MODELING THE IMPACT OF EXPLOSIONS IN ENCLOSED SPACES USING DISCONTINUOUS BOUNDARY ELEMENTS AND FINITE VOLUME METHODS

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ABSTRACT

This paper is aimed at studying the influence of loading due to shock (strike) waves in an enclosed space; underground parking is a typical example. The problem which is to be solved is divided into the description of motion and pressure in the air and in the solid phase, separately; the interaction of the effects arising in the air and the solid phase are concentrated along the boundary interface between the two media. As the free hexagon method (discontinuous boundary element method) has proved to be well applicable in solving nonlinear problems in structures, it is also used, slightly adjusted, here. In order to connect both mediums (structure, air), the gas-dynamics in the air is described, based on equations of conservations, by a simplified finite volume elements, their shapes are also hexagonal, to be in geometrical compliance with the rock and structure. The hexagonal shapes are arbitrarily, i.e. not necessarily honey combs, like mostly used in connection with finite elements. Note that such a shape of elements complies with the requirements for the finite volume method. The BEM describes the elasticity, or nonlinear behavior inside of each hexagonal element of the solid phase, while the mechanical behavior on the interfaces between the adjacent elements obeys the laws of Mohr-Coulomb localized damage.

Keywords: discrete element method, free hexagon method, discontinuous boundary element method, finite volume method, impact of explosion in closed space, underground parking.

1 INTRODUCTION

The aftermath of explosion due to various causes, either intentional (bomb attacks, VBIED, suicide bomber), or accidental (gas explosion) are very dangerous if occurring in a free space. The solution of such a problem is complicated because of nonlinear nature of equations of conservations. On the other hand, the problem becomes much more complex, if the explosion appears in a closed space, as the shock waves interfere with reflected waves and local aftershocks. Consequently, the question, if the structure is safe against the influence of the pressure induced by the explosion and if so to which extent, is naturally much more difficult. This question belongs to the most serious tasks for engineers (underground engineers, for example) because of the commonly fatal consequences of explosion. This paper is focused on an assessment of impact of shock waves propagating in underground spaces, which can be underground parking, chambers, underground power stations, and such. Two basic problems are to be solved. The propagations of shock waves in a solid medium (structure, rock surrounding the structure), and in the air are considered.

Geodynamical problems (structure, rock) are often solved by discrete element methods. The obvious advantages of such models consist in possibility of introducing nonlinear phenomena to the elements and also concentrate on development of mechanical behavior along the boundaries between adjacent elements. In this way, both the local damage and plasticity in the solid phases can be taken into account. The large deformations also play an important role, as the aftermath of explosion causes fast changes of mechanical properties and large movements in both the solid phases and the air. The free hexagon method, as one of a discrete element method, seems to be appropriate for solving such problems.

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Note that the basic theory of propagation of shock wave and its impact on various structures is described in the Landau and Lifshitz (LL) book, [1]. The most inceptive solution is the Godunov approach starting with finite differences, [2], [3]. The latter and [4] proposed a powerful method: discontinuous finite elements.

In previous papers of the author, propagation of shock wave in a closed air space was discussed, based on nonlinear Godunov's equations and experimental studies obtained in cooperating institutes, [1], [2].

The free hexagons are published in [5], where the solution of statical case is proposed. Extension to dynamics with application in the effects of explosions initiated in a closed space can be found in [6]–[8], for example.

Interaction of pressure due to the shock wave and an arch shell is found in [9]. The effect of explosion in underground spaces was also published in WIT conferences; see [10], [11].

A comprehensive book on finite volumes is [12], for example. The paper [13] is concerned with cell-centers finite volumes, which approximation is also used here for LL equations.

Other publications on the topic studied in this paper are worth noting [14]–[18].

2 FORMULATION AND NUMERICAL IMPLEMENTATION OF BASIC EQUATIONS

As said before, the problem of interaction of the structure and the air the shock wave through which propagates is split into two sub-problems. The first one formulates and solves the mechanical behavior in a structure (rock, lining) and the second is concern with description of the change of the air movement and pressure based on LL gas- (fluid-, hygro-) dynamics. The first problem, the effect of sudden change of load of the solid structure, has previously been addressed in several papers based on the free hexagon method. Then the solution is relatively simple. It consists of prescribed behavior inside of the elements (plasticity, hereditary problems), and of given nonlinearities along the interfaces between the elements. On the other hand, the basic model considered hereinafter is pseudo-linear (pseudo-elastic) and the nonlinear Mohr-Coulomb localized damage is applied on the common interface between adjacent particles.

The solution of shock wave propagation in the air is more complicated, as the nonlinear behavior due to the effect of kinetic energy and other influences turn the problem to strongly nonlinear. Moreover, the set of basic equations of conservation has to be completed by additional relations, which involve chemical nature of the charge, thermodynamic constraints, etc.

Formulations of both sub-problems and their solutions in the frame of hexagonal elements are presented in the next text. As usual, the behavior of elements in the solid phase is described by boundary elements while in the air finite volumes are applied and the solution, in contrast to the solid phase behavior, is continuous.

The problem of propagating shock wave is focused on the situation in a tunnel. The charge is located at the bottom of the semi-circular cross-section of the tunnel, which is assumed sufficiently long to avoid the longitudinal effects of the propagating wave, and either narrow enough or wide enough, so the problem can be understood as two-dimensional. In Fig. 1 an expected behavior of the system shock wave – structure (rock) in the early stage of time is depicted. The objective of this paper is to show what happens after the shock wave arrives at the interface air – the upper part of the tunnel lining.

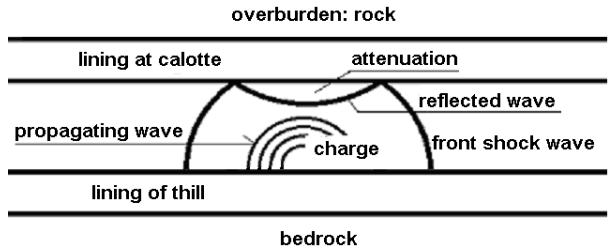

Figure 1: Situation in the tunnel after shock wave touches the lining.

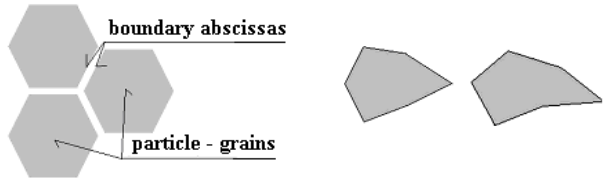

Figure 2: Set up of adjacent particles and their possible shapes.

2.1 Solution in one solid particle for position iteration

First, basic ideas of the free hexagon method are briefly recalled. Both the solid phase and the air are defined in Cartesian coordinate system Ox_1x_2, as the propagation of movements allows for narrowing the problem to two dimensions. Let the domain $S \in R_2$ describing the solid phase be partitioned into hexagonal particles $S_i, i = 1, \cdots, n$, elements, which are a priori mutually not connected. Possible geometry of particles and the set up of adjacent elements are seen in Fig. 1, where along the boundary abscissas of the interface springs in both normal and tangential directions are assumed.

Generally, inside each element non-linear mechanics can be introduced (plasticity, hereditary properties, viscous material).

In order to define the mechanical behavior on the interfacial boundary of arbitrary adjacent elements, a pseudo-cone *K* is introduced, which obeys the natural requirements on the interfaces of adjacent elements at any time due to the effect of shock wave (air movement) transferred to the solid phase:

$$\begin{aligned} &K \equiv \{\boldsymbol{u} \in V, [u]_n \geq 0,\ p_n \leq p_n^+, \text{if } p_n > p_n^+ \Rightarrow p_n = 0, \\ &\left|p_t\right| \leq c\kappa(p_n^+ - p_n) - p_n \tan \text{ø}, \\ &\text{if } \left|p_t\right| > c\kappa(p_n^+ - p_n) - p_n \tan \text{ø} \Rightarrow p_t > p_n \tan \text{ø} \operatorname{sgn}[u]_t\}, \end{aligned} \tag{1}$$

where V is the set of regular functions in S, $\boldsymbol{u}$ is the vector of displacements, $[u]_n$ and $[u]_t$ are jumps of displacements in normal and tangential directions, respectively, p_n and p_t are tractions in normal and tangential directions, respectively, p_n^+ is the normal strength,

$\varnothing$ (the angle of internal friction) and c (shear strength) are positive material constants, κ is the Heaviside function. The pseudo-cone expresses the following requirements:

- There is no penetration (mutual overlapping) of the elements,
- Tensile strength cannot be exceeded on the interfaces,
- Mohr–Coulomb hypothesis is fulfilled in tangential direction.

Free hexagon method provides a particle model, similar to classical PFC (Particle Flow Method), containing the following assumptions used specifically in this paper:

- All particles are hexagonal, and either regular partition of the domain is considered (honey combs), or the hexagons possess arbitrary convex shape.
- Linear material behavior is assumed inside of each particle, and is described by boundary element method, with uniform distribution of both displacements and tractions along the boundaries of the particles.
- The contacts are specified along the interfaces – abscissas – between adjacent elements.
- Behavior along the contacts is considered as soft, based on the idea of a spring connection in both the normal and tangential directions with respect to the boundaries between adjacent elements.
- Bonds of a specified tensile strength p_n^+ exist at contacts between particles. The tensile strength decides whether the split occurs in direction normal to the interfaces.

It is worth noting that in comparison to the classical PFC, the free hexagon method allows for the definition of stresses along the interfaces.

The solution of such a strongly non-linear problem is decomposed into two steps: the first one monitors the movements of the vertices in any specific particle at a time attained, which generates the deformation of the current particle in the fixed close ambience, and the second problem consists of a time-incremental scheme, which is discussed in the next section. In this way, the time dependent development is split into the solution of pseudo-static phases.

The algorithm for the static cases is fully described in [5]. Knowing the form of kernels for 2D elasticity, the approximations of boundary displacements, tractions and volume are derived from the boundary element method leading to the following equations:

$$\boldsymbol{K}\boldsymbol{u} = \boldsymbol{F} + \boldsymbol{Q}\,, \tag{2}$$

where $\boldsymbol{K}$ is a square matrix (12 * 12), not necessarily symmetric, $\boldsymbol{u}$ is the vector of displacement approximations, $\boldsymbol{F}$ is the vector of approximated tractions, $\boldsymbol{Q}$ involve the influences of volume weight and here, moreover, involve also the effect of inertia forces. Both the latter vectors posses dimension (12*1).

For the next purpose, write the matrix eqn (1) as (indices 1 and 2 indicate that the system (1) is transformed to the directions x_1 and x_2):

$$K_{ij}^{11}u_1^j + K_{ij}^{12}u_2^j = F_1^i + Q_1^i,\quad K_{ij}^{21}u_1^j + K_{ij}^{22}u_2^j = F_2^i + Q_2^i,\quad i,j = 1,...,6\,. \tag{3}$$

Considering volume weight and dynamical forces in the particle s and displacements u_1^{is}, u_2^{is} in adjacent elements, one gets for unknown u_1^{sj} and u_2^{sj} at the nodal points (k_{ij} are components of transformed spring stiffnesses, δ_{ij} is the Kronecker delta, j is the number of adjacent element, being enumerated from 1 to 6, as the number of neighbors is at most 6):

$$\begin{aligned}
&\sum_{j=1}^{6} (K_{ij}^{s11} + k_{11}^{sj}\delta_{ij})u_1^{sj} + (K_{ij}^{s12} + k_{12}^{sj}\delta_{ij})u_2^{sj} = k_{11}^{si}u_1^{is} + k_{12}^{si}u_2^{is} + F_1^{si} + Q_1^{si}, \\
&\sum_{j=1}^{6} (K_{ij}^{s21} + k_{21}^{sj}\delta_{ij})u_1^{sj} + (K_{ij}^{s22} + k_{22}^{sj}\delta_{ij})u_2^{sj} = k_{21}^{si}u_1^{is} + k_{22}^{si}u_2^{is} + F_2^{si} + Q_2^{si}.
\end{aligned} \tag{4}$$

The latter system of equations is a system of 12 equations for 12 unknown displacements, six in x_1 direction and six in x_2 direction. This system is always solved in any iteration step under assumption that the neighboring elements are considered fixed and the increment of displacements is taken from their values in the previous step.

2.2 Algorithm for time involvement into the free hexagons

The first level calculation cycle is time-stepping algorithm that requires the repeated application of the law of motion to each particle and a force-displacement law to each contact at the second level. Contacts, which may exist between the elements are formed and broken automatically during the course of simulation. At the start of each time-step, the set of contacts is updated from the known particles positions. The forces displacements law is then applied to each contact to update the contact forces based on the relative motion the two entities at the contact and the contact constitutive model. Next the law of motion is applied to each particle to update its velocity and position based on the resultant force and moment arising from the contact forces and any body forces acting on the particle. The computed solution will remain stable only if the time step does not exceed a critical time step, see [19]:

$$t_{\text{crit}} = \min\{\sqrt{m/k^{\text{n}}}, \sqrt{I/k^{\text{s}}}\}, \tag{5}$$

where k^{n} and k^{s} are the average normal and shear stiffnesses, m is the mass of particle and I is the polar moment of inertia of the particle, calculated once in the beginning of calculation for each time-step. It means that I belong to the geometry in the undeformed state.

A critical time step cannot be stepped over for each particle separately to each degree of freedom. The final critical time step is taken to be the minimum of all critical time steps computed for all degrees of freedom of all particles.

The equations of motion can be expressed as two types of vector equations, one of which relates the resultant forces to the normal (translational) and tangential motion and other of which relates the resultant moment to the shear (rotational) motion. If the global coordinate system is $0x_1x_2$, in each particle the equation for translational motion and rotation have to be fulfilled and can be written in the vector form:

$$F_i = m(\ddot{x}_i - g_i)\,,\ M = I\dot{\omega}\,, \tag{6}$$

where F_i are the resultant forces, the sum of all externally applied forces acting on the particle in i -th direction, m is the total mass of the particle, $\ddot{x}_i$ is the acceleration and g_i is the body force acceleration vector (e.g. loading due to gravity), I is the principal moment of inertia of the particle considered, $\dot{\omega}$ is the angular acceleration. The translation and angular accelerations are calculated as:

$$\ddot{x}_i^t = \frac{1}{\delta t}(\dot{x}_i^{t+\delta t/2} - \dot{x}_i^{t-\delta t/2})\,,\ \dot{\omega}^t = \frac{1}{\delta t}(\omega^{t+\delta t/2} - \omega^{t-\delta t/2})\,. \tag{7}$$

Inserting these velocities to the equations for motions and solving these equations for the velocities at time $(t + \delta t/2)$ results in

$$m[(\dot{x}_i^{t+\delta t/2} - \dot{x}_i^{t-\delta t/2})/\delta t - g_i] = F_i^t\,,\quad I(\omega^{t+\delta t/2} - \omega^{t-\delta t/2})/\delta t = M^t\,. \tag{8}$$

Finally, the velocities are used to update the position of the particle centre:

$$\dot{x}_i^{t-\delta t/2} = (x_i^t - x_i^{t-\delta t})/\delta t\,,\ \ \dot{x}_i^{t+\delta t/2} = (x_i^{t+\delta t} - x_i^t)/\delta t\,, \tag{9}$$

where x_i are the coordinates of the nodes moved in time t .

Our aim is to calculate new positions of nodes in each particle. For this aim it is necessary to carry out transformations of coordinates from global and local coordinate systems, as described in [6], to get, among others (3). Note that in contradiction to PFC equilibrium of forces is attained on the boundary of the particles. The inertia forces have to be calculated from the displacements.

2.3 Computational scheme for time iteration

At the time t we know rotation φ^t , displacements x_i^t and the same quantities at the previous time-step $\varphi^{t-\delta t}$ and $x_i^{t-\delta t}$. From this it is also known $\omega^{t-\delta t/2} = (\varphi^t - \varphi^{t-\delta t})/\delta t$, F_i^t and M^t .

Now applying the former forces and the time-increment of loading we get $x_i^{t+\delta t} \equiv x_i^T$ and $\varphi^{t+\delta t} \equiv \varphi^T$ from the algorithm mentioned in the previous section. Using the second formula (8), $v^{T+\delta t/2} = \dot{x}_i^{T+\delta t/2}$ immediately follows, v is the vector of velocity. In order to get new inertia forces from the known position dependent iteration, [6], implicit time dependent iteration can be established. Since the number of degrees of freedom may be very extensive, for calculation of accelerations at time T the forces F_i^t and moments M^t are utilized. It means that (7) is applied in the sense of the mentioned assumption:

$$m[(\dot{x}_i^{T+\delta t/2} - \dot{x}_i^{T-\delta t/2})/\delta t - g_i] = F_i^t\,,\quad I(\omega^{T+\delta t/2} - \omega^{T-\delta t/2})/\delta t = M^t \tag{10}$$

and the new velocities of the particles $\dot{x}^{T+\delta t/2}, \omega^{T+\delta t/2}$ are available from the previous formulas. Since the new positions of the nodes x_i^T and accordingly the rotations φ^T are known from the position iteration. The first relation (7) delivers $\dot{x}_i^{T-\delta t/2} = (x_i^T - x_i^t)/\delta t$, and moreover from definition $\omega^{T-\delta t/2} = (\varphi^T - \varphi^t)/\delta t$. Now $F_i^{t+\delta t} \equiv F_i^T$ and M_i^T is obtainable from (6) setting $t+\delta t := T+\delta t$ and $t-\delta t := T-\delta t$. Updating all particles, the position iteration can take part according to [6].

2.4 Solution of the propagation of shock wave

Mathematical model of the movements of points positioned in the air is based on the solution of equations of gas dynamics, which, for two-dimensional problem in Cartesian system of coordinates are listed as:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v_x)}{\partial x} + \frac{\partial(\rho v_y)}{\partial y} = 0 \ , \tag{11}$$

$$\frac{\partial(\rho v_x)}{\partial t} + \frac{\partial(p + \rho v_x^2)}{\partial x} + \frac{\partial(\rho v_x v_y)}{\partial y} = 0 \ , \tag{12}$$

$$\frac{\partial(\rho v_y)}{\partial t} + \frac{\partial(\rho v_x v_y)}{\partial x} + \frac{\partial(p + \rho v_y^2)}{\partial y} = 0 \ , \tag{13}$$

$$\frac{\partial e}{\partial t} + \frac{\partial[(e+p)v_x]}{\partial x} + \frac{\partial[(e+p)v_y]}{\partial y} = 0 \ , \tag{14}$$

where:

x, y	Cartesian coordinates [m]
v_x, v_y	components of the velocity vectors $\boldsymbol{v}$, [m/msec]
$\lvert v\rvert^2 = v_x^2 + v_y^2$	norm of the vector of velocity
$\rho = \rho(x,y,t)$	density of gas [kg/m^3]
$p = p(x,y,t)$	pressure of gas [MPa]
$\varepsilon = \varepsilon(p,\rho;x,y,t) =$ $= \dfrac{p - c_0^2(\rho - \rho_0)}{(\gamma-1)\rho}$	potential energy [m^2/ms^2]
$\dfrac{1}{2}\rho(v_x^2 + v_y^2)$	kinetic energy [m^2/ms^2]
$e = \rho[\varepsilon - (v_x^2 + v_y^2)/2]$	full energy of a unit of mass of the gas, [MPa]

The pressure due to explosion of TNT (Trinitrotoluene is a chemical compound with the formula $C_6H_2(NO_2)_3CH_3$) charge can be recorded as:

$$p = (\gamma - 1)\rho\varepsilon \ , \tag{15}$$

where γ is the exponent of adiabatic process. For the air it is mostly selected $\gamma = 1.4$, and in

the case of explosion the exponent of adiabatic process becomes only mass density dependent, i.e. $\gamma = \gamma(\rho)$.

Let us denote

$$s = \begin{Bmatrix} v_x \\ \rho v_x \\ \rho v_y \\ e \end{Bmatrix}, \quad a = \begin{Bmatrix} \rho v_x \\ p + \rho v_x^2 \\ \rho v_x v_y \\ (e+p)v_x \end{Bmatrix}, \quad b = \begin{Bmatrix} \rho v_y \\ \rho v_x v_y \\ p + \rho v_y^2 \\ (e+p)v_y \end{Bmatrix}. \tag{16}$$

The equations of conservation (10)–(13) in vector form are then recorded as:

$$\frac{\partial s}{\partial t} + \frac{\partial a}{\partial x} + \frac{\partial b}{\partial y} = 0. \tag{17}$$

Let $A_i, i = 1,...,m$ be a non-overlapping set in $A \in \Omega$, where $\Omega \in 2D$ and the closure $\sum_{i=1}^{m} A_i$ completely covers A. The time interval is divided into non-overlapping subintervals $\delta t^n = [t^{(n-1)}, t^{(n)}], n = 1,...,T$. The classical finite volume approximation of (16) relies on the change of the control volumes between time $t^{(n-1)}$ and $t^{(n)}$. Select arbitrary admissible i and j in such a way that $K_i \equiv S$ and $K_j \equiv Z$ are adjacent elements with common boundary P. Eqn (16) is then defined on $S \times [t^{(n-1)}, t^{(n)}]$ and $Z \times [t^{(n-1)}, t^{(n)}]$, respectively. Integrating them one obtains:

$$\begin{aligned} &\frac{1}{\delta t^n}\int_S (s(x,t^n) - s(x,t^{n-1}))\mathrm{d}x + \sum_{j=1}^{6} \int_{t^{n-1}}^{t^n}\int_{\partial S_j} (a^n(x,t)n_{S,1}^j(x,t) + b^n(x,t)n_{S,2}^j(x,t))\,\mathrm{d}\gamma_S(x)\mathrm{d}t = 0, \\ &\frac{1}{\delta t^n}\int_Z (s(x,t^n) - s(x,t^{n-1}))\mathrm{d}x + \sum_{j=1}^{6} \int_{t^{n-1}}^{t^n}\int_{\partial Z_j} (a^n(x,t)n_{Z,1}^j(x,t) + b^n(x,t)n_{Z,2}^j(x,t))\,\mathrm{d}\gamma_Z(x)\mathrm{d}t = 0, \end{aligned} \tag{18}$$

because the number of boundaries (boundary abscissas) ∂S_j of S as well as ∂Z_j of Z is exactly six, where $\mathrm{d}\gamma_S$ and $\mathrm{d}\gamma_Z$ denote the integration with respect to the one-dimensional measure of the boundary of S and Z, respectively, $n_{S,1}^j$ and $n_{S,2}^j$ are directional cosines of the outward unit normals to ∂S_j and similarly $n_{Z,1}^j$ and $n_{Z,2}^j$ are directional cosines of the outward unit normals to ∂Z_j. If $\|.\|$ is the one or two-dimensional measures, eqns (24) may be written as:

$$\|S\|(s_S^n - s_S^{n-1}) + \delta t^n \sum_{j=1}^{6} \|S_j\| F_S^{j,n} = 0, \quad \|Z\|(s_Z^n - s_Z^{n-1}) + \delta t^n \sum_{j=1}^{6} \|Z_j\| F_Z^{j,n} = 0, \tag{19}$$

where s_S^n (resp. s_S^0) is an approximation of $\frac{1}{\|S\|}\int_S (s(x,t^n)\mathrm{d}S$ (resp. $\frac{1}{\|S\|}\int_S (s(x,t^0)\mathrm{d}x$), s_Z^n (resp. s_Z^0) is an approximation of $\frac{1}{\|Z\|}\int_Z (s(x,t^n)\mathrm{d}Z$ (resp. $\frac{1}{\|Z\|}\int_Z (s(x,t^0)\mathrm{d}x$), and $F_S^{j,n}$ is an approximation of $\frac{1}{\delta t^n \|\partial S_j\|}\int_{t^{n-1}}^{t^n}\int_{\partial S_j} (a^n(x,t)n_{S,1}^j(x,t)+b^n(x,t)n_{S,2}^j(x,t))\,\mathrm{d}\gamma_S(x)\mathrm{d}t$, and similar relations are valid, if substituting S for Z.

For two adjacent faces of two control volumes S_j and S_k it holds at any time t, $n_1^j(x,t)+n_1^k(x,t)=0$, $n_2^j(x,t)+n_2^k(x,t)=0$, $x\in \overline{S}_j \cap \overline{S}_k$, hence the normal fluxes.

The time iteration has to be in compliance with the free hexagons, so that it obeys the rules defined in the section 2.2 with the exception that different variables appear in finite volume method. The nodal points are centered at the hexagonal elements.

3 EXAMPLE

An explosion in underground parking is supposed with the charge located on the bottom of a tunnel at the center of symmetry. The values of the radius of charge is 0.25 m for its mass q = 50 kg and the density of TNT ρ_{TNT} = 1620 kg/m^3. Initial pressure of the charge is 1.3 kN/cm^2.

A simplification is introduced here that the material properties of the granite rock remain stable inside of the particles and are given in a standard way by modules taken for linear elasticity: E = 38 × 10^9 N/m^2, ρ = 7833 kg/m^3, ν = 0.17, lining properties are: E = 14 × 10^9 N/m^2, ρ = 2250 kg/m^3, ν = 0.16, friction angle is 35^0, tensile strength is 1.26 × 10^6 N/m^2 and shear strength is 0.37 × 10^6 N/m^2. It is assumed that the rock will not suffer form damage.

In Fig. 3 the geometry of the problem is seen together with the regular particles set up for the computation. Total amount of particles in our problem is 3800, from that 1400 in the rock, 300 in the lining and 2100 in the air. Internal diameter of each particle is 0.25 m. The time step is introduced by the value of 0.1 msec. This can be considered as sufficient approximation.

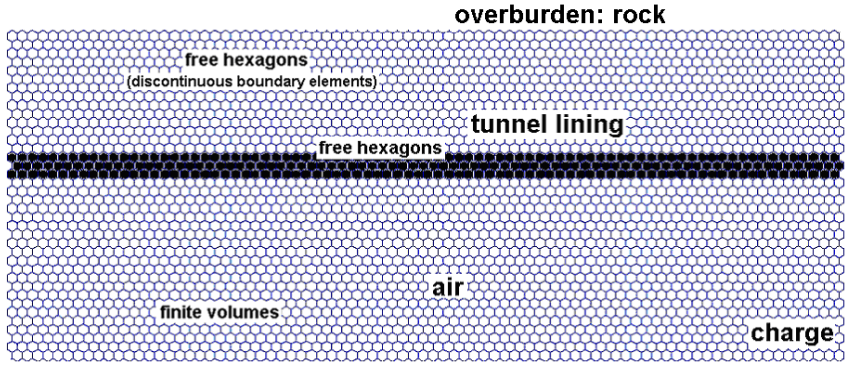

Figure 3: Geometry of the tunnel and grains set up.

The movement in the air after 7 msec is seen in Fig. 4, together with its vectors.

After 8 msec the shock wave attains the upper lining of the tunnel and causes small deflection of the concrete. This is shown in Fig. 5. In Fig. 6, after 10 msec the shock wave propagates in the horizontal direction and lefts behind concentration of air pressure at approximately ¾ of the height of tunnel, attenuation above this concentration and spalling of concrete due to vertical deflection.

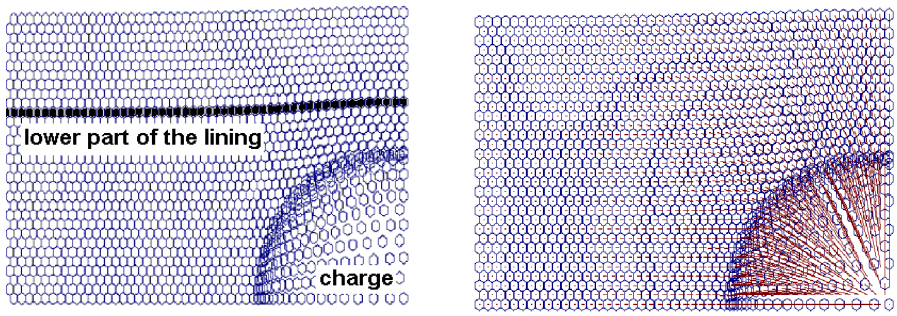

Figure 4: Movements 7 msec after the initiation of the explosion.

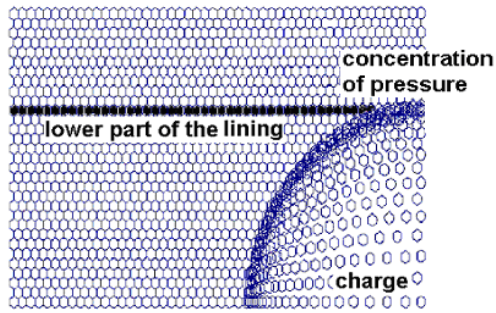

Figure 5: After 8 msec the shock wave reached the lining at the calotte.

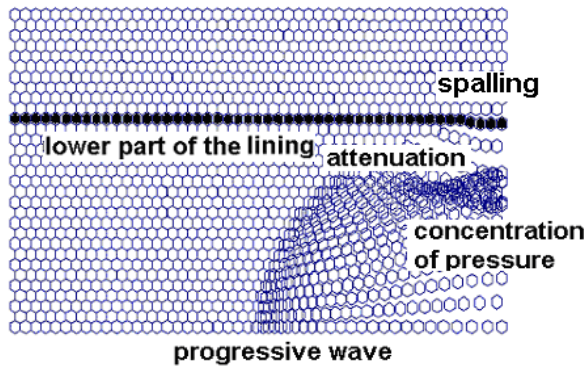

Figure 6: Situation after 10 msec.

4 CONCLUSION

In this paper a very topical problem is proposed. The effect of explosion in a closed space – tunnel – is formulated and solved. The hexagon method (discontinuous boundary element method) is proposed as a numerical tool and developed for time dependent D'Alembert principle involved to the formulation of the numerical procedure. The approximation of equations of conservation in the air is proposed finite volumes, which seem to be one of the best decisions. It appears that in such an application of the methods envisaged is very powerful and efficient as the computer time consumed for computation is relatively small.

The concrete lining is treated as linear elastic inside of the elements and obeys Mohr-Coulomb damage rules along the particle interfaces, while the functions in the air are linear, without any upheaval. The great advantage of the free hexagons is the acceptance of unpredictability of local damage occurrence. Although short time steps seem to be required, the results of the example presented in the frame of this paper are reasonable for time step equal 0.1 msec. This relatively rough discretization of time interval does not harm seriously the results obtained by this procedure, as shorter time interval does not show greater variations in the solution.

In this study the bearing system (side walls, ceiling of the tunnel lining) was constructed from concrete of specific material properties. After the shock wave starts interacting with the linear elastic lining, the material exhibits breakage by damage law. As the tunnel has rectangular cross-section, both tensile and shear spalling can occur. Rapid attenuation below the lining at the calotte appears and concentrated pressure is registered inside of the tunnel. If the front of the shock wave propagates in the horizontal direction, the front wave in the vertical direction changes and is hurled against the upper lining again. In this problem a great interaction of distinctive shock waves is not supposed because of the nature of the tunnel geometry in 2D. Although the numerical solution in the air and in the solid body is relatively fast, the procedures in both domains is quadratic more exacting. If the solution in the air consumes time n and in the solid bodies m then the resulting time is less than $n \times m$, because of faster solution in the air (caused by finite volumes). The problem of correct definition of the contact conditions between the air and the solid body, such as accurate definition of refractions, part absorption of the kinetic energy, and such phenomena, can the consumed time basically increase.

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