

Virtual inversion of environment vibration sources caused by rail traffic in urban areas

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Abstract

The environment vibration source of rail traffic has been dealt with by many researchers, mainly by railway spectra from observation of the vibration. From the 3D point of view, the real source function can never be observed, even though seismometers are installed at points very close to the railway, since the observation must be mixed with the vibration from many other sources along the way. In order to validate the possibility of inverting the source function from ground observation, a virtual inversion is presented in this paper. The vertical displacement time histories at an array from a series of triangular pulses are synthesized to simulate the impacts of wheels and rail joints while a train goes through the railway. The cross power spectra are calculated from the displacements at each two points of the array, and are taken as the objective functions to inverse the intensity and the duration of the source function by means of the micro-Genetic algorithm. The inversion result is quite close to the given values, and provides a good foundation for further study on inversion of random vibration source function.

Keywords: environmental vibration, rail traffic, source model, virtual inversion, micro-genetic algorithm.

1 Introduction

In almost all studies, the vibration source caused by urban rail traffic is modelled by vibration at the nearest point from the rail, and the effect on buildings and human beings is calculated by a two-dimensional analysis [1]. That vibration, however, is actually not only the source, but also another vibration. The vibration is mainly engendered from the bump between the steel wheel and railway. The



vibration will be strong if there are discontinuities, such as a joint. In this situation, the vibration source comes from many joints along the rail while the wheels move through rapidly. The most difficult problem is no one can observe the source pulse clearly even at a very near point from rail, because of the mixture of pulses before and after this source. The recorded vibration is always the combination of many pulses, among which the nearer the stronger the pulse, the further away the weaker the pulse. A new method, virtual inversion, is presented in this paper to detect the intensity and duration of the source function, in which the source function at each joint is given as a triangular pulse. The vibration time histories at six ground points are synthesized by superposition of vibrations from many source functions. The vibration from impact at a joint is calculated by an analytical solution. The triggering time of individual pulses are determined by the train's velocity, given length of the rail segment and the number of segments from the start point, and also the length between the wheels. Then, the cross power spectrum of vibrations at each of two points is calculated by means of the periodogram method, and is taken as the objective function of the inversion. The micro-Genetic algorithm is adopted in inversion of the intensity and the duration of the source function. The results of 15 inversions are validated with the given values. Finally, the idea to inverse the source spectrum for the problem of rails without joints is further discussed.

2 The given source function and other parameters

The basic idea of virtual inversion is to remove the effect of ignorance about the inversion input, and check the result by the given input [2]. So it is called virtual, since the input is known, but never considered in the inversion process. From an observation on impact vibration between the wheel and the rail joint [3], the given axial load is 107kN and the velocity of the train is 54km/h, the pulse at the joint observed is very similar to a triangular pulse, as shown in figure 1.

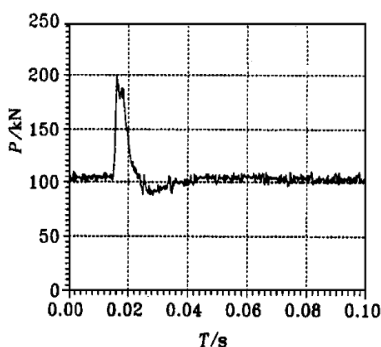


Figure 1: The observed impact between wheel and rail joint.

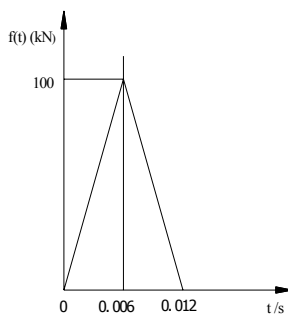


Figure 2: The source function of this paper.

Therefore, the source is taken as a triangular pulse, as shown in figure 2, and the source function is defined as the following.

$$f(t) = \begin{cases} 1666.67 \cdot t & 0 < t < 0.006s \\ 200 - 1666.67 \cdot t & 0.006s < t < 0.012s \\ 0 & t < 0, t > 0.012s \end{cases} \quad (1)$$

The intensity 200kN and duration 0.012s are the key parameter values to be checked in the inversion.

A YZ25G locomotive with air-conditioner is referenced, 25 cars in the train are assumed and the other detail parameters are shown in figure 3 [4]. For simplification, 16 rail segments are considered; the length of each is 25.0m. The velocity of the train is taken as 52km/h.

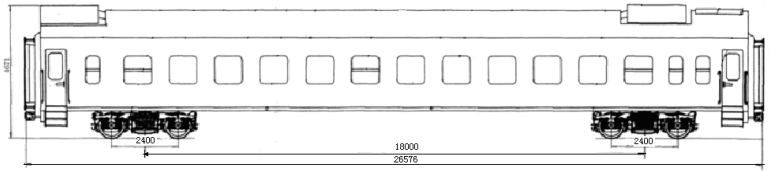


Figure 3: The size of the YZ25G locomotive.

3 Numerical solution of displacement on half space surface caused by a triangular pulse

Based on the Cagniard-Hoop method, an accurate solution of the vertical vibration source for a rectangular pulse acting on the surface of an elastic half space was derived in [5], as shown in equation 2.

$$u_z(r, 0, \tau) = \begin{cases} 0, & (\tau < b) \\ \frac{p_0}{32(1-b^2)\pi\mu r} \left\{ -4 + a_1 \sqrt{\frac{b^2-\gamma^2}{\tau^2-\gamma^2}} + a_2 \sqrt{\frac{b^2-p}{\tau^2-p}} + a_3 \sqrt{\frac{b^2-q}{\tau^2-q}} \right\}, & (b \leq \tau < 1) \\ \frac{p_0}{16(1-b^2)\pi\mu r} \left\{ -4 + a_1 \sqrt{\frac{b^2-\gamma^2}{\tau^2-\gamma^2}} H(\gamma-\tau) \right\}, & (1 \leq \tau < b+\tau_0) \\ \frac{p_0}{32(1-b^2)\pi\mu r} \left\{ -4 + 2a_1 \sqrt{\frac{b^2-\gamma^2}{\tau^2-\gamma^2}} H(\gamma-\tau) - a_1 \sqrt{\frac{b^2-\gamma^2}{(\tau-\tau_0)^2-\gamma^2}} \right. \\ \quad \left. - a_2 \sqrt{\frac{b^2-p}{(\tau-\tau_0)^2-p}} - a_3 \sqrt{\frac{b^2-q}{(\tau-\tau_0)^2-q}} \right\}, & (b+\tau_0 \leq \tau < 1+\tau_0) \\ \frac{p_0 a_1}{16(1-b^2)\pi\mu r} \left\{ \sqrt{\frac{b^2-\gamma^2}{\tau^2-\gamma^2}} H(\gamma-\tau) - \sqrt{\frac{b^2-\gamma^2}{(\tau-\tau_0)^2-\gamma^2}} H(\gamma-\tau+\tau_0) \right\}, & (1+\tau_0 < \tau) \end{cases} \quad (2)$$

where

$$p, q = \frac{1}{32(b^2-1)\gamma^4} \left[8\gamma^4 - 1 \pm \sqrt{(8\gamma^4-1)^2 + 64(b^2-1)\gamma^6} \right] \quad (3)$$

$$a_1 = \frac{(2\gamma^2 - 1)^2}{(\gamma^2 - p)(\gamma^2 - q)} ; a_2 = \frac{(2p - 1)^2}{(p - \gamma^2)(p - q)} ; a_3 = \frac{(2q - 1)^2}{(q - \gamma^2)(q - p)} \quad (4)$$

$$\gamma = \frac{c_s}{c_R} \quad (5)$$

$$c_R = \sqrt{-z_1 c_s} \quad (6)$$

where c_s is shear wave velocity, c_R is the Rayleigh wave velocity, z_1 is a root of the Rayleigh function, and

$$\tau = \frac{c_s t}{r} ; \quad \tau_0 = \frac{c_s t_0}{r} \quad (7)$$

where t_0 is duration of the pulse.

In order to synthesize the displacement field by means of the above equations, the triangular impulse is discretized into a series of thin rectangular impulses. The vertical displacement on the half space is synthesized numerically, with shear wave velocity of the half space $v_s=250.0\text{m/s}$, and the velocity of the car 14.4 m/s . An example of the response pulses is shown in figure 4.

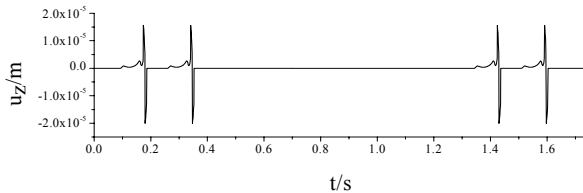


Figure 4: Ground vibration from impacts of the car with 4 wheel-pairs through a joint.

4 Synthesized vibrations at a ground array and their spectra

In order to describe the spatial feature of the displacement field on the surface caused by the sources, the vibrations at a rectangular array, which consist of six points, are synthesized by means of the superposition of vibrations at each point from all sources. The array configuration adopted and the spatial relation to the railway (source) is shown in figure 5.

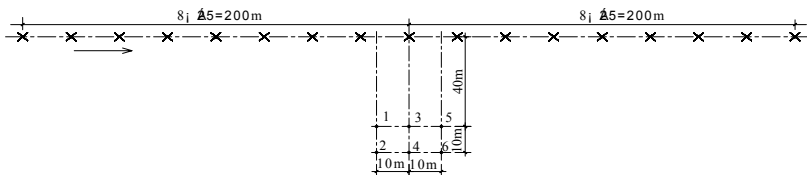


Figure 5: Configuration of the array.

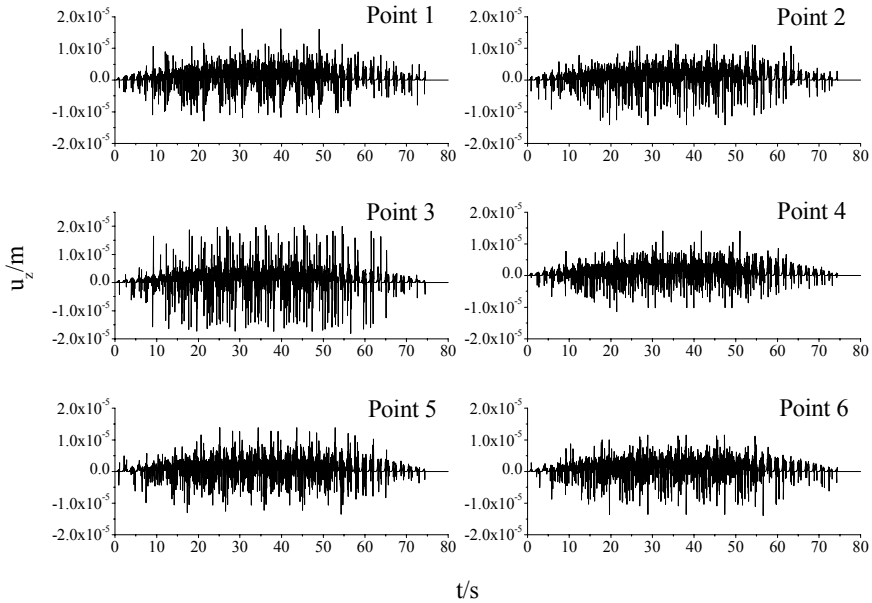


Figure 6: Vertical displacement time histories at the six points of the array.

The vertical displacement time histories at the six points of the array are shown in figure 6.

From this figure, one can see that there is a vibration increasing segment at the beginning of each history, after that is a stable vibration segment, and finally a decreasing segment. This shows the three stages where the train is getting on the rail, is totally on the rail and is getting off the rail respectively.

The cross power spectra of each pair of two points are calculated to acquire the spatial features on the vertical displacement field in the frequency domain.

The spectra are calculated by the periodogram method, the so called the direct method, in which each of the two histories is smoothed by the window function, and then the time histories are transformed into the frequency domain by fast Fourier transform. The power spectrum can be worked out by equation as follows:

$$S_{ij}(f) = R_j^*(f) \times R_i(f) \quad (8)$$

where $R_i(f)$ and $R_j(f)$ are the Fourier spectra at point i and j respectively.

In the calculation, the sampling time step is 0.001 seconds and the number of amplitudes in the frequency domain is 1024. The total of 15 cross power spectra of time histories at every two points in the array are shown in figure 7.

From this figure, it can be easily found that the predominant frequency is not high, just in between 1 to 3Hz, since the train velocity is slow.

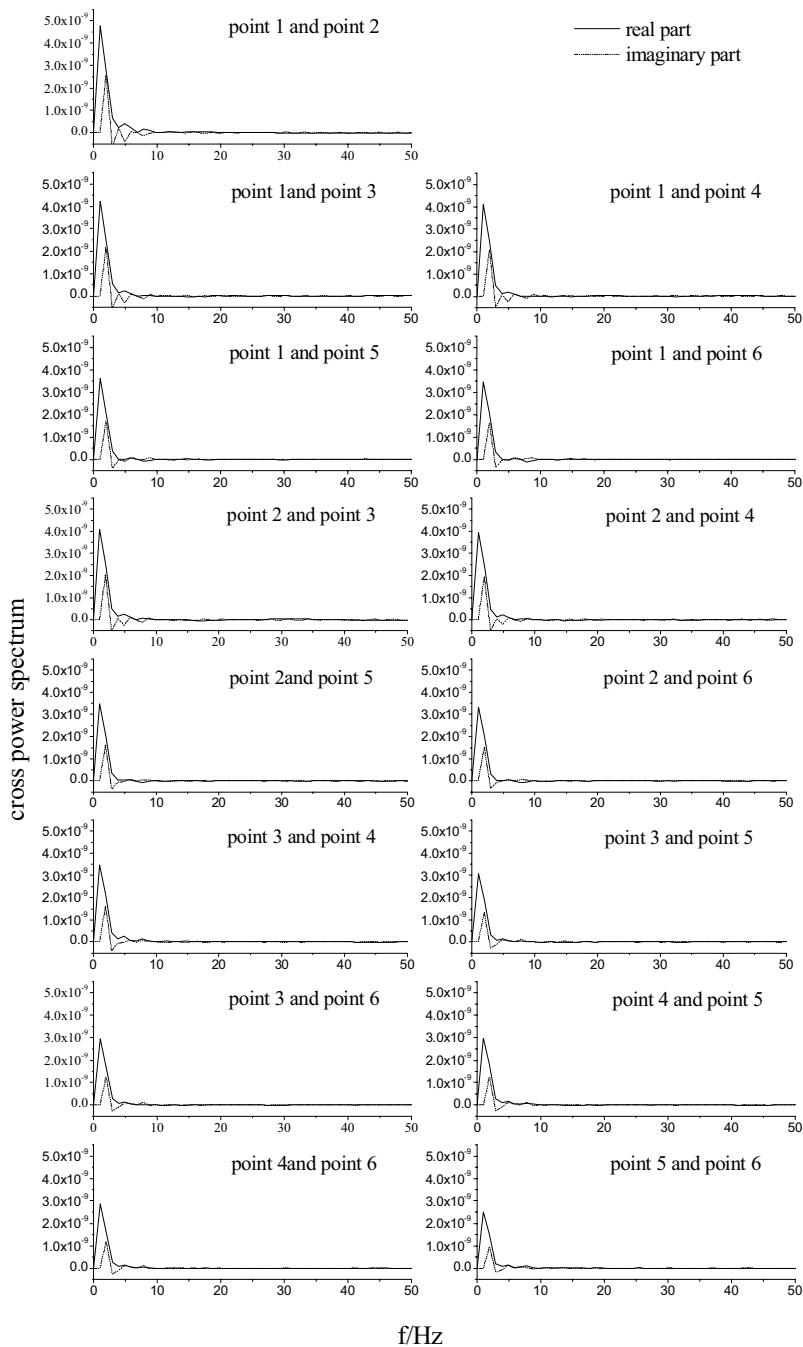


Figure 7: 15 cross power spectra at every two points in the array.



5 The inversion strategy for the Genetic Algorithms

The Micro-GA is adopted in the inversion of the source function. In Micro-GA, selection and crossover is the fundamental operation, mutation is cancelled, and the strategy of preserving the most excellent and initializing the popularity are added.

The second last one integrally preserves the most excellent individual for the next generation, which will ensure no loss of the excellent information gained during evolution. The last one ensures the diversity of the genes.

The inversion flowchart is shown in figure 8.

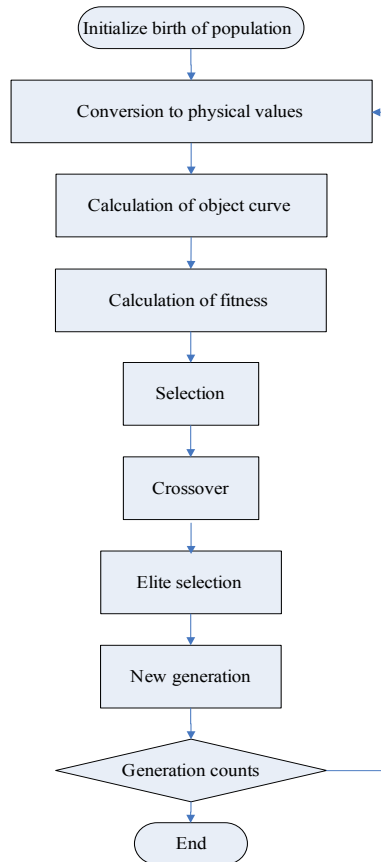


Figure 8: The flowchart of the inversion in this paper.

The object function of the inversion is selected as follow for the j^{th} intensity value.

$$\phi_j = \sum_{i=1}^n [S_e(i) - S_{cj}(i)]^2 \quad (9)$$

where $S_e(i)$ and $S_{ej}(i)$ are the i^{th} cross power spectra amplitudes of expected and are calculated from the j^{th} values of intensity and duration of the source function, respectively.

The fitness function is selected as following equation (9), to assure the most fitted individual is copied to the next generation with a high probability.

$$F_j = \exp(-5 \times \phi_j) \quad (10)$$

The binary coding is adopted for its simplicity and to make more searching patterns, the selection operator is the league match and the cross operator is the consistent cross.

The population size is 5, the cross probability is taken as 0.6, and the maximum genetic generation is 2000.

The upper and lower limits of the search range of the triangular pulse intensity are taken as 1000kN and 10kN and the upper and lower limits of the search range for the duration of the triangular pulse are taken as 0.12s and 0.0012s, respectively.

Table 1: Comparison of the inversed results of the source pulse intensity and duration with the given ones.

| No. | Intensity (kN) | | | Duration(s) | | |
|-----|---------------------|-----------------|--------------------|---------------------|-----------------|--------------------|
| | The inversed result | The given value | Relative error(%) | The inversed result | The given value | Relative error(%) |
| 1,2 | 102.05 | 100 | 2.05 | 0.0119 | 0.012 | -0.83 |
| 1,3 | 100.20 | 100 | 0.20 | 0.0118 | 0.012 | -1.67 |
| 1,4 | 101.19 | 100 | 1.19 | 0.0121 | 0.012 | 0.83 |
| 1,5 | 101.45 | 100 | 1.45 | 0.0123 | 0.012 | 2.50 |
| 1,6 | 103.87 | 100 | 3.87 | 0.0117 | 0.012 | -2.50 |
| 2,3 | 98.48 | 100 | -1.52 | 0.0123 | 0.012 | 2.50 |
| 2,4 | 103.87 | 100 | 3.87 | 0.0119 | 0.012 | -0.83 |
| 2,5 | 100.83 | 100 | 0.83 | 0.0125 | 0.012 | 4.17 |
| 2,6 | 104.45 | 100 | 4.45 | 0.0123 | 0.012 | 2.50 |
| 3,4 | 101.57 | 100 | 1.57 | 0.0117 | 0.012 | -2.50 |
| 3,5 | 100.37 | 100 | 0.37 | 0.0121 | 0.012 | 0.83 |
| 3,6 | 101.27 | 100 | 1.27 | 0.0123 | 0.012 | 2.50 |
| 4,5 | 102.35 | 100 | 2.35 | 0.0117 | 0.012 | -2.50 |
| 4,6 | 102.14 | 100 | 2.14 | 0.0121 | 0.012 | 0.83 |
| 5,6 | 101.07 | 100 | 1.07 | 0.0121 | 0.012 | 0.83 |

6 The result of the virtual inversion and conclusions

It can be seen from table 1 that the result of virtual inversion is stable, and very close to the target function. It is proven to be feasible to inverse the source function from vertical displacement time histories at a dense array.

It may be reasonably believed that it is possible to inverse the source spectrum from the cross power spectrum or dispersion curve of the vertical displacement field at an array from random vibration analysis. There are two key points, one is on the rail spectrum, and the other is on the random vibration solution of the half space. The result will be published at the next conference.

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