Operational modal analysis and assessment of an historic RC arch bridge

C. Gentile & N. Gallino

Department of Structural Engineering, Polytechnic of Milan, Italy

Abstract

This paper summarizes the dynamics-based assessment of a historic reinforced concrete arch bridge. The bridge, known as *Victory bridge* and originally constructed in 1923, spans 75 m over a deep valley in the Province of Lecco, Italy. The outlined investigation included ambient vibration testing, output-only modal identification, finite element modelling and updating of the uncertain structural parameters of the model.

Keywords: ambient vibration testing, finite element model updating, output-only modal identification, arch bridge.

1 Introduction

A great number of reinforced concrete (RC) arch bridges were erected in Italy in the first half of the 20th century, as the concrete arch was the most common solution adopted for bridges spanning over 40 m or more. Of course, these bridges were designed for live loads quite different from automobile traffic they carry today and often require a structural condition assessment. A good example is the *Victory bridge* [1], completed in 1923 and located about 75 km from Milan.

Within an advanced maintenance program, the Province of Lecco, owner of the bridge, planned extensive investigations of the bridge, including:

- a) review of the bridge history, with an emphasis on the structural modifications of 1984. A great amount of documents and design drawings has been found in the archives and both the original and the current geometry of the bridge were reconstructed with fair accuracy;
- b) visual inspection and topographic survey of the bridge;
- c) ambient vibration testing;
- d) live load static tests and laboratory tests on few cored samples of the bridge concrete to estimate the strength and the elastic properties;

- e) finite element modelling of the bridge in its present configuration, model updating and use of the updated model as a baseline model for the future monitoring of the structure;
- f) design and installation on the bridge of a permanent Structural Health Monitoring (*SHM*) system.

Tasks (a)-(c) and (e) have been completed to date and are addressed in the paper.

Figure 1: *Victory Bridge* (Cremeno, Italy): geometry of the original structure.

2 Description of the bridge and historic background

The investigated bridge [1] was designed in 1922 by Prof. A. Danusso and has been considered for a long time one of the most appreciable Italian examples of RC arch bridges. The structure is known as *Victory Bridge* because it is dedicated to the Italian soldiers who fell in the 1st World War.

The bridge is 75 m long and connects the small towns of Cremeno and Maggio, about 75 km from Milan; the bridge overpasses the deep and impervious valley of the Pioverna Creek and substituted a pre-existent wooden suspension footbridge. Plan and elevation of the original bridge are shown in Fig. 1. The deck was 4.80 m wide and consisted of a cast in place RC slab supported by 4 longitudinal girders and 17 transverse floor beams. The parabolic arch structure consists of 2 independent solid arch ribs, transversally connected together with cross struts, having rise of about 20.0 m and clear span of 53.6 m. The two arches, with variable rectangular cross section, are canted inward to better resist to wind actions. The main girders of the deck are connected to the arches and to the foundation system by 7 couples of slender columns.

The structural architecture of the bridge was highly appreciated especially for the aesthetic and structural shape of the arches, considered as brilliant and dared. In addition, the construction process was very fast since a provisional timber bridge was employed to support the centring of the arches: the construction of the bridge officially began on June 11th, 1923 and the centrings were removed on December 9th of the same year. The archives reported that the deflection measured after the removal of the centrings was only 2.0 mm. The 28-days compressive strength of the concrete was in the range of 37.3–40.1 N/mm². The reception load test consisted in the uniform loading of the deck by sand and gravel and the maximum load of 6.30 kN/m² was attained with a deflection of about 1.5 mm.

In 1984 Victory Bridge underwent important structural modifications, mainly consisting of enlargement of the deck (from 4.80 m to 8.90 m) and strengthening of the arches. Furthermore, horizontal and transverse struts were added to improve the connection between the columns and to limit their slenderness. The main geometrical characteristics of the bridge in its current configuration are shown in Figs. 2 and 3.

3 Ambient vibration tests and modal identification

Two series of ambient vibration tests were carried out, in April and May 2006, using a 16-channel data acquisition system with 14 piezoelectric accelerometers (model WR-731A, each with a battery power unit WR-P31).

Two-conductor cables connected the accelerometers to a computer workstation with a data acquisition board for A/D and D/A conversion of the transducer signals and storage of digital data. The vertical response of the bridge was measured at 24 selected points (Fig. 2). Since the tests were conducted using 14 accelerometers simultaneously, a series of two set-ups was required to cover all the measurement points and 4 reference sensors (located at points 19–22 in Fig. 2) were considered in the dynamic tests. In both tests, the ambient acceleration time-histories were recorded for 3000 s per channel, using a sampling rate of 200 Hz to provide good waveform definition.

The modal identification was carried out by using the *Frequency Domain Decomposition (FDD)* [2] technique implemented in the ARTeMIS software [3]. The *FDD* technique involves the following main steps: (a) the estimate of the spectral matrix (i.e. the matrix of cross-spectral densities) in the frequency domain from the measured signals; (b) the Singular Value Decomposition (*SVD*) of the spectral matrix at each frequency; (c) the inspection of the curves representing the singular values to identify the resonant frequencies and estimate the corresponding mode shape using the information contained in the singular vectors of the *SVD*.

Since the first (and largest) singular value at each frequency represents the strength of the dominating vibration mode at that frequency, plotting the first singular value yields the resonant frequencies as local maxima. It is worth noting that the *FDD* technique is able to detect closely spaced modes; in such instances, more than a singular value will reach a maximum in the neighbourhood of a

Figure 2: Plan and elevation of *Victory Bridge* after the strengthening (dimensions in cm), and instrumented points in the dynamic tests.

Figure 3: Views of the bridge (on the right, during the test).

given frequency and every singular vectors corresponding to a non-zero singular value is a mode shape estimate.

The two sets of mode shapes resulting from the tests of April and May 2006 were compared using the modal assurance criterion (*MAC*) [4]. The *MAC* is a coefficient analogous to the correlation coefficient in statistics and ranges from 0 to 1. In general, a *MAC* value greater than 0.80 is considered a good match while a *MAC* value less than 0.40 is considered a poor match.

To correlate the results of finite element analysis and operational modal analysis, in addition to the *MAC*, the normalized modal difference (*NMD*) [5] was used. The *NMD* is related to the *MAC* by the following [6]:

$$NMD(\phi_{A,k}, \phi_{B,j}) = \sqrt{\frac{1 - MAC(\phi_{A,k}, \phi_{B,j})}{MAC(\phi_{A,k}, \phi_{B,j})}} \tag{1}$$

Figure 4: Average of normalized singular values (SV) of the spectral matrix of all data sets and selected modes (2nd test, 18.05.06).

Figure 5: Identified vibration modes (2nd test, 18.05.06).

In practice, the *NMD* is a close estimate of the average difference between the components of the two vectors $\phi_{A,k}$, $\phi_{B,j}$ for example, a *MAC* of 0.950 implies a *NMD* of 0.2294, meaning that the components of vectors $\phi_{A,k}$ and $\phi_{B,j}$ differ on average of 22.94%. The *NMD* is much more sensitive to mode shape differences than the *MAC* and hence is used to better highlight the differences between highly correlated mode shapes.

The results of the output-only modal analysis in terms of natural frequencies can be summarized through the spectral plots of Fig. 4, showing the averages of the first 3 normalized Singular Values of the spectral matrices of all data sets recorded in the 2nd ambient vibration test (May 2006). The inspection of Fig. 4 yields to the identification of 11 normal modes (placed at 5.13, 7.08, 8.40, 10.11, 11.08, 11.48, 13.62, 14.65, 16.14, 17.48 and 18.85 Hz) in the frequency interval of 0–20 Hz. It is worth noting that the natural frequencies estimated in the two tests are almost coincident and a similar correspondence was found also for the mode shapes, with the *MAC* values ranging from 0.924 to 0.993.

The observed modes are represented in Fig. 5 and can be basically arranged as bending (B) and torsion (T) modes of the deck.

Figure 6: 3D view of the FE model of *Victory Bridge*.

4 Finite element modelling and structural identification

A 3D finite element model of the bridge was created using the commercially available MIDAS/Civil computer program. The linear elastic model (Fig. 6), based on the design drawings of the original bridge [1] and of the strengthening intervention, was formulated using the following assumptions:

- a) four-node shell elements were used to represent the deck slab and the stiffening diaphragms connecting the deck to the arches;
- b) arches, girders, floor beams, columns and bracing members were modelled as tapered beam elements;
- c) the arch footings and the outer columns footings were considered as fixed;
- d) since the longitudinal translation of the deck is partially prevented by the presence of asphalt backfill between the deck and the abutments, the boundary conditions between the deck and the abutments were modelled by a series of uniaxial springs oriented in the longitudinal direction and attached to both ends of the deck along each node;

Table 1: Correlation between experimental and theoretical modal behaviour.

Experimental		Base Model				Updated Model			
Mode	f_{FDD}	f_{FEM}	D_F	MAC	NMD	f_{FEM}	D_F	MAC	NMD
Type	(Hz)	(Hz)	(%)	(%)	(%)	(Hz)	(%)	(%)	(%)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
B ₁	5.127	5.014	2.20	0.9717	17.06	5.113	0.27	0.9722	16.93
B ₂	7.080	6.798	3.98	0.9797	14.39	7.080	0.00	0.9805	14.12
T ₁	8.398	8.049	4.16	0.9218	29.12	8.396	0.02	0.9221	29.06
T ₂	10.107	9.931	1.74	0.8895	35.24	10.126	0.19	0.9136	30.75
B ₃	11.084	10.448	5.74	0.8909	35.00	10.908	1.59	0.8915	34.88
B ₄	11.475	11.213	2.28	0.8467	42.54	11.669	1.69	0.8533	41.47
T ₃	13.623	13.301	2.36	0.6648	71.01	13.735	0.82	0.8597	40.39
B ₅	14.648	13.959	4.70	0.9060	32.20	14.515	0.91	0.9160	30.29
B ₆	16.138	15.664	2.94	0.8930	34.61	16.251	0.70	0.8966	33.96
T ₄	17.480	16.901	3.31	0.8356	44.36	17.527	0.27	0.8455	42.74
B ₇	18.848	18.128	3.82	0.8899	35.17	18.849	0.01	0.9011	33.13

- e) the Poisson's ratio of the concrete was held constant and equal to 0.20;
- f) a weight per unit volume of 24.0 kN/m^3 was assumed for the concrete of arches, girders, floor beams, columns, bracing members and stiffening diaphragms;
- g) an "equivalent" weight per unit volume of 27.0 kN/m^3 was assumed for the deck concrete slab in order to account for the effects of the asphalt pavement and walkways.

The model results in a total of 1438 nodes, 1448 beam elements and 593 shell elements with 8576 active degrees of freedom.

A preliminary dynamic analysis, was performed to check the similarity between experimental and theoretical modal parameters; in this analysis, the concrete Young's modulus was estimated at 35 GPa while, after some manual tuning, the resultant stiffness K of the longitudinal springs, placed at both ends of the deck, was assumed equal to $8.8 \times 10^5 \text{ kN/m}$.

The correlation between the dynamic characteristics of the base model and the experimental results is shown in columns (4)-(6) of Table 1 via the absolute frequency discrepancy $D_F = |(f_{FEM} - f_{FDD})/f_{FDD}|$, the MAC and the NMD . Table 1 shows a reasonable correlation for all modes, being a one-to-one correspondence between the mode shapes with MAC values greater than 0.66; in addition, the frequency discrepancy ranges between 2.20% and 5.74%. Hence, the correlation between theoretical and experimental behaviour seems to provide a sufficient verification of the main assumptions adopted in the base model.

Successively, the distribution of the Young's modulus over the entire bridge was updated. In order to limit the number of parameters in the structural identification procedure and to have a well-conditioned updating problem, the bridge was divided in 6 regions (Fig. 7), with the elastic modulus of concrete being assumed as constant within each zone. In order to assess whether the parameters E_i ($i = 1, 2, \dots, 6$) and the stiffness K would be identifiable from the measured natural frequencies, the base FE model was checked through a sensitivity analysis [6]. The sensitivity analysis computes the sensitivity

coefficient as the rate of change of a particular response of the model with respect to a change in a structural parameter. Since the natural frequencies are assumed as reference responses, the sensitivity coefficients are defined as $s_{ik} = \partial f_i^{FEM} / \partial X_k$, where f_i^{FEM} ($i=1,2,\dots,M$) is the i -th natural frequency of the model and X_k ($k=1,2,\dots,N$) is the k -th parameter to be identified. Fig. 8 shows the sensitivity coefficients as $100(X_k/f_i^{FEM})(\partial f_i^{FEM} / \partial X_k)$, representing the percentage change in mode frequency per 100% change in the parameter, and clearly reveals that:

- 1) all reference responses are highly affected by E_1 (i.e. the elastic modulus of the deck, see Fig. 7);
- 2) all the parameters E_i ($i = 1, 2, \dots, 6$) affects, with different extent, more natural frequencies of the models so that are good candidate to be selected as updating parameters.

Furthermore, since the stiffness K only affects the frequency of the first mode, this parameter was not included in the updating procedure.

Figure 7: Regions selected for the Young's modulus updating.

The structural parameters were updated by minimising the difference between theoretical and experimental natural frequencies through the procedure [7]. However, after the updating, a complete correlation analysis was carried out between theoretical and experimental modal parameters.

The structural identification procedure converged to the following values:

$$\begin{array}{lll} E_1 = 37.80 \text{ GPa} & E_2 = 38.19 \text{ GPa} & E_3 = 35.94 \text{ GPa} \\ E_4 = 37.93 \text{ GPa} & E_5 = 38.09 \text{ GPa} & E_6 = 38.75 \text{ GPa} \end{array}$$

suggesting a satisfactory quality of the concrete and a good state of preservation of the bridge.

The dynamic characteristics of the updated model are compared with the experimental data in columns (8)-(10) of Table 1. It can be observed that the updated model exhibits an excellent correspondence of its natural frequencies with the measured ones, since the frequency discrepancy $D_F = |(f_{FEM} - f_{FDD}) / f_{FDD}|$ is lower than 1.0% for most modes while the maximum D_F is about 1.70%. Furthermore, the good match between measured and computed dynamic properties is confirmed by the MAC , with values ranging from about 0.846 to 0.981. As a further demonstration, Fig. 9 shows the vibration modes of the model, corresponding to the experimental ones, previously shown in Fig. 5.

Based on the above results, it can be stated that, despite both the complex structural layout of the bridge and the simplifying assumptions of the model, there is an excellent agreement with the experimental results for all identified modes.

Figure 8: Sensitivity coefficients.

Figure 9: Vibration modes of the updated FE model.

5 Conclusions

A rational methodology for the dynamics-based assessment of RC arch bridges has been presented and applied in the paper. The outlined approach (based on ambient vibration testing, output-only modal identification, F.E. modelling, sensitivity analysis and updating of the uncertain structural parameters of the model) seems to be especially suitable for arch bridges (where visual inspection is usually very difficult) and in the framework of continuously operating *SHM* systems. The following conclusions can be drawn:

- a) the ambient vibration tests allowed the identification of 11 normal modes in the frequency range 0–20 Hz;
- b) an excellent agreement was found between the modal estimates obtained from two successive series of tests;
- c) in the analytical study, the application of the simple system identification technique provided a linear elastic model, adequately representing the dynamic characteristics of the bridge;
- d) the values of the identified Young's modulus, ranging between 35.94 GPa and 38.75 GPa, suggested a satisfactory quality of the concrete and a good state of preservation of the bridge.

Acknowledgements

The research was supported by the MIUR (Italian Ministry of University and Research), under grant Cofin 06, and the Lecco Province. Furthermore, the authors would like to thank Dr. Arch. E. Tosca for the valuable help in the historic studies of the bridge.

References

- [1] Santarella, L. & Miozzi, E., *Italian bridges in reinforced concrete* (in italian), Hoepli, Milano, 1924
- [2] Brincker, R., Zhang, L.M. & Andersen, P., Modal identification from ambient responses using Frequency Domain Decomposition, *Proc. XVIII Int. Modal Analysis Conf.*, pp. 625-630, 2000.
- [3] ARTEMIS Program Overview, <http://www.svibs.com>, 2006.
- [4] Allemang, R.J. & Brown, D.L., Correlation coefficient for modal vector analysis, *Proc. I Int. Modal Analysis Conf. (IMAC)*, pp. 110-116, 1983.
- [5] Waters, T.P., *Finite element model updating using measured frequency response functions*, Ph.D. Thesis, Department of Aerospace Engineering, University of Bristol, 1995
- [6] Maia, N.M.M. and Silva, J.M.M. (Eds.), *Theoretical and experimental modal analysis*. Research Studies Press Ltd, 1997.
- [7] Douglas, B.M. & Reid, W.H., Dynamic tests and system identification of bridges, *J. Structural Div.*, ASCE, **108**(10), pp. 2295-2312, 1982.