(\lambda, P) = Lik(x \mid E(X \mid P, \lambda), \Sigma_X(P, \lambda)) \tag{16}$$

is only computationally feasible for very small-scale examples. When dealing with large transportation systems, a natural way of attempting to overcome computational problems is to use a multivariate normal approximation.

Taking log of the likelihood function of link flows and omitting additive constants, the following objective function is obtained:

$$\log L(\lambda, P) = -\frac{1}{2}\sum_{n=1}^{N} (x^{(n)} - E(X \mid P, \lambda))^T (\Sigma_X(P, \lambda))^{-1} (x^{(n)} - E(X \mid P, \lambda)) \tag{17}$$

subject to $1 \geq p \geq 0$ and $\lambda \geq 0$. $x^{(n)}$ is the link traffic count for sample n. Then, equation (17) is simplified as:

$$\log L(\lambda, P) = -\frac{1}{2}\sum_{n=1}^{N} (x^{(n)} - E(X \mid P, \lambda))^T S^{-1} (x^{(n)} - E(X \mid P, \lambda)) \tag{18}$$

subject to $1 \geq p \geq 0$ and $\lambda \geq 0$.

3 Numerical testing

The network in Figure 1 has 1 O–D pair, 5 links and 3 routes.

The sampling period is 5 minutes, while each sampling interval (h) is 1 minute. We generated N = 1, 3 and 5 samples of link flow data, with parameter values set at

$$\lambda_{true} = (20,25,30,35,25)',$$

$$P_{true} = \begin{pmatrix} 0.2 & 0.3 & 0.5 & 0.25 & 0.1 \\ 0.5 & 0.3 & 0.5 & 0.45 & 0.5 \\ 0.3 & 0.4 & 0 & 0.3 & 0.4 \end{pmatrix},$$

The link flow vector is $X = [x_a(h)]^T$, where a: 1 ~ 5, h: 1 ~ 5. And the route flow vector is $F = [f_k(t)]^T$, where k: 1 ~ 3, t: 1 ~ 5. Hence the link-route incidence matrix is a 25×15 matrix with each entry determined by the sampled link flow data and the BPR link performance function:

$$T_a(h) = t_{aff}[1 + A_a(\frac{x_a(h)}{c_a})^{P_a}]$$

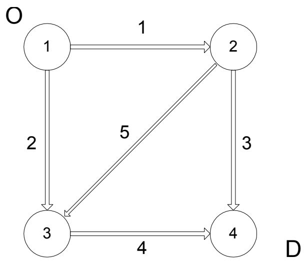

Figure 1: Network topology.

where a = 1 ~ 5, h = 1 ~ 5, t_{aff} = free flow travel time in minutes on link a, c_a = link capacity, A_a and P_a are constants. There are 15 columns in the matrix, corresponding to 3 routes repeated for 5 departure time intervals; and there are 25 rows in the matrix, corresponding to 5 links repeated for 5 sampling time intervals. For example, the intersection of the first row and first column states that, during sampling time interval 1, link 1 which belongs to route 1 takes all the traffic of route 1 departing the origin at time interval 1. We know that

$$E(F) = [\lambda(t) p_k(t)]' \text{, where t: 1 ~ 5, k: 1 ~ 3.}$$

$$\Sigma_F = diag(\lambda(t) p_k(t)) \text{, where t: 1 ~ 5, k: 1 ~ 3.}$$

Thus, we can derive $E(X)$ and Σ_X according to Equations (12) and (13). Note that Σ_X has many non-zero off-diagonal elements, which indicates that correlation between different link flows is represented in our model.

3.1 Case I – no measurement errors

Compare with N = 1 and N = 3, the five groups of simulated data (corresponding to one sample per week for five weeks) randomly collected for computing the GLS estimates have even larger mean square errors of 176.84, 130.06, 217.75, 186.86 and 344.05 from the expected link flow, respectively. However, the GLS estimates for O–D flows are at the same closeness or closer to the true values than N = 1 and 3 as summarized in Figures 2 and 3.

The GLS estimates for O–D flows are within [-2.82, 0.29] (flow units) range of the true means and the mean square error equals 20.57. The route choice parameter estimates are within [-0.08, 0.10] range of the true values with the mean square error equal to 0.03. From Figure 3, it is evident that larger the sample size, the closer the route choice parameter estimates to true values. This trend is not so evident for O–D flow estimates in Figure 2. The reason to choose the real small sample sizes of 1, 3 or 5 for this study is to test the robustness of the methodology. The test has shown that small sample sizes are working consistently, which will help reduce the extensive real data collection.

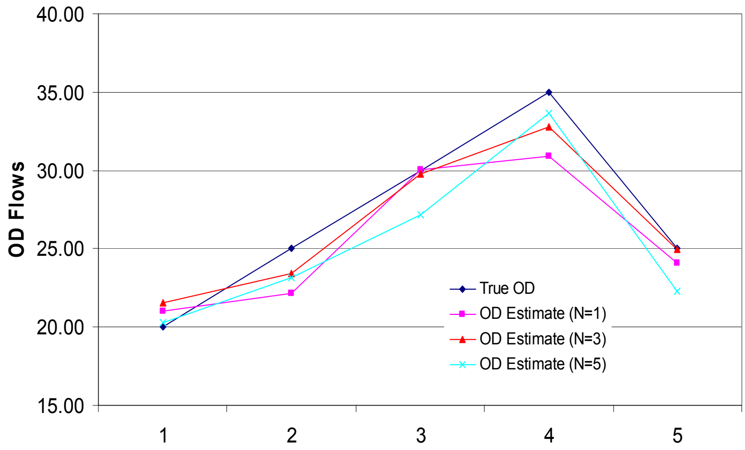

Figure 2: O–D demand estimations from error-free link flows.

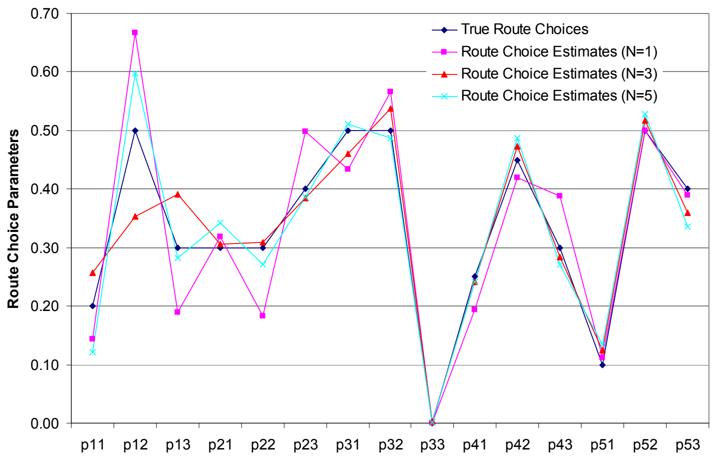

Figure 3: Route choice estimations from error-free link flows.

3.2 Case II – including measurement errors

When N = 3, the three groups of error-included simulated data have large mean square errors of 245.55, 163.00 and 261.30 from the expected link flow, respectively. However, the GLS estimates for O–D flows and route choice parameters are closer to the true values than N = 1 as summarized in Figures 4 and 5.

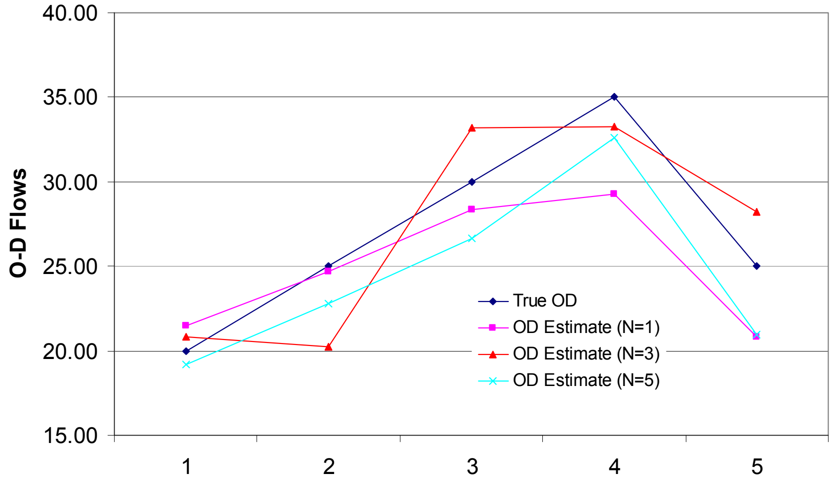

Figure 4: O–D demand estimations from error-included link flows (error s.d. = 2 vehs/link).

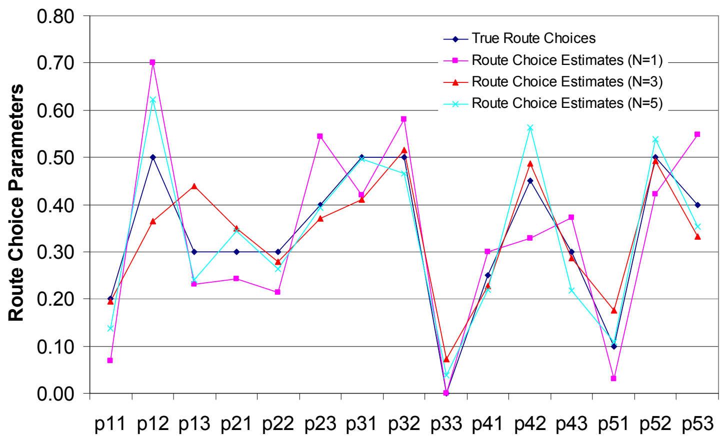

Figure 5: Route choice estimations from error-included link flows (error s.d. = 2 vehs/link).

The GLS estimates for O–D flows are within [-4.74, 3.24] (flow units) range of the true means and the mean square error equals 46.70. The route choice parameter estimates are within [-0.13, 0.14] range of the true values with the mean square error equal to 0.07. Compare with N = 1 and N = 3, the five groups of error-included simulated data have even larger mean square errors of 354.75, 163.57, 515.26, 408.43 and 449.13 from the expected link flow, respectively. However, the GLS estimates for O–D flows are closer to the true values than N = 1 and 3 as summarized in Figure 4 and 5. The GLS estimates for O–D flows are within [-4.03, -0.8] (flow units) range of the true means and the mean square error equals 38.99. The route choice parameter estimates are within [-0.08, 0.12] range of the true values with the mean square error equal to 0.05. From Figure 4 and 5, it is evident that larger the sample size, the closer the O–D flow and route choice parameter estimates to true values. The reason to choose the same small sample sizes of 1, 3 or 5 for this part of the study is to test the sensitivity of the methodology to measurement errors. The test has shown that when N = 1 and 3, the effects of measurement errors are strong that the parameter estimates are far away from the true value. However, when N = 5, the effects of measurement errors are mostly balanced out by averaging over samples that the O–D flow and route choice parameter estimates are consistently close to the true values. This finding shows that approximately 5 samples are required to reduce the effects of measurement errors with standard deviation of 2.

3.3 Case III – more O–D pairs

Last, the model's sensitivity to the number of O–D pairs is tested. Up till now, only one O–D pair (from node 1 to node 4) has been considered. Now the following three additional O–D pairs are considered: 2–4 (2 routes), 1–2 (1 route) and 3–4 (1 route). When five groups of simulated link data (corresponding to one sample per week for five weeks, N =5) are randomly collected for computing the GLS estimates have very large mean square errors of 1123.25, 1472.75, 301.25, 977.75 and 1514.75 from the expected link flows, respectively. The GLS estimates for O–D flows are relatively close to the true values as summarized in Figures 6 and 7.

The GLS estimates for 4 pairs of O–D flows are within [-11.10, 5.82] (flow units) range of the true means and the mean square error equals 469.24. The route choice parameter estimates for 4 pairs of O–D flows are within [-0.38, 0.37] range of the true values with the mean square error equal to 0.86. The test

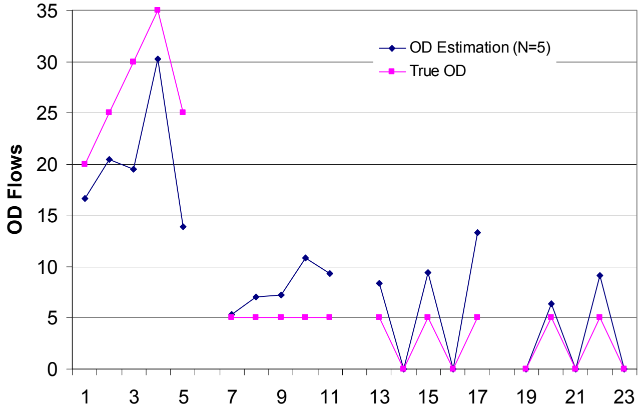

Figure 6: Multiple O–Ds estimations.

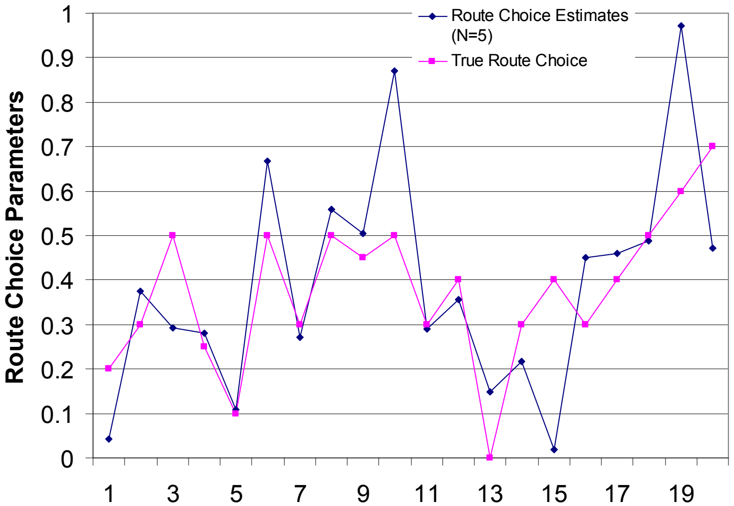

Figure 7: Route choice estimations from multiple O–Ds.

has shown that the effects of number of O–D pairs are noticeable while the O–D flow and route choice parameter estimates are relatively close to the true values. One probable reason is the network's traffic condition is getting congested and the study time period needs to be expanded for completing trips. The other probable reason is the five sets of simulated random link flow samples are too far away from the expected values, it is expected that the effects of number of O–D pairs will be mostly balanced out by averaging over samples with the increase of sample size, the reduction of sample errors and the inclusion of historical information.

4 Concluding remarks

In this paper, a new methodology based on repeated sampling has been developed to estimate dynamic O–D demand and route choice parameters from link flow data. Our approach can incorporate uncertainty about O–D demand and route choice probabilities, and prior information about model parameters and measurement error in the link flow data. More importantly, our methodology can consistently estimate O–D demand and route choice from link data alone. The exact likelihood of our model is too complicated for practical use, but inference is possible using either a multivariate normal likelihood approximation or a related generalized least squares technique. It should be noted that more research must be conducted in the reduction of computational efforts to solve constrained multivariate minimization problems for real-time ITS application.

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