A fixed-point model and algorithms for simulating urban freight distribution in a multimodal context with crossed congestion

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Abstract

Urban road networks are generally shared by at least three different kinds of vehicles: private cars, buses and freight vehicles. These vehicle categories influence their travel times reciprocally (the *crossed congestion* phenomenon). Hence, in order to simulate the performance of urban road networks, it is worth developing a multimodal assignment model that takes into account the effects of reciprocal influences among vehicles on user travel choices (at least on mode and path choices). In this paper, we extend the model proposed in a previous paper to the case of urban freight distribution simulation, considering crossed congestion among all considered transportation systems. The proposed model is tested on a real dimension network adopting three algorithmic approaches (external, internal and hyper-network) for its solution and comparing their performances in terms of calculation times and result reliability. Finally, we discuss the effects of crossed congestion on freight path choices and the influences of freight vehicle presence on (road and transit) user choices.
Keywords: multimodal traffic assignment, crossed congestion, urban freight distribution, non-separable cost functions.

1 Introduction

Generally, a multimodal assignment problem has to be adopted when it is necessary to estimate user flows on different transportation systems under two assumptions: that modal split, as well path choice, is affected by network costs

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and at least two transportation modes are congested (i.e. their performance depends on user flows). This problem can be considered an elastic equilibrium problem and can therefore be formulated as a fixed-point problem.

Elastic assignment problems can usually be classified into deterministic or stochastic equilibrium models according to the assumption on the random residual of the path choice model. In both cases, Cantarella [2] and Cascetta [3] proved that, with suitable assumptions on cost functions, demand functions and path choice models, it is possible to state the existence and uniqueness of the equilibrium solution.

In this paper, we propose an assignment model where the elasticity of the demand is associated to a mode choice model, considering the phases of trip generation and distribution as rigid. Moreover, in order to apply assumptions suitable in urban contexts, we hypothesise that travel times of transit systems depend on flows of private cars and freight vehicles in the case of shared lanes, and are constant (with respect to the flows) in the case of exclusive lanes.

It is worth noting that the proposed model introduces some theoretical complications because it does not satisfy some assumptions, proposed by Cantarella [2], Sheffi [4] and Oppenheim [5], for stating the uniqueness of the equilibrium solution in the case of elastic demand. This problem was analysed and solved by D'Acierno *et al.* [1], extending cost function assumptions in the case of a bimodal (road and transit) context. Hence, the aim of this paper is to extend the previous model to the case of the presence of a third congested system, i.e. the freight distribution system.

The proposed model is solved by adopting the three different approaches (external, internal and hyper-network) used to solve elastic demand assignment problems and analysing effects of freight distribution on (road and transit) user choices.

The paper is structured as follows: the proposed fixed-point problem is analysed in section 2; extensions of solution algorithms for managing elastic demand problems are proposed in section 3; section 4 describes the application of the algorithms in the case of a real dimension network and compares them in terms of calculation times and reliability of solutions; conclusions and research prospects are summarised in section 5.

2 The proposed fixed-point problem

The assignment problem proposed in this paper for simulating user flows on an urban network is based on the following assumptions:

- there are three analysed transportation systems: road, transit and freight systems;
- demand is elastic at mode choice level for the road and transit systems (i.e. a user can choose whether to travel by road or on the transit system);
- demand is rigid for the freight system (i.e. network performances affect only path choice and not travel demand or freight quantities);
- the path choice behaviour of the transit user is pre-trip/en route (hyper-path approach);

- road, transit and freight systems are affected reciprocally in terms of travel times in the case of shared lanes;
- transit travel times are constant (with respect to road and freight vehicles) in the case of exclusive lanes;
- the assignment model is multi-class with undifferentiated congestion;
- the average occupancy index for the road system is constant for all user categories;
- capacity constraints are neglected in all transportation systems.

With the above hypotheses, the user flows on the network can be obtained by means of the interaction between two kinds of models: a *supply model*, that simulates transport performances depending on user flows, and a *demand model*, that imitates user choice influenced by performances of the three considered transportation systems.

The supply model can be formulated as:

$$\overline{C}_m^i = \beta^i \cdot \left(\overline{A}_m^i\right)^T \overline{c}_m\left(\overline{f}_c, \overline{\varphi}_t, \overline{f}_f\right) + \overline{C}_m^{i,NA}\ . \tag{1}$$

where $\overline{C}_m^i$ is the vector of path generalised costs for mode m associated to user category i; m is the generic transportation system, where m=c for the road system, m=t for the transit system and m=f for the freight system; β^i is a non-negative coefficient associated to user category i, which allows the multi-class approach to be considered; $\overline{A}_m^i$ is the link-path incidence matrix for mode m associated to user category i; $\overline{c}_m$ is the vector of link generalised costs for mode m; $\overline{f}_c$ is the link flow vector of the road system (m=c); $\overline{\varphi}_t$ is the frequency vector of the transit system, which is equal to the vehicle flow vector associated to the transit system (m=t); $\overline{f}_f$ is the link flow vector of the freight system (m=f); $\overline{C}_m^{i,NA}$ is the vector of non-additive path costs for mode m associated to user category i, which expresses costs that depend only on paths of mode m (such as road tolls at motorway entrance/exit points).

Likewise, the demand model, based on the assumption that users are rational decision-makers maximising the utility associated to their choices, can be formulated as:

$$\overline{f}_m = \sum_i \overline{f}_m^i = \sum_i \overline{B}_m^i \overline{P}_{k,m}^i\left(\overline{C}_m^i\right)\overline{d}_m^i\left(\overline{C}_c^i, \overline{C}_t^i\right). \tag{2}$$

where $\overline{f}_m$ is the link flow vector for mode m; $\overline{f}_m^i$ is the link flow vector for mode m associated to user category i; $\overline{B}_m^i$ is the link-path incidence matrix for mode m associate to user category i, which is equal to matrix $\overline{A}_m^i$ only in the case of road and freight system; $\overline{P}_{k,m}^i$ is the path choice matrix for mode m associated to user category i; $\overline{d}_m^i$ is the travel demand vector for mode m associated to user category i.

Combining eqn (1) with eqn (2), the demand-supply interaction model, known as *assignment model*, is obtained as a fixed-point model (Cantarella [2]).

In particular, taking into account the assumptions of the proposed model, the fixed model can be specified by means of the following relations:

$$\bar{f}_c^* = \sum_i \bar{A}_c^i \bar{P}_{k,c}^i \left(\beta^i \cdot \left(\bar{A}_c^i\right)^T \bar{c}_c\left(\bar{f}_c^*, \bar{\varphi}_t, \bar{f}_f^*\right) + \bar{C}_c^{i,NA}\right) \cdot \bar{d}_c^i \left(\beta^i \cdot \left(\bar{A}_c^i\right)^T \cdot \bar{c}_c\left(\bar{f}_c^*, \bar{\varphi}_t, \bar{f}_f^*\right) + \bar{C}_c^{i,NA}, \beta^i \cdot \left(\bar{A}_t^i\right)^T \bar{c}_t\left(\bar{f}_c^*, \bar{\varphi}_t, \bar{f}_f^*\right) + \bar{C}_t^{i,NA}\right). \tag{3}$$

$$\bar{f}_t^\wedge = \sum_i \bar{B}_t^i \bar{P}_{k,t}^i \left(\beta^i \cdot \left(\bar{A}_t^i\right)^T \bar{c}_t\left(\bar{f}_c^*, \bar{\varphi}_t, \bar{f}_f^*\right) + \bar{C}_t^{i,NA}\right) \cdot \bar{d}_t^i \left(\beta^i \cdot \left(\bar{A}_c^i\right)^T \cdot \bar{c}_c\left(\bar{f}_c^*, \bar{\varphi}_t, \bar{f}_f^*\right) + \bar{C}_c^{i,NA}, \beta^i \cdot \left(\bar{A}_t^i\right)^T \bar{c}_t\left(\bar{f}_c^*, \bar{\varphi}_t, \bar{f}_f^*\right) + \bar{C}_t^{i,NA}\right). \tag{4}$$

$$\bar{f}_f^* = \sum_i \bar{A}_f^i \bar{P}_{k,f}^i \left(\beta^i \cdot \left(\bar{A}_f^i\right)^T \bar{c}_f\left(\bar{f}_c^*, \bar{\varphi}_t, \bar{f}_f^*\right) + \bar{C}_f^{i,NA}\right) \cdot \bar{d}_f^i. \tag{5}$$

In particular, the proposed assignment problem can be formulated via three sub-problems: a fixed-point problem with elastic demand for the road system, whose solution is vector $\bar{f}_c^*$; a stochastic network loading for the transit system (due to the fact that transit user flows do not affect network performances), whose solution is vector $\bar{f}_t^\wedge$; and a fixed-point problem with rigid demand for the freight system (due to the fact that freight demand is not affected by network performances), whose solution is vector $\bar{f}_t^*$.

However, it is worth noting that in order to solve eqns (3), (4) and (5) it is necessary to solve jointly eqn (3) and eqn (5) and then implement eqn (4).

Extending the analysis developed by D'Acierno *et al.* [1], we may state that the fixed-point problem described by eqns (3), (4) and (5) has at least one solution if: path choice functions of the road and freight systems (i.e. matrices $\bar{P}_{k,c}^i$ and $\bar{P}_{k,f}^i$) are continuous; travel demand functions of the road and the transit systems (i.e. vector $\bar{d}_c^i$ and $\bar{d}_t^i$) are continuous and upper bounded; cost functions of all transportation systems (i.e. vectors $\bar{c}_c$, $\bar{c}_b$ and $\bar{c}_f$) are continuous; each OD pair is connected.

The first three assumptions are generally satisfied by almost all functions proposed in the literature, whereas the last hypothesis is related to the network framework analysed and is generally respected.

Likewise, it is possible to extend the considerations of D'Acierno *et al.* [1] in order to state the uniqueness of the equilibrium solution. In particular, we can show that the fixed-point problem described by eqns (3), (4) and (5) has at most one solution if: path choice functions of all transportation systems (i.e. matrices $\bar{P}_{k,c}^i$, $\bar{P}_{k,t}^i$ and $\bar{P}_{k,f}^i$) are additive, non-increasing with respect to generalised path costs (i.e. matrices $\bar{C}_c^i$, $\bar{C}_t^i$ and $\bar{C}_f^i$), continuous with continuous first partial derivative; demand functions of the road system (i.e. vector $\bar{d}_c^i$) are non-negative, upper bounded and non-increasing with respect to generalised path costs (i.e. matrices $\bar{C}_c^i$, $\bar{C}_t^i$ and $\bar{C}_f^i$); cost functions of the road and freight systems (i.e. vectors $\bar{c}_c$ and $\bar{c}_f$) are increasing with respect to link flows (i.e. $\bar{f}_c$ and $\bar{f}_f$).

The first and second assumption are satisfied by almost all functions proposed in the literature, whereas the second hypothesis is generally not satisfied, even if D'Acierno *et al.* [1] stated that in the case of small networks this property is always satisfied if cost functions are similar to those reported in fig. 1 and fig. 2.

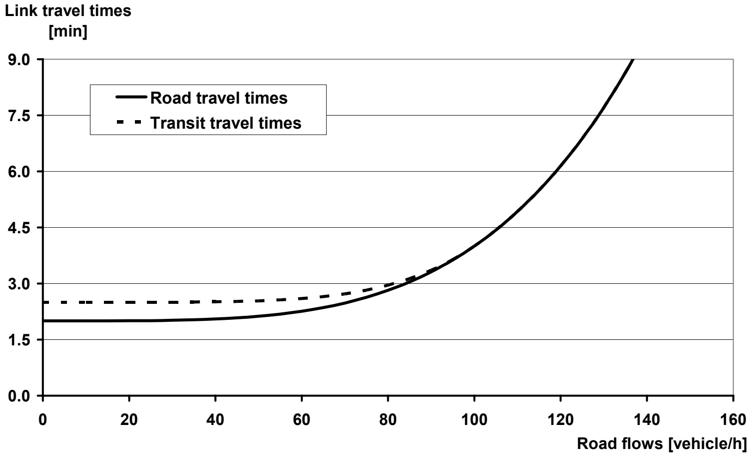

Figure 1: First case of cost functions.

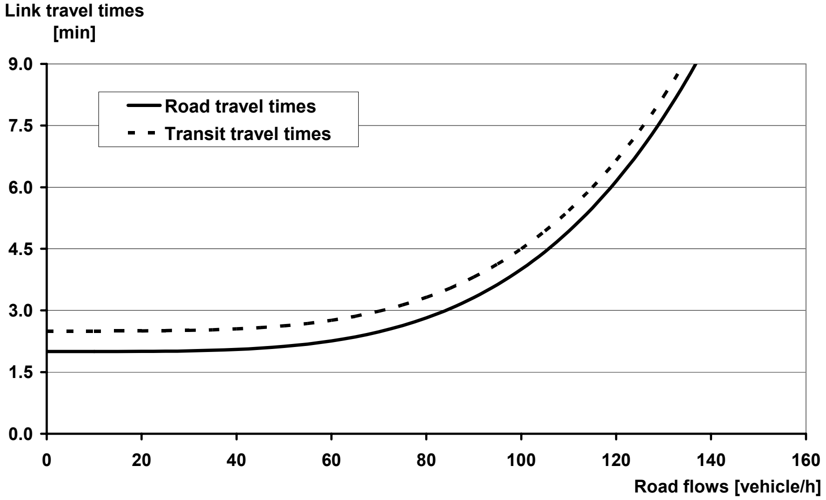

Figure 2: Second case of cost functions.

In particular, in fig. 1 when road flows are low, bus speeds are lower than car speeds (bus travel times are higher than those of cars); when the road flows increase, the speeds of the two systems tend to converge. Whereas, fig. 2 represents the case when the bus travel time on a link is equal to the road travel time plus a constant term that expresses the dwelling time at bus stops. However, it is worth noting that in urban contexts it is possible to have the first diagram, the second one or a combination of them. Therefore, the proposed condition could be always satisfied.

3 Solution algorithms

In order to solve the proposed multimodal assignment problem for the simulation of urban freight distribution, we developed four solution algorithms based on the three algorithmic approaches (external, internal and hyper-network) for the solution of elastic demand assignment problems.

Convergence of internal approach algorithms was stated under some quite general assumptions (Cantarella [2] and Cascetta [3]). Likewise, Cascetta [3] and Sheffi [4] showed that hyper-network approaches are theoretically equivalent to internal ones, and hence the general assumptions of Cantarella [2] and Cascetta [3] allow their convergence to be stated. By contrast, the convergence of external approach algorithms was not proved in the literature. In any event, if it is possible to state that the solution exists and is unique and the external approach algorithm converges, it converges just to the equilibrium solution.

The solution algorithm for all proposed approaches is based on an MSA-FA framework (Cantarella [2]); inside this framework the stochastic network loading algorithm adopted is that proposed by Dial [6] for road and freight systems and the modified version by Nguyen *et al.* [7] for the transit system. These algorithms generate implicitly paths (a necessary requirement on real dimension networks) and the immediate calculation of the EMPU (*Expected Maximum Perceived*) variables, which allow us to calculate the demand component in the case of elastic demand (i.e. in the case of the road and transit systems).

The algorithm proposed by Dial [6] (as well as its modified version proposed by Nguyen *et al.* [7]) consists of three phases:

- phase 1:Dial-efficient paths generation;
- phase 2:weight calculation (by which it is possible to calculate also the EMPU variables and hence travel demand vectors);
- phase 3:network loading.

In all three algorithmic approaches adopted phase 1 is performed only once, since the assumption of additive path (hyper-path) choice models means that the choice set is independent of the values of generalised costs. Moreover, we applied the property of independence of network costs with regard to transit user flows. Therefore, the loading of the transit network can be performed only once, after the equilibrium flows on road and freight systems have been calculated.

Once a starting modal split has been fixed, the external approach algorithm calculates the EMPU variables of the road system by an MSA-FA algorithm (phase 2 and phase 3 cycles) applied on road and freight systems and the EMPU variables of the transit system by means of only one implementation of phase 2 applied on the transit system. These EMPU variables are used to calculate a new modal split, thereby recalculating the costs on the networks and the new values of the EMPU variables. When a stop test on modal split is satisfied, the algorithm ends and the corresponding flows of the road and the freight systems are the multimodal equilibrium ones, that is the solution of eqn (3) and eqn (5), while it is necessary to perform a last transit network loading (phase 3) in order to obtain the transit flows, that is the solution of eqn (4).

The version (1st) of the external-approach algorithm differs from the version (2nd) for the starting flow values adopted in MSA-FA algorithms to calculate the rigid demand flows on road and freight networks. In the first case the starting flows are equal to zero; in the second, instead, they are equal to the flows obtained in the previous assignments.

Once a null starting flow vector has been fixed the internal approach algorithm calculates the EMPU variables of both road and transit systems, applying phase 2 to all three (road, transit and freight) systems. The algorithm calculates the stochastic network loading flows on the road and the freight networks (phase 3) after the calculation of the modal split. With the obtained flows, it recalculates the EMPU variables until convergence on road and freight flows is reached, that is the solution of eqn (3) and eqn (5) are obtained. Also in this approach, it is necessary to perform a last transit network stochastic loading (phase 3) in order to obtain the (equilibrium) transit flows, that is the solution of eqn (4).

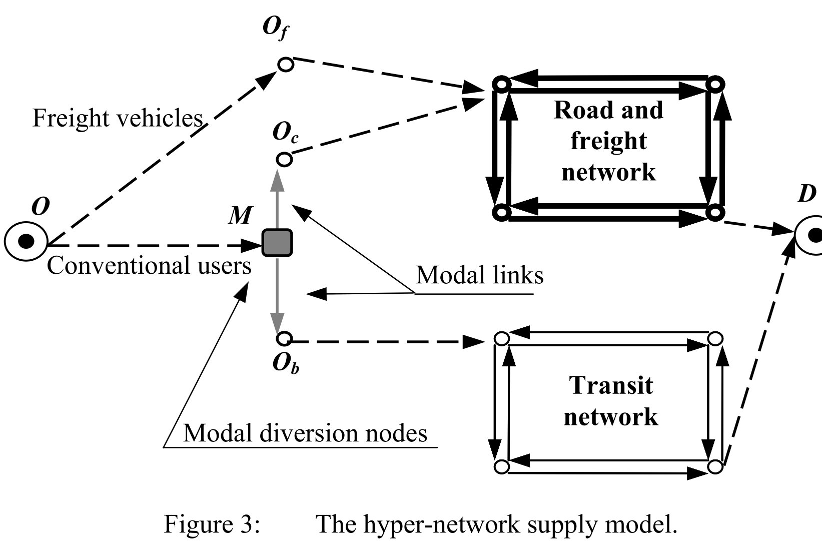

Figure 3: The hyper-network supply model.

The hyper-network approach algorithm carries out a rigid demand assignment model on a multimodal supply network model (hyper-network) that allows the simulation of the mode choice on network by the use of specific dummy links (modal diversion links). Therefore this algorithm consists in the use of a rigid demand MSA-FA algorithm (phase 2 and phase 3 cycles). This particular network framework is an extension of the hyper-network model proposed by D'Acierno *et al.* [8]. A pattern of the adopted hyper-network supply model is reported in fig. 3.

4 First results on a real network

The proposed algorithms were tested on a real dimension network, whose features are reported in table 1. Comparing the transit system and the road networks, it may be noted that the ratio of the link number is about 7:1. This justifies the huge difference (about 363 times) between the elaboration times (proposed in table 2) of the network loading for the road and freight system (phase 2 and phase 3 in the case of these transportation systems) and demand model performance, which also incorporates the weight calculation of the transit system (phase 2).

A comparison among the algorithms is reported in table 3. An important result is that the three algorithms converge to the same solution. Indeed, the maximum percentage error is never exceeds 5.0% which represents the threshold

Table 1: Test network features.

	Links	Line links	Pedestrian links	Nodes	Transit lines
Road/freight network	1,133	–	–	529	–
Transit network	7,124	3,216	1,326	3,779	75

Table 2: Calculation times (Pentium IV – 3.00 GHz).

Approach	Network loading (road & freight)	Demand model performing	Fixed times
External	0.74 s	26.89 s	106.45 s
Internal	0.74 s	26.89 s	43.38 s
Hyper-network	33.40 s	–	53.44 s

Table 3: Comparison among algorithm approaches.

Approach	Network loading (road & freight)	Demand model performing	Maximum percentage error	Calculation time [min.]
External (1st)	172	22	0.492%	13.75
External (2nd)	50	21	1.740%	11.80
Internal	9	9	Reference	4.87
Hyper-network	9	–	0.000%	5.88

used in the stop tests of the algorithms. However, though it was not possible to demonstrate the uniqueness of the solution and the consequent convergence of the algorithms on a real network, we numerically proved the convergence of the proposed algorithms.

The comparison between the two versions of the external algorithm shows that the second version obviously requires less stochastic loading on road and freight systems.

In terms of calculation times, the internal approach algorithm requires much lower elaboration times than the external ones (about 3 times), which can be explained by the different number of iterations required to converge (see table 3).

Moreover, the internal and the hyper-network approaches coincide both in terms of iterations and results, but in the hyper-network one the elaboration times are slightly higher (about 1.20 times), since the dummy links increase the number of total links in the network to be examined.

Finally, in order to verify the usefulness of a multimodal approach in urban freight distribution two experiments were performed.

The first set out to analyse the presence of freight vehicles on trip choices of 'conventional' (i.e. road and transit) users. In particular, the simulation obtained considering the proposed three-system assignment model was compared with those obtained by neglecting freight vehicles. Our results showed that the maximum difference in user flows is 60.00% in the case of the road system and 4.44% in the case of the transit system; likewise the average difference in user flows is 6.42% in the case of the road system and 0.28% in the case of the transit system.

Instead, the second trial was based on the analysis of the influence of road and transit system vehicles on freight vehicle route choices. The simulation obtained by considering the proposed three-system assignment model was compared with a monomodal assignment model with rigid demand applied only on the freight system. In this case, results showed that the difference in freight vehicle flows is 169.39% in the maximum case and 4.94% in the average case.

Thus the two experiments showed that neglecting freight vehicle presence can substantially modify link flows in the case of the road system. Likewise, neglecting the multimodal aspects in the urban context, that is the presence of a road and transit system, can considerably change the minimum cost path used by freight vehicles.

5 Conclusions and research prospects

The proposed multimodal assignment problem allows transportation systems in the urban context to be simulated by considering the effects of private cars and freight vehicles on congestion.

Although it is not possible to state the convergence theoretically, the developed algorithms converge to the same solution in reasonable times. Moreover, comparison among the different approaches showed that, at least in the case of the analysed network, the internal-cycle algorithms are more efficient in terms of calculation times.

Future research should address the use of the proposed model as a simulation tool in network design problems in order to take account of all transportation components in urban contexts.

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