# Investigating the traction system in trains

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## Abstract

The paper looks at the traction system with the view on the interaction between the wheel and the rail. A theoretical model is proposed which includes all the components in the traction system and their dynamic influence on each other. The programming is carried out in MatLab.

With flat wheel condition, the actual measurement of the rail strain during the impact is made and the stress level obtained. The results are compared with the stress levels calculated from the model. The verified model will enable one to study the system vibration/performance for any design modification on the components and the condition on the rail affecting the traction.

*Keywords: traction system, modelling rail-wheel interaction, flat wheel, bogie gearbox.* 

# 1 Introduction

The development in the rail technology has created a relatively safe and economical transport system. The rail-wheel interface is considered to be a major part in the rail traction system. Problems such as rail head fatigues, rail surface spalling, rail surface roughness and wheel flat cracks may lead to failing contacts between the rail and the wheel resulting in an accident.

As the wheels in the bogie are driven by a motor through a gearbox, it becomes necessary to analyse the whole driving system in assessing the interaction between the rail and the wheel. Any input force at the interface will influence the dynamics of the driving system which in turn would affect the behaviour of the rail-wheel contact.



There are different possibilities of failure on the rail as explained by Cannon [1]. A complete cross section break may result from lower toughness due to cold weather and the fact that the longitudinal stress is large due to wheel sliding on the rail. The defect inside the rail head known as a 'piping cavity' or a horizontal cracking with transverse cracking in the rail head usually results from a manufacturing defect. Foot transverse fatigue cracks are usually initiated from galling (wear and corrosion) at a rail support (chair or base-plate). The fatigue cracks are often difficult to detect and rail fracture commonly occurs as suggested by Cannon [1].

Wu and Thompson [2] say that wheel flats fatigue is caused by worn flats on wheel tread. This kind of situation usually happens in a poor adhesion condition at wheel-rail interface, such as leaves covered railhead during the autumn. Wheel flats introduce a relative displacement input to the wheel-rail system in the same way as roughness causing high levels of noise or impact loading that leads to damage of the components. The impact from flats would increase the stress levels to twice the nominal wheel load. This in return increases the contact pressure-Hertzian stress resulting in plastic yield in the rail-wheel material producing excessive noise and discomfort for passengers and eventually fast deterioration of the railway infrastructure.

A typical bogie system, which is used to carry the coach, is shown below.



Figure 1: Typical bogie.

The system is complicated with a gearbox located on the wheel axle asymmetrically. A motor fixed with bogie is connected to the gearbox. The bogie is supported by 4 pairs of springs and dampers on the wheels. The bolster springs (between bogie frame and bolster) allows the bogie to rotate relative to the train body, isolating the body from vibration generated by the bogie, and transmitted traction force from the bogie to the body.



Different models of the bogie have been studied. Figure 2 below presented by Shimamune et al. [3], shows a DDM (Direct Drive Motor) system for the JR East conventional commuter train.

In order to prevent the DDM from being rotated by reaction force, a link-like reaction force receiving rod is used to connect the motor enclosure with the bogie frame. This model focuses on rotation of the wheel axle, so that the system can be treated as a linear spring system.



Figure 2: Typical model for bogie.

Gearbox dynamics due to elasticity and backlash in the gear has often played an important role on the wheel axle between two wheels. The gearbox includes a gear which is fixed on the axle of the wheel shaft and pinion which is excited by asynchronous motor always engaged together inside the gearbox.

# 2 Theoretical analysis/modelling

The proposed model of the bogie including all the components is shown in Figure 3.

All the major components are marked. X and  $\Phi$  indicating linear and angular displacement respectively. Linear and torsional stiffness are shown as k and K respectively with mass moment of inertia as J.

The lumped mass parameter model proposed for the bogie is shown in Figure 4.

The effective mass of Left Hand Side (LHS) wheel  $m_1$  and Right Hand Side (RHS) wheel  $m_2$  are calculated as

$$m_{1} = m_{w_{1}} + m_{sh_{1}} + \frac{l_{1}}{L}m_{GB}$$
(1)

$$m_2 = m_{w_2} + m_{sh_2} + \frac{l_2}{L} m_{GB}$$
(2)

where suffix w, sh, GB indicate wheel, shaft and gearbox respectively.

Then the equations for linear and angular displacement of the LHS wheel could be written by substituting in equation (1)

$$m_1 \ddot{x}_{w_1} = k_s (x_1 - x_{w_1}) + c_{w_1} (\dot{x}_1 - \dot{x}_{w_1}) - R_1$$
(3)

$$J_{w_1}\ddot{\varphi}_{w_1} = -k_{w_1} \left( \varphi_{w_1} - \varphi_{GB} \right) - c_{sh_1} \left( \dot{\varphi}_{w_1} - \dot{\varphi}_{GB} \right) - T_1$$
(4)

Similarly, these equations for the RHS wheel would be by substituting in equation (2)

$$m_2 \ddot{x}_{w_{2_1}} = k_s \left( x_2 - x_{w_2} \right) + c_{w_2} \left( \dot{x}_2 - \dot{x}_{w_2} \right) - R_2$$
(5)

$$J_{w_2}\ddot{\varphi}_{w_2} = -k_{w_2} \left( \varphi_{w_2} - \varphi_{GB} \right) - c_{sh_2} \left( \dot{\varphi}_{w_2} - \dot{\varphi}_{GB} \right) - T_2 \tag{6}$$



Figure 3: Arrangement of the components.



Figure 4: Proposed model.

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For reactions on rail-wheel interface  $R_1$  and  $R_2$  a contact stiffness between the two components is considered. The deflection of the contact spring, where *r* is the roughness relative displacement input between the wheel and rail,  $x_r$  and  $x_w$  are displacement of rail and wheel,  $C_H$  is the Hertzian constant which depends on the radii of curvature of the surfaces and their material properties.

$$f(x) = C_H \delta^{\frac{3}{2}} \text{ if } \delta < 0$$

$$f(x) = 0 \text{ if } \delta \ge 0$$
(7)

where  $\delta = x_w - x_r - r$ .

If the relative motion in the contact spring in a small range, the equation can be written as f=W+df, W is the nominal preload, df is the fluctuating part of the contact force

$$df(x) \approx \frac{3}{2} C_H \delta_0^{\frac{1}{2}} d\delta = k_H d\delta$$
(8)

The linear contact stiffness

$$k_{H} = \frac{3}{2} C_{H} \delta_{0}^{\frac{1}{2}}$$
<sup>(9)</sup>

Hertzian constant

$$C_H = \frac{2}{3} E^* R_e^{\frac{1}{2}}$$
(10)

from equation (10)

$$E^* = \frac{E}{1 - v^2} = 2.3 \times 10^{11} Pa$$

is the plane strain Young's Modules,  $R_e = R_{wheel}$  is an equivalent radius of curvature from equation (8)

$$k_{H} = E^{*} R_{e}^{\frac{1}{2}} \delta_{0}^{\frac{1}{2}}$$
(11)

from equation (9)

 $\delta_0 = x_w - x_r - r = 0$ ,  $k_H = 0.9 GN / m$ 

It is known that the vehicle weight 40Tonnes, with 6 axles per vehicle, the axle load is 6.7Tonne and the wheel load M is 3.35Tonne, (fully laden axle is 9.9Tonne and fully laden wheel M is 4.95Tonne).

The inertial reaction force comes from the static loading condition

$$R_1(0) = (M + m_1)g$$
(12)

$$R_2(0) = (M + m_2)g$$
(13)



where M is the mass of the main body that loads on the wheel (M=3350kg for empty, M=4950kg for fully laden). The instantaneous reaction forces then would be

$$R_{1} = R_{1}(0) + k_{H} \left( x_{w_{1}} - x_{r_{1}} - r \right)$$
(14)

$$R_2 = R_2(0) + k_H \left( x_{w_2} - x_{r_2} - r \right)$$
(15)

Traction force T is affected by  $R_1$  and  $R_2$  and coefficient of friction  $\mu$ 

$$T_1 = \mu R_1 \tag{16}$$

$$T_2 = \mu R_2 \tag{17}$$

According to the test on rail, it is found that even in a better railway condition that has no obvious cracks, there is still a head checking, which is defined as traffic induced angled cracks, on a fine rail surface. This surface rolling contact fatigue will be modelled as a sin wave input at the rail surface in the form of  $x_r = 0.001 \sin(\omega t)$  when rail roughness r=0.

The maximum measured speed on the rail was 81 km/h (22.5 m/s). Considering the distance between each sleeper being 0.75,  $\omega$  value would be

$$\omega = 2\pi f = 2\pi \frac{v}{0.75} = 60\pi \tag{18}$$

resulting in

$$x_r = 0.001 \sin(60\pi t)$$
 (19)

The displacement of LHS Bogie x1 and RHS Bogie x2 can be written as:

$$M\ddot{x}_{1} = -k_{s}\left(x_{1} - x_{w_{1}}\right) - c_{w_{1}}\left(\dot{x}_{1} - \dot{x}_{w_{1}}\right)$$
(20)

$$M\ddot{x}_{2} = -k_{s}(x_{2} - x_{w_{2}}) - c_{w_{2}}(\dot{x}_{2} - \dot{x}_{w_{2}})$$
(21)

The gear and pinion connected by their teeth, it can be modelled as a mesh spring system on the contact surface.

The gear has a mesh stiffness  $k_m$  that comes from the connection between pinion and gear, angular rotation of gear is  $\phi_g$ , pinion is  $\phi_p$ , gearbox is  $\phi_B$ , inertial of the gear is  $J_g$ , radius of gear is  $r_g$ , pinion is  $r_p$ 

$$J_{g}\ddot{\varphi}_{g} = -k_{w_{1}}(\varphi_{g} - \varphi_{w_{1}}) - c_{sh_{1}}(\dot{\varphi}_{g} - \dot{\varphi}_{w_{1}}) - k_{w_{2}}(\varphi_{g} - \varphi_{w_{2}}) - c_{sh_{2}}(\dot{\varphi}_{g} - \dot{\varphi}_{w_{2}}) - k_{m}r_{g}(\varphi_{g}r_{g} + \varphi_{p}r_{p} - \varphi_{B}r_{g})$$
(22)

The pinion gets mesh stiffness  $k_m$  that comes from the connection between pinion and gear, angular rotation of gear is  $\phi_g$ , pinion is  $\phi_p$ , gearbox is  $\phi_B$ , inertial of the pinion is  $J_p$ , and the tortional vibration from the motor



$$J_{p}\ddot{\varphi}_{p} = -k_{m}r_{p}\left(\varphi_{g}r_{g} + \varphi_{p}r_{p} - \varphi_{B}r_{g}\right) + k_{M}\left(\varphi_{M} - \varphi_{p}\right) - c_{M}\left(\dot{\varphi}_{p} - \dot{\varphi}_{M}\right)$$
(23)

Motor which is joined with pinion by motor shaft has the inertial  $J_M$ , and torsional stiffness  $K_M$ .

$$J_{M}\ddot{\varphi}_{M} = -k_{M}\left(\varphi_{M} - \varphi_{p}\right) - c_{M}\left(\dot{\varphi}_{M} - \dot{\varphi}_{p}\right)$$
(24)

There is a spring with the linear stiffness  $k_B$  and damping  $c_B$  between the bogie frame and gearbox itself, the inertial of the gearbox is  $J_B$ 



Figure 5: Interaction between gear and pinion.

Vertical mesh force  $F_N = F_m * \cos 20^\circ$ , pressure angle between gear and pinion is  $20^\circ$ 

$$F_N = k_m r_g (\varphi_g r_g + \varphi_p r_p - \varphi_B r_g) \cos 20$$
(25)

$$J_B \ddot{\varphi}_B = -k_B r_B \varphi_B - c_B r_B \dot{\varphi}_B - \frac{r_g}{r_B} F_N$$
(26)

The linear spring model of the whole system is therefore, defined by the equations given. Vondrich [4] explained that these equations could be solved using Matlab with state-space representation.

## **3** Loading conditions

#### 3.1 Out of balance motor coupling

When motor is accelerated to its maximum speed and then decelerated again, there will be an excitation at the rotational speed assuming a unbalance at the coupling. This force P will cause the gearbox to be forced sideways on the central shaft as it rotates, causing additional stress. P as a function of time can be described by  $P = P_0 \sin(\omega_M t)$  assuming an unbalance of mass m at a distance



of e, P<sub>0</sub> will be  $P_0 = me\omega_M^2$ . When m =0.15kg, e =0.15m,  $\omega_M$  = (gear radius/pinion radius)\* (train speed V/wheel radius).

$$P = 880\sin(198t) \quad v = 22.5m/s \tag{27}$$

Equations (23), (24) and (26) for pinion, motor and gearbox respectively, will then be changed to;

$$J_{p}\ddot{\varphi}_{p} = -k_{m}r_{p}\left(\varphi_{g}r_{g} + \varphi_{p}r_{p} - \varphi_{B}r_{g}\right) + k_{M}\left(\varphi_{M} - \varphi_{p}\right) - c_{M}\left(\dot{\varphi}_{p} - \dot{\varphi}_{M}\right) + P$$

$$(28)$$

$$J_M \ddot{\varphi}_M = -k_M \left( \varphi_M - \varphi_p \right) - c_M \left( \dot{\varphi}_M - \dot{\varphi}_p \right) + P \tag{29}$$

$$J_B \ddot{\varphi}_B = -k_B r_B \varphi_B - c_B r_B \dot{\varphi}_B - \frac{r_g}{r_B} F_N - \frac{r_g + r_p}{r_B} P$$
(30)

#### 3.2 Worn Wheel flats

As discussed before, there will be a worn flat in some poor adhesion conditions at wheel-rail interface. In this case, having the length of the worn area L as 0.15m, the rotating time (free fall) of the worn wheel  $t_s$  can be calculated:



Figure 6: Flat wheel simulation.

Considering a worn flat wheel running on the rail, when it comes to the worn area, there may be no contact between the wheel and rail for a short while, as a wheel flat flying on the air, followed by a large impact at the next moment. Figure 7 shows the orientation of the wheel and the impact loading belonging to the corresponding scenario.



Figure 7: Impact force produced by flat wheel.

### 4 Preliminary results and discussions

Strain gauge calibration has shown that for impact load of 72kN, an output voltage of 0.703 was obtained. Therefore, using a calibration factor of 102.4 kN/volt the measured load during impact was 82 kN.



Figure 8: Measured track load (left) and track load from the model rail-wheel Interface reaction force r obtained from the model for the same two speed conditions are displayed above (LHS R1—'blue' line, RHS R2—'green' line).

For the empty vehicle, travelling with 81 km/h speed the maximum calculated impact force was about 75 kN. This compares with the measured value of 81kN.

# 5 Conclusions

A computer model is presented for the traction system. It includes the interaction of the gearbox with the chassis as well as the interaction between the rail and the wheel. The model would enable the investigators to determine the response of each component for excitation from different sources.

The preliminary results obtained from the model on the load produced from a flat wheel, do compare with the measured values on the track.

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