

# A novel approach for seismic analysis of multi-storey buildings

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## Abstract

In this paper a novel equivalent planar-frame model with openings is presented. The model deals with seismic analysis using the Pushover method for masonry and reinforced concrete buildings. Each wall with openings can be decomposed into parallel structural walls made of an assemblage of piers and a portion of spandrels. As formulated, the structural model undergoes inelastic flexural as well as inelastic shear deformations. The mathematical model is based on the smeared cracks and distributed plasticity approach. Both zero moment location shifting in piers and spandrels can be evaluated. The constitutive laws are modelled as bilinear curves in flexure and in shear. A biaxial interaction rule for both axial force – bending moment and axial force – shear force are considered. The model can support any shape of failure criteria. An event-to-event strategy is used to solve the nonlinear problem. Two applications are used to show the ability of the model to study both reinforced concrete and unreinforced masonry structures. Relevant findings are compared to analytical results from experimental, simplified models and finite element models such as Drain3DX and the ETABS finite element package.

*Keywords: seismic analysis, unreinforced masonry, reinforced concrete, structural wall, equivalent frame.*

## 1 Introduction

Earthquakes are considered to be the major cause of structural failure of buildings in Europe. Despite their rarity and moderate intensity, earthquakes in the interior of northwest and central Europe have the potential to cause extensive damage and associated financial losses, due to the vulnerability of the local



building stock. In almost all countries, the majority of the building stock is classified as existing buildings. This is why extensive assessment of such structures is motivated since they have been generally designed to resist gravity loads.

This paper makes a contribution to the seismic vulnerability assessment of existing buildings through the development of a simplified analytical model. The need for such models is always motivated by first, the large amount of structures that should be analyzed in a very short time and second, the search for optimal solutions for structural retrofiting. A widely used model for structural analysis is the line elements or the equivalent frame models. Despite of some limitations in the equivalent frame model, it is very attractive in comparison to complex finite element models [1–5]. Moreover, they have shown satisfactory results particularly for RC structures. In this context, the proposed model is based on beam-column element and distributed of non-linearity approaches.

## 2 A model for structural walls with openings

The mathematical model can represent solid walls, frame structural elements, coupled walls and perforated walls [12, 13]. The structural model consists of an assemblage of vertical plane walls with openings that form a single perforated wall. Each structural wall is made of pier elements with or without rigid offsets and a portion of spandrels such that there are two kinds of individual walls: exterior walls and interior walls (Fig. 1). The length of these parts of spandrels is equal to the zero moment length, and can be updated at each step depending on the bending moments at the spandrel ends.

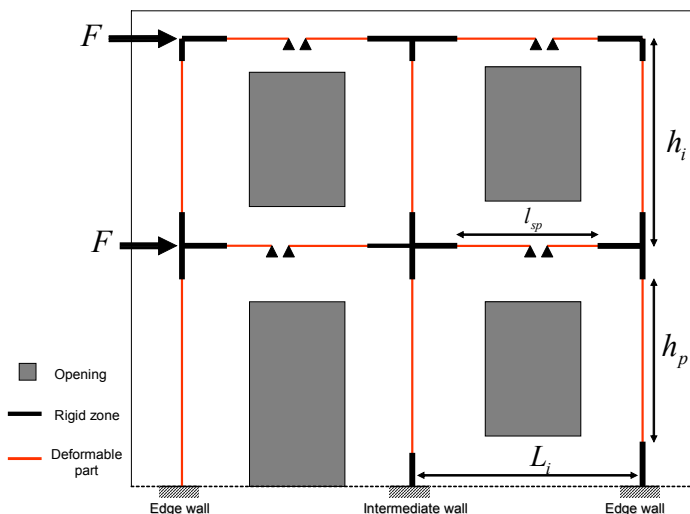


Figure 1: A representation of the equivalent frame model for planar walls with openings.

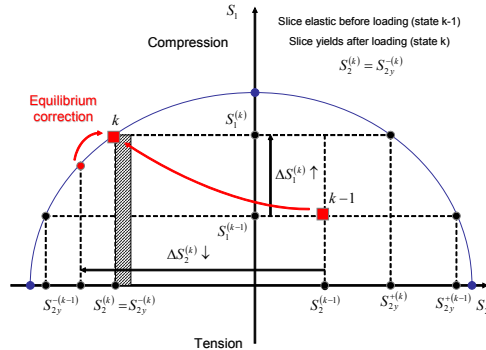


Figure 2: General representation of an interaction process between  $S_1$  and  $S_2$  parameters.

The proposed model is based on the spread nonlinearity approach. Each pier and spandrel can be discretized into a series of slices [6] while cross-sections are considered as homogeneous. The mechanical model undergoes flexural as well as shear deformation. In the current formulation, the model only considers a biaxial interaction (Fig. 2) between axial forces – bending moments (N-M) and axial forces – shear forces (N-V). The so-called shifting of the primary curve technique is used in a simple manner. The wall formulation permits the capture of the coupling effect in elevation due to the nonlinearity distribution in both piers and spandrels. Thus, the zero moment location in both piers and spandrels can be mitigated during the nonlinear analysis. The variation of the axial vertical loads is considered for piers only. The nonlinearity is treated using a smeared plasticity approach [6]. The present formulation deals with a Pushover analysis. It is based on the well-known event-to-event strategy. A simplified algorithm for systems with interaction effect is used through an equilibrium correction at each step of calculation. Finally, the sum of all generated capacity curves of planar frames permits to analyze an entire building.

The storey moment-lateral force formulation of a structural wall element is expressed by:

$$\{M_{bs}\} = [K_{Frame}] \{P\} \quad (1)$$

$$\{P\} = [L_{P-T}] [L_{T-F}] \{F\} \quad (2)$$

$$[K_{Frame}]^{-1} = \begin{bmatrix} B_1 & C_1 & 0 & 0 & \dots & 0 & 0 & 0 \\ A_2 & B_2 & C_2 & 0 & \dots & 0 & 0 & 0 \\ 0 & A_3 & B_3 & C_3 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \dots & A_{N-1} & B_{N-1} & C_{N-1} \\ 0 & 0 & 0 & 0 & \dots & 0 & A_N & B_N \end{bmatrix} \quad (3)$$

$$P_n = D_n T_{n-1} + E_n T_n \quad (4)$$

$\{M_{bs}\}$  represents the base storey bending moments vector,  $[K_{Frame}]$  is the equivalent frame stiffness matrix,  $\{P\}$  is the reduced shear forces vector,  $[L_{P-T}]$  is the reduced storey shear forces – storey shear force transformation matrix,  $[L_{T-F}]$  is the storey shear force – applied lateral force transformation matrix in absence of vertical distributed loads,  $\{F\}$  is the lateral load pattern vector. The equivalent frame matrix and the reduced shear forces vector are defined by the expressions  $A_n$ ,  $B_n$ ,  $C_n$ ,  $D_n$ , and  $E_n$  (with  $n=1,N$ ;  $N$  is the number of storeys) (Eq. (3),(4)). These functions are defined with regards to the equivalent stiffnesses of piers and spandrels [12,13].

### 3 Pushover analysis of a RC building

A three-dimensional multi-storey building made in RC structural walls is studied (Fig. 3) [12, 13]. The structure was modelled on both Drain3DX [7] using a fibre beam element (type 15) and ETABS [8] using a point hinge beam element. The use of fibres to model cross-sections accounts rationally for axial force – biaxial bending moments. On the other side, ETABS provides a flexural point hinge finite element model (PHFE) called P-M2-M3. This model considers an interaction between two-way moment curves and axial forces. Since the structural model behaves in the in-plane direction, the point hinge model performs with a biaxial interaction rule. In the equivalent frame model (EFM), the nonlinear behaviour for each slice is defined by a moment-curvature relationship in compression only. Three cases are investigated to study the axial force redistribution and the axial force – bending moment interaction rule. They are: (1) rigid floor-type structure with 100% of the floor stiffness,  $I_{Floor}$ , (2) semi rigid-floor type with 50%  $I_{Floor}$  and (3) flexible floor-type with 10%  $I_{Floor}$ . For each floor-type model, two cases were studied for the EFM and four cases for the PHFE model on ETABS. The case studies are defined as follows:

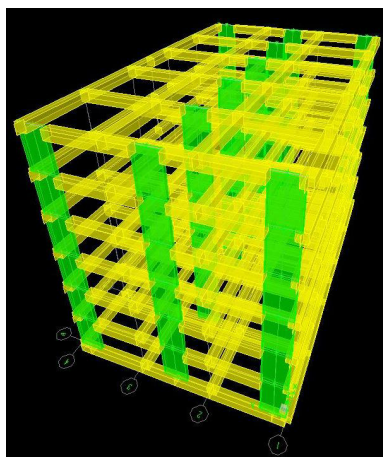


Figure 3: A view of the structural model developed on ETABS.

1. PHFE M1 and M2: Bilinear and elastic-plastic moment-rotation law respectively, without (N-M) interaction,
2. PHFE M3 and M4: Bilinear and elastic-plastic moment-rotation law respectively, including (N-M) interaction,
3. EFM 1 and EFM2: Without and with (N-M) interaction respectively.

Figures 4, 5 and 6 display capacity curves for the three floor-type models analyzed by Drain3DX, ETABS and the proposed EFM. The (N-M) interaction effect increases with the total base shear. The (N-M) interaction has small effect in the first stage of the analysis. As the floor stiffness increases, the force

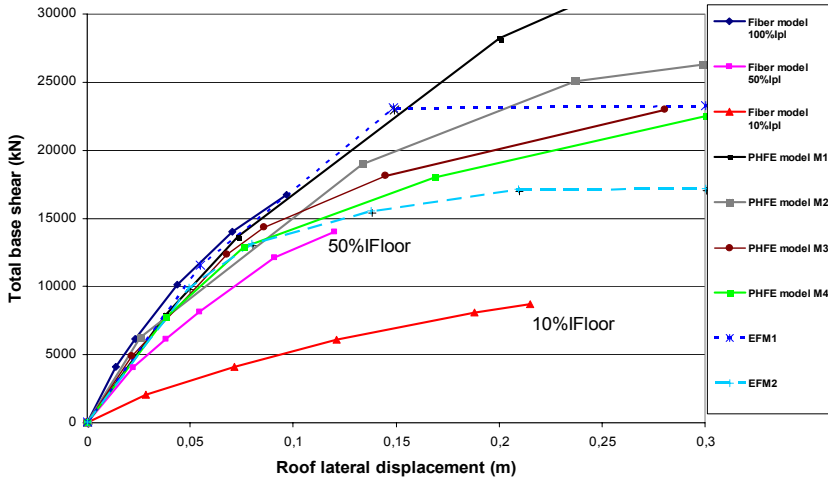


Figure 4: Capacity curves for rigid-floor type model.

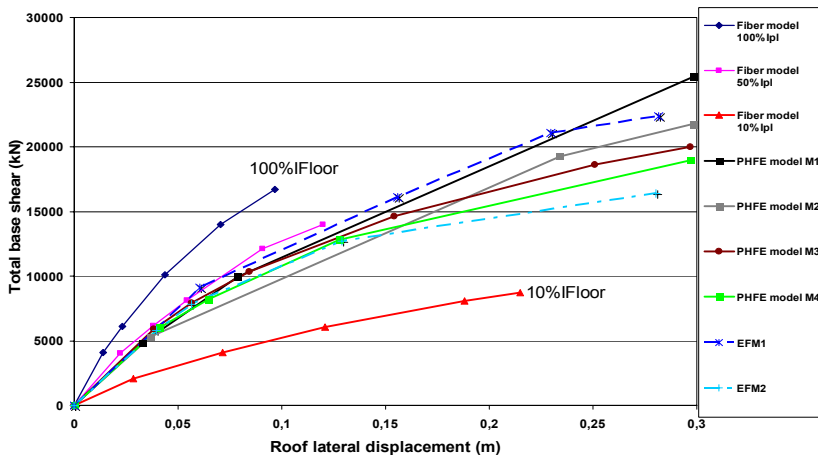


Figure 5: Capacity curves for semi rigid-floor type model.



redistribution capacity of the structure increases, the normal forces increase, and then the effect of (N-M) interaction becomes significant. When axial force is still small, the (N-M) interaction is negligible. In other words, the (N-M) interaction rule has no effect for flexible floor-type structures (Fig. 6). This application tends to demonstrate the ability of the EFM, in comparison to ETABS's results, to reproduce the interaction between the floor stiffness, the structural wall coupling, the force redistribution, and the failure criteria on the global response of the building.

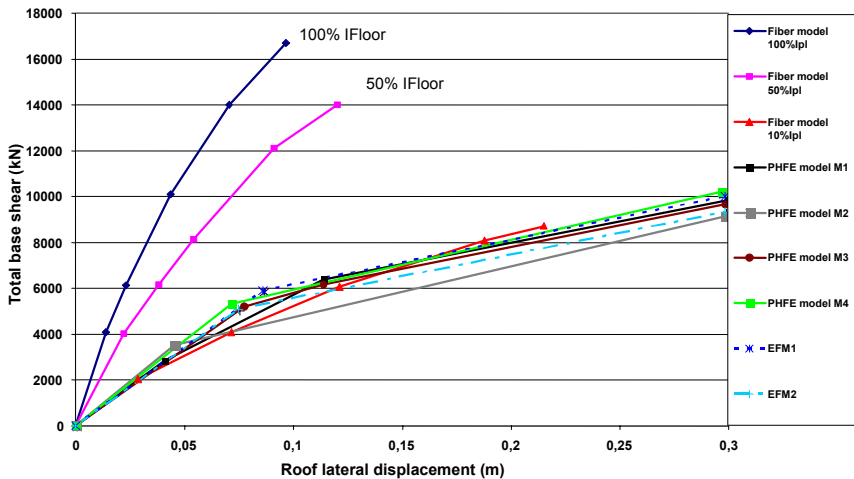


Figure 6: Capacity curves for flexible-floor type model.

#### 4 Pushover analysis of an URM building

The proposed model can be used also for URM structures modelling [12, 13]. A full-scale two-storey unreinforced masonry tested at the Pavia University was chosen for model validation (Fig. 7). The URM piers and spandrels are subdivided into a series of slices. The slices represent a homogeneous bricks and mortar one-phase material. The yield criteria considered are expressed for flexure (Eq. 5) and for shear behaviour (Eq. 6) according to the Magenes model [9–11] as follows:

$$f(N, M) = M + \alpha N + \beta N^2 \leq 0 \quad (5)$$

$$f(N, V) = \min \left( V - \frac{\alpha_1 N + \beta_1 N^2}{\gamma_1 + N} \quad V + \alpha_2 + \beta_2 N \quad V - \alpha_3 \sqrt{1 + \beta_3 N} \right) \leq 0 \quad (6)$$

Two constants  $\alpha$  and  $\beta$  are required for flexure failure criteria, while nine constants  $\alpha_i$ ,  $\beta_i$  and  $\gamma_i$  (for  $i=1,3$ ) are required for shear failure criteria [9–11].  $N$  is the axial compressive load acting on a pier element. The elastic properties of the structure used in the model are given in the reference [11–13].



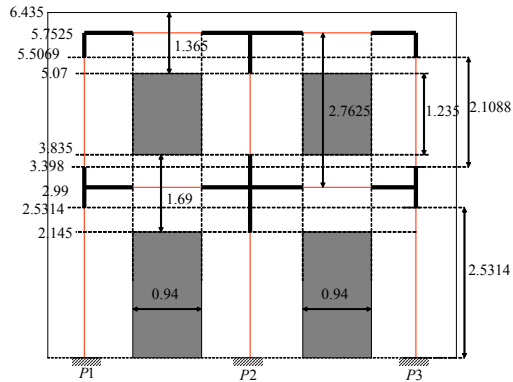


Figure 7: Elevation view of the wall D and geometry (in m).

Table 1: Case studies for both EFM and PHFE models.

Case	Model type	Rigid Zone in Pier	Rigid Zone in Spandrel	(N-M) failure criteria	V Shear effect	Maximum strength (*)
1	PHFE	-	-	-	-	-20.1%
2	PHFE	-	-	✗	-	-15.5%
3	EFM	-	-	✗	-	-10.7%
4	PHFE	-	-	-	✗	-21.7%
5	PHFE	-	-	✗	✗	-22.4%
6	EFM	-	-	✗	✗	-20.5%
7	EFM	✗ ( $2E_m$ )	-	✗	✗	-9.9%
8	EFM	✗ ( $2E_m$ )	-	✗	-	+18%
9	EFM	✗ ( $4E_m$ )	-	✗	✗	-9.9%
10	EFM	✗ ( $10E_m$ )	-	✗	✗	-9.0%
11	EFM	✗ ( $2E_m$ )	✗ ( $2E_m$ )	✗	✗	-8.5%
12	EFM	✗ ( $10E_m$ )	✗ ( $10E_m$ )	✗	✗	-7.1%
13	PHFE	✗ ( $10E_m$ )	✗ ( $10E_m$ )	✗	✗	-9.3%

Legend: PHFE: Point Hinge Finite Element model, EFM: Equivalent Frame Model, (-) Option considered, (✗) Option not considered, (\*) The maximum strength ratio=analytical /experimental maximum strengths %,  $E_m$  is the masonry Young Modulus.

The use of rigid offsets is a crucial issue in equivalent frame modelling. In this study, full rigid offsets are considered. The capacity curves (total base shear versus top lateral displacement) are developed for different cases (Table 1).

In the light of the obtained results, the following recommendations are made:

1. The effect of the axial force - bending moment, (N-M), interaction is showed by the case '1' and '2'. As displayed in figure 8, as axial compressive load increases, flexural strength of the piers also increases with regards to the failure criteria (Eq. 5).
2. The nonlinear effect of shear mechanism is illustrated by cases '3' and '4' in the absence of rigid offsets, and by cases '7', '8' and '9' in the presence of

- rigid offsets (Fig. 9). As expected, the contribution of shear mechanism tends to decrease the capacity of the structure due to the occurrence of shear damage. This feature is successfully captured by the simplified model.
3. As displayed in figures 8 and 9, the rigid offsets have a significant effect on the global response not only on stiffness, but also on strength capacity of the structure. This is expected as the horizontal element stiffness closely affects the contribution of the frame mechanism to structural response (cases '10', '11' and '12'). The capacity curves obtained from EFM (case '12') versus PHFE model (case '13') are satisfactory.
  4. In cases '12' and '13', the two capacity curves are close to a certain extent in spite of the smeared approach in the EFM. Both cases '5-6', and '12-13', show the comparison of the modelling performance, including shear effect and (N-M) interaction rule and using either the EFM and the PHFE model with or without rigid zones.

## 5 Conclusions

This paper presents a simplified formulation of an equivalent frame model. The model permits to consider many relevant features of structural behaviour such as structural wall coupling, zero moment location shifting, axial force-bending moment interaction, axial force-shear force interaction, and failure modes prediction without the use of finite element method. However, in the case of URM buildings, it is well known that smeared crack approach suffers from a few limitations. The smeared crack model is enable to represent effectively the rocking and bed joint sliding mode of failures.

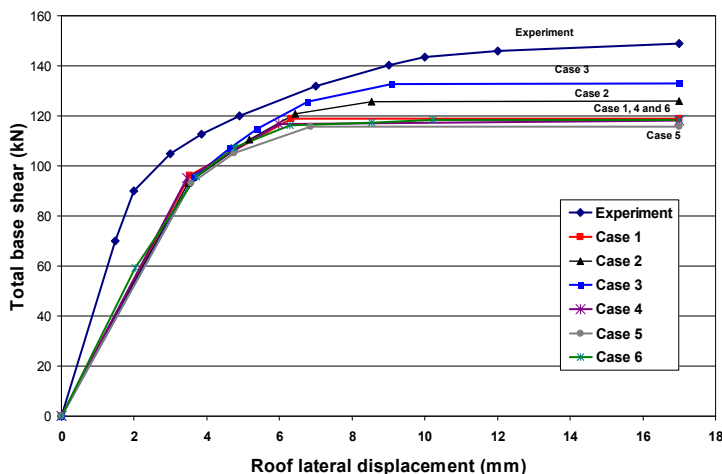


Figure 8: Capacity curves of the wall D with no rigid offsets.



For the development of capacity curves, the obtained results from the proposed model show good agreement with experiment and numerical results (Fig. 8, 9). The model has proven its capability to satisfactorily predict the maximum strength. The calculated maximum strengths, in particular for the masonry structure (in the range of 9%), could be judged as good results since the model is based on simplified approaches in comparison to finite element models. In all cases, obtained results should be considered from an engineering point of view as is generally done for all simplified existing models.

Finally, the proposed model is formulated in order to extract capacity curves with damage identification. The model can be used to assess URM structures, RC structures as well as dual structures that are commonly adopted in many countries.

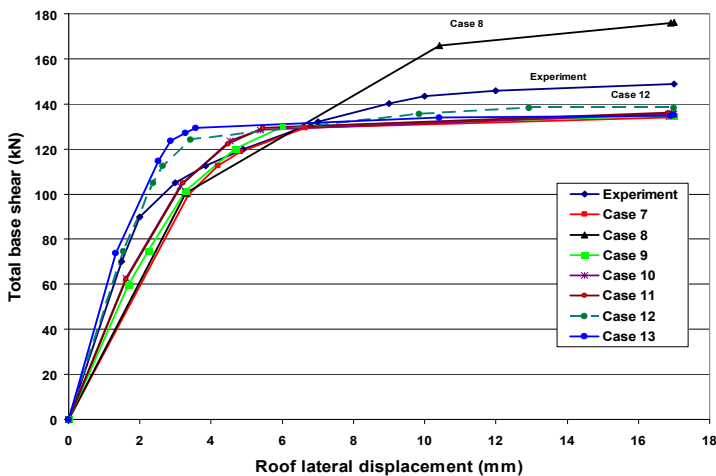


Figure 9: Capacity curves of the wall D with rigid offsets.

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