Structural junction identification methodology

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Abstract

The focus of this work is to present, from a practical point of view, a methodology able to tune the dynamic behaviour of complex assembled structures in frequency domain and optimizing the parameters, in terms of stiffness and damping, of lumped elements at junction points among structural components. Performing sensitivity studies through evaluating the impact of a set of modifications in the dynamic behaviour of complex structures by means of running several FEM models, requires significant computational effort and even if it is accepted, it is often not able to fit the experimental data adequately. In this context, the Direct Structural Dynamic Modification Method is defined as the procedure which permits one to evaluate the impact of a set of changes on the structural dynamic behaviour, without the need to continuously re-run the FEM Model. The Inverse SDM problem aims to identify in the framework of physical compatible sets of modifications, the most appropriate in order to fit the desired dynamic behaviour. In this study the ISDM problem is completed in order to be implemented efficiently in MATLAB and is applied to fit the analytical Frequency Response Functions (FRFs) with the experimental results. The full aircraft model and the Ground Vibration Test of the A340-600 are considered in order to test the power of the method when applied to a real and complex structure. From the results it can be seen that the parameters of the lumped elements at the interfaces among components are efficiently optimized in order to improve the dynamic response of the structure. The physical understanding of junction behaviour permits appropriate definition of the constraints of the optimization problem and to get a global minimum of the objective function. The results are shown in terms of FRFs and in terms of global FRF indicators
Keywords: frequency response functions, junction identification, structural dynamic modification, complex mode indicator function, frequency domain assurance criteria.

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1 Introduction

Finite Element Models are widely used in structural dynamics in order to study and predict the behaviour of real structures. Today the demand for high-quality models and the accompanying processing techniques are growing fast and, at the same time, the products get more complex and intricate. It is possible to affirm that, whilst the modelling and dynamic response prediction techniques for individual structural components have been well developed, when a similar procedure is extended to structural assemblies, the prediction quality deteriorates quickly. For example as the number of components in the assembly increases the calculation quality declines because the connection mechanisms between components are not represented sufficiently. For a long time the lack of reliability in modelling the junctions in complex assemblies has been under estimated, hence its effect on the global dynamic behaviour is neglected.

2 Objective of the work

The focus of this work is to present, from a practical point of view, a methodology able to tune the dynamic behaviour of complex assembled structures in frequency domain and optimizing the parameters of lumped elements at junction points among structural components. From a modelling and computational point of view, if it is true that most of the junctions can be seen as localized sources of stiffness and damping, these can be modelled as lumped spring/damper elements. On the other hand a lack of reliability occurs because the properties, in terms of stiffness and damping, are normally unknown and just assumed on the basis of experience.

Performing sensitivity studies through evaluating the impact of a set of modifications in the dynamic behaviour of complex structures by means of running several FEM Models, requires significant computational effort and even if it is accepted, it is often not able to fit the experimental data adequately.
In this context the Structural Dynamic Modification (SDM) Method is defined as the procedure which permits one to evaluate the impact of a set of changes on the structural dynamic behaviour, without the need to continuously re-run the FEM Model.

In this study the DIRECT SDM problem has been completed in order to be implemented efficiently in MATLAB but the focus is on the Inverse Structural Dynamic Modification (ISDM) Method, which has been applied to fit the analytical Frequency Response Functions (FRFs) with the experimental results. The full aircraft model and the Ground Vibration Test of the A340-600 have been considered in order to test the power of the method when applied to a real and complex structure.

3 Scenario

3.1 Experimental data: A340-600 ground vibration test (GVT)

Dedicated sensor plans for each structural part and, at interfaces among components, are used during the A340-600 GVT. Several load condition are applied during the test and some of them are selected as the most appropriate for this study. They are: Vertical Tail-Plane (VTP) loaded in X and Y directions and Horizontal Tail-Plane (HTP) loaded in X and Z directions. Particularly the VTP – X loading condition is considered in order to show the procedure and the results. The Frequency Response Functions (FRFs) of the sensors of interest are analyzed in order to find out what physical phenomena happen across the junctions. The frequency range covered is between 2.5 and 25 Hz.

Figure 1: A340-600 GVT.

The focus of this study is on the interfaces between the Auxiliary Power Unit (APU) and its Suspension System. The APU is installed in the A346 Tail-Cone by its Suspension System, which has a double purpose: to sustain the inertia loads at which the APU is submitted and to isolate the airframe from the APU's vibrations.

The Suspension System consists of 3 principals subassembly called:

- Left-Hand: 3 rods, 3 APU lugs on structure side, 1 Rubber Mount
- Right -Hand: 2 rods, 2 APU lugs on structure side, 1 Rubber Mount
- Aft –Hand: 2 rods, 2 APU lugs on structure side, 1 Rubber Mount

Each rubber mount is done by a steel isolator housing with an elastomeric inside.

Couples of tri-axial accelerometers are installed across each rubber mount, one on the APU bracket and the other on the isolator housing.

The following pictures show the geometrical configuration of the Suspension System (fig. 2) and the experimental FRFs across one of the rubber mounts (fig. 3).

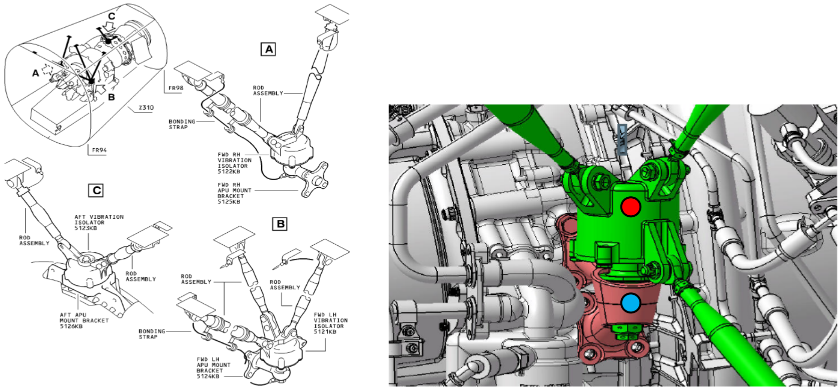

Figure 2: APU suspension system and detail of the LH rubber mount.

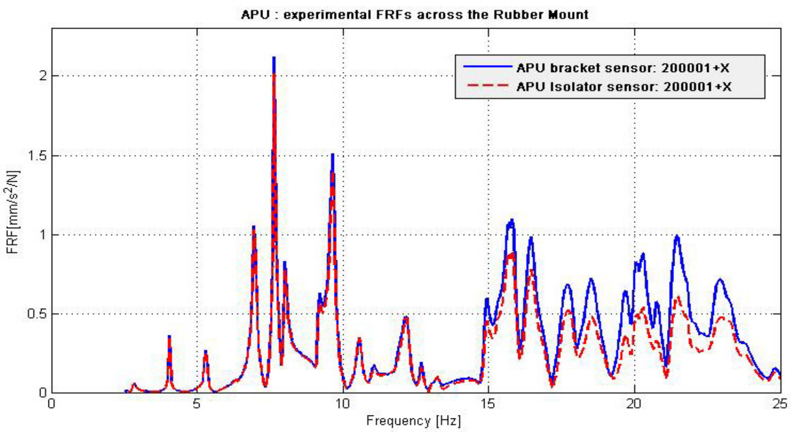

Figure 3: Experimental FRFs across the LH rubber mount.

The analysis of experimental FRFs of the sensors located at rubber mounts shows that hardening, softening and dissipation phenomena happen across the junction. These effects arise for frequency values above the 15 Hz. It is expected because only above this frequency value the local APU modes become relevant. Hence the junction dynamic behaviour is firstly driven by the global modes and then, when local modes arise, the proper modelling of junction properties became relevant.

3.2 A340-600 finite element model and comparison

For this study the full A340-600 Finite Element Model (FEM) is used. The Global FE Model has been properly validated and tuned for the low frequency range between 2 and 14 Hz. It means that for degrees of freedom close to the excitation point no big discrepancy among experimental and theoretical FRFs is expected. Nevertheless, for the sensors of the APU Rubber Mounts, particularly above the 15 Hz, a certain degree of discrepancy is expected, due to the local behaviour.

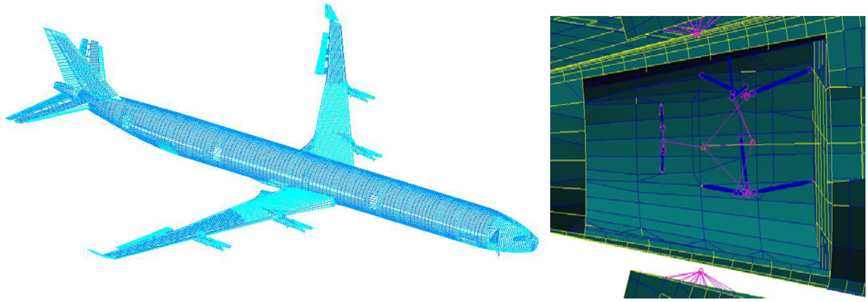

Figure 4: A340-600 Finite Element Model.

In order to compare analytical and experimental results:

- The coherence between experimental parameters and nodes, in terms of geometrical position and reference coordinate systems, is defined.
- The same experimental load conditions are applied to the FE Model and the analytical FRFs are calculated.
- The full set of FRFs, experimental versus analytical, are compared in terms one-to-one FRF and global FRF indicators. Generally speaking the indicators permit to combine the FRFs in order to have a clear understanding about the degree of reliability of the FE Model. They are: the Complex Mode Indicator Function (CMIF) and the Frequency Domain Assurance Criteria (FDAC).

3.3 Global FRF indicators

3.3.1 Complex mode indicator function

The concept of CMIF is developed by performing Singular Value Decomposition (SVD) of the Transfer Function Matrix (TFM) at each spectral line. The CMIF returns the eigenvalues, which are the square of singular values, associated with the Modal Matrix. It is a simple and efficient method for identifying the modes of a complex system. The peaks detected in the CMIF indicate the existence of modes, and the corresponding located frequencies of these peaks give the damped natural frequency for each mode.

If the CMIF of the experimental sensors of the VTP (where the load is applied) is compared against the CMIF of the experimental sensors of the rubber mounts (fig. 6), it can be highlighted again, that there is a low frequency range where the dynamic behavior of the junction is driven by the global modes, and above the 15 Hz local modes arise (fig. 5).

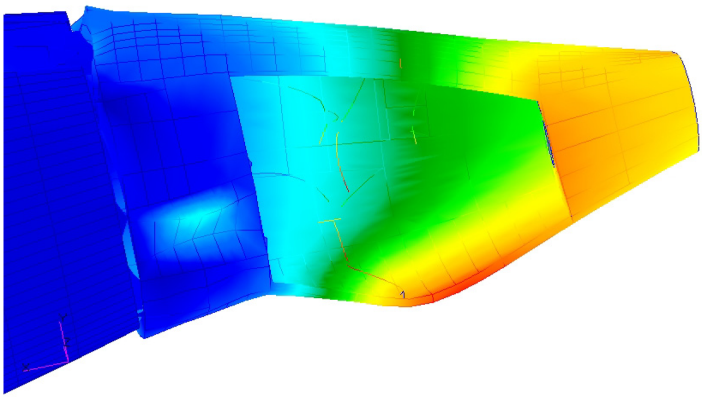

Figure 5: Tail-Cone and APU local mode at 15 Hz.

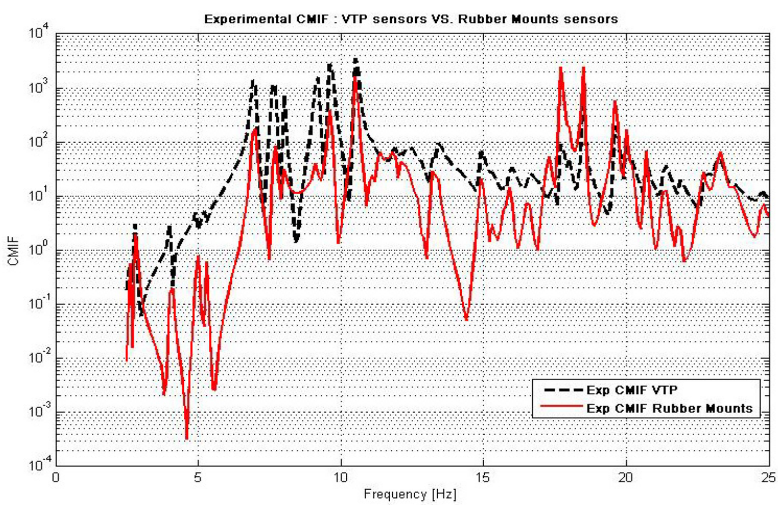

Figure 6: Experimental CMIF: VTP vs. rubber mounts

3.3.2 Frequency domain assurance criteria

The FDAC coefficients represent the correlation between two sets of FRFs at specific frequencies across the full spatial/coordinate domain.

It expresses the shape correlation between measured and predicted response. Because FDAC evaluates the shape of an FRF, which is mainly determined by the position and amount of resonance peaks, this function is most sensitive to changes of mass and stiffness modeling.

4 Structural dynamic modification

4.1 Overview of the SDM method

Structural Dynamic Modification (SDM) Method is defined as the procedure which permits one to evaluate the impact of a set of changes on the structural dynamic behaviour, without the need to continuously re-run the FEM Model.
The modified dynamic behaviour can be expressed as function of the baseline FEM Dynamic Database and the set of modifications.

Many authors have formulated and completed the theoretical problem, highlighting that the method becomes particularly efficient if into the modification lumped elements are involved. Lumped modifications consist of whatever relationship between two degrees of freedom, both of the structure or one degree of freedom belonging to the structure and another one belonging to an external fixed point. Usually the relationship is expressed as a combination of lumped masses, spring and damper elements.

The baseline FEM Dynamic Database can be expressed by the modal database made of eigenvalues and eigenvectors (real or complex), or by the frequency response function (FRF) database, in which case the Transfer Function Matrix (TFM) is available.

Two problems could be faced:

- The Direct SDM (DSDM) problem consists in evaluating the effect of a given set of changes on the dynamic behaviour of the structure
- The Inverse SDM (ISDM) problem, more complex, aims to identify in the framework of physical compatible sets of modifications, the most appropriate in order to fit the desired dynamic behaviour.

Sestieri [1] exhaustively explains and completes the theoretical DIRECT SDM problem, giving a complete and critical overview about approaching the DSDM by the use of the modal database and the FRF database.

In this study the DIRECT SDM problem has been completed in order to be implemented efficiently in MATLAB but the focus is on the Inverse Structural Dynamic Modification (ISDM) Method, which has been applied to fit the analytical Frequency Response Functions (FRFs) with the experimental results.

4.2 Mathematical formulation of the direct structural dynamic modification method

The dynamic equation of the baseline model, in frequency domain and physical coordinates, is:

$$u_0(\omega) = H_0(\omega)F(\omega) \tag{1}$$

Being $H_0(\omega)$ the well-known Transfer Function Matrix (TFM).

$$H_0(\omega) = \left[-\omega^2 M + j\omega C + K\right]^{-1} \tag{2}$$

where M, C, K, are the Mass, Damping and Stiffness Matrices of the Baseline Finite Element Model.

The Dynamic Stiffness Matrix of the baseline structure is defined as:

$$B_0(\omega) = H_0(\omega)^{-1} = \left[-\omega^2 M + j\omega C + K\right] \tag{3}$$

And being the modifications described by the following matrix:

$$\Delta B(\omega) = \left[-\omega^2 \Delta M + j\omega \Delta C + \Delta K\right] \tag{4}$$

The dynamic equation of the modified model, under the same loading conditions, in frequency domain and physical coordinates, is:

$$F(\omega) = \left[B_0(\omega) + \Delta B(\omega)\right] u_{\mathrm{mod}}(\omega) \tag{5}$$

Finally combining the two equations, (1) and (5), the relationship, which expresses the modified TFM as function of the baseline TFM and the Modification Matrix, is obtained:

$$u_0(\omega) = H_0(\omega) F(\omega) = \left[I + H_0(\omega)\Delta B(\omega)\right] u_{\mathrm{mod}}(\omega) \tag{6}$$

$$H_{\mathrm{mod}}(\omega) = \left[I + H_0(\omega)\Delta B(\omega)\right]^{-1} H_0(\omega) \tag{7}$$

If the modification matrix involves only few degrees of freedom of the totals the Equation (7) can be efficiently rearranged in order to obtain a significant advantage from a computation point of view.

$$\begin{aligned} &\begin{bmatrix} H_{\mathrm{mod},AA} & H_{\mathrm{mod},AB} \\ H_{\mathrm{mod},BA} & H_{\mathrm{mod},BB} \end{bmatrix} = \\ &= \begin{bmatrix} H_{0,AA} & H_{0,AB} \\ H_{0,BA} & H_{0,BB} \end{bmatrix} - \begin{bmatrix} H_{0,AB} \\ H_{0,BB} \end{bmatrix} \Delta B_{BB} \left[I_{BB} + H_{0,BB}\Delta B_{BB}\right]^{-1} \left[H_{0,BA} \quad H_{0,BB}\right] \end{aligned} \tag{8}$$

where

$$\Delta B(X,\omega) = \begin{bmatrix} 0 & 0 \\ 0 & \Delta B_{BB}(X,\omega) \end{bmatrix} \tag{9}$$

is the Modification Matrix of the degrees of freedom involved into the modification and X is the vector of the parameters which describe the modification.

The computation performed by means of the Equation 4 must be repeated for all the frequency values of interest.

The Modification Matrix, containing the whole set of modifications, is a function of:

- the coordinates of the degrees of freedom where each modification will be applied,
- the parameters which define the lumped modification
- the frequency values of interest.

More details can be found in references Sestieri [1] and Sanliturk [2].

4.3 Inverse structural dynamic modification

The great computational efficiency of the SDM method makes it suitable to be implemented into an optimization problem.

The parameters of the problem, in terms of lumped mass, stiffness and damping, can be optimized in order to minimize the error between the analytical and the experimental Transfer Function Matrices. Hence the Inverse SDM (ISDM) problem can be properly planned in order to identify in the framework of physical compatible sets of modifications, the most appropriate for fitting the experimental dynamic behaviour.

The following Objective Function (OF) can be defined:

$$OF(\Delta B(X)) = \sum_{n=1}^{Np} \sum_{\omega=\omega_{in}}^{\omega_f} \left\{ \begin{array}{l} WF * \left(\dfrac{H_{\exp,n}^{ABS}(\omega) - H_{\mathrm{mod},n}^{ABS}(X,\omega)}{H_{\exp,n}^{ABS}(\omega)} \right)^2 + \\ + (1-WF) * \left(\dfrac{H_{\exp,n}^{PHASE}(\omega) - H_{\mathrm{mod},n}^{PHASE}(X,\omega)}{H_{\exp,n}^{PHASE}(\omega)} \right)^2 \end{array} \right\} \tag{10}$$

The sum is extended to all the experimental sensors and to the whole frequency range of interest.

Being the Transfer Function Matrices complex, both, magnitude and phase need to be taken into account. The Weight Factor (WF) can be used to change the priority of the optimization between the magnitude and the phase.

Clearly, the vector X, containing the parameters which describe the modifications, is the unknown of the problem.

5 Optimization of rubber mounts parameters

The parameters, in term of stiffness and damping, of the lumped elements at the interfaces between the APU and its Suspension System are optimized in order to improve the dynamic response of the structure when compared with the experimental data. The frequency range considered is between 13–25 Hz, because only above the 15 Hz the APU modes become relevant. These spring/damper elements are intent to simulate the rubber mounts of the APU suspension system, hence the physical understanding of the rubber behaviour permit to define properly the constraints of the optimization problem and to get a global minimum of the objective function.

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Each lumped element, connecting pair of degrees of freedom, has two parameters, one defining the stiffness and the other defining the hysteretic damping ratio:

$$\Delta B_{elem}(K,h,\omega) = K_{elem}(\omega)\left(1 + jh_{elem}(\omega)\right) \tag{11}$$

The Modification Matrix for all the structure is:

$$\Delta B_{BB}(K,h,\omega) = \sum_{p=1}^{N_{elem}} \Delta B_p(K_p,h_p,\omega) \tag{12}$$

Considering all the translational degrees of freedom and the three subassemblies of the Suspension System, finally 18 variables need to be optimized for each frequency value.

The constraints of the optimization problem come from the physical and geometrical properties of the rubber mounts. For each rubber mount, the stiffness in X and Y directions is supposed to be the same, and the Z-axial stiffness is generally higher than the laterals stiffness in X and Y directions. The Right-Hand and Left-Hand Isolators are geometrically identical and consequently they are supposed to have the same mechanical properties. The stiffness properties of the Aft-Hand Isolator are higher than the stiffness properties of the Right-Hand and Left-Hand Isolators. All these equalities and inequalities constraint equations are considered within a 15% of uncertainty.

This permits the appropriate definition of the constraints of the optimization problem and to get a global minimum of the objective function. A nonlinear least square method has been applied in MATLAB environment through the use of the function ‘fmincon’.

6 Summary of results

From the results of the optimization it can be seen that the dynamic behaviour is efficiently improved, even more if it is considered that the rubber mounts are particularly complex due to several dependencies, such as: frequency, dynamic amplitude, preload, temperature, which strongly affect the dynamic behaviour.

The value of the Objective Function, which quantifies the error between the experimental and the analytical Transfer Function Matrices, for the FE Baseline Model is about 10^4. After the optimization, this value, clearly cannot go to zero, but reaches a very low value, less than 10^2.

The results are shown below in terms of Frequency Response Functions, both magnitude and phase have been considered (fig. 7). This permits a direct comparison of each experimental value versus the analytical one, and in terms of Global Indicators (in which all the FRFs are combined in order to obtain one representative value), such as Frequency Domain Assurance Criteria (FDAC, fig. 8) and Complex Mode Indicator Function (CMIF, fig. 9), allowing determination of how the global dynamic behaviour of the system has been improved.

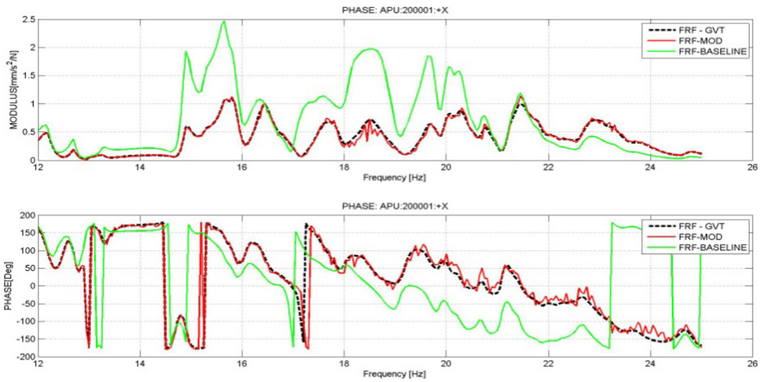

Figure 7: Inverse SDM: FRF results of sensor APU: 200001+X.

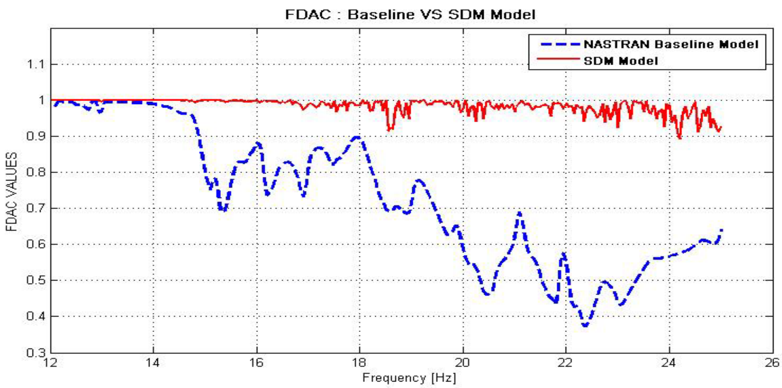

Figure 8: Inverse SDM: FDAC results.

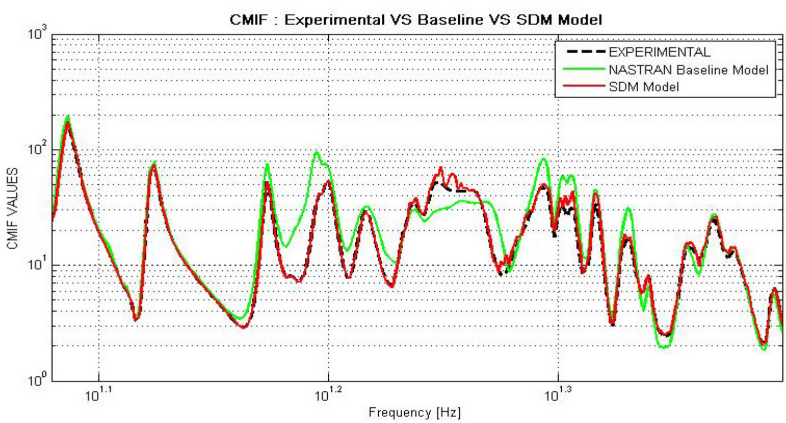

Figure 9: Inverse SDM: CMIF results.

7 Conclusion

The Structural Junction Identification Methodology is a process which permits:

- to understand what is the frequency range where the local dynamic behaviour of the junctions becomes relevant. The use of the CMIF, calculated for the experimental sensors of interest, can be particularly useful in order to achieve that understanding. It is worth pointing out that when large FE Models are used, the dynamic behaviour of the junction is firstly driven by the global modes and then, above a certain value of frequency, local modes arise. It means that a local correlation cannot be satisfactory gotten if, previously, the Global FE Model behaviour has not been properly validated and tuned;
- to perform, with a very low computational cost (minutes), sensitive analysis of the TFM against a change of the stiffness and damping properties of the lumped elements at junction interfaces. That is achieved through the use of the SDM method;
- to set-up the optimum values for the lumped elements, basically in terms of stiffness, mass and damping, through the use of the Inverse SDM. Once the global tuning of the model is completed, the Inverse SDM can be efficiently implemented in order to find the optimum set of lumped modifications and improving, consequently, the local dynamic behaviour. The physical understanding of junction behaviour permits appropriate definition of the constraints of the optimization problem and to get a global minimum of the objective function.

During all the process, the Global FRF Indicators (CMIF and FDAC) can be successfully used in order to find out the degree of correlation between analytical and experimental results.

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