Theoretical basis and significance of variance of discharge as a bidimensional variable in microirrigation lateral design

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Abstract

In order to support the theoretical basis and contribute to the improvement of educational capability issues relating to irrigation systems design, this work presents an alternative deduction for variance of the discharge as a bidimensional and independent random variable. Then, a subsequent brief application of an existing model is applied for statistical design of laterals in micro-irrigation. The better manufacturing precision of emitters allows lengthening of a lateral for a given soil slope, although this does not necessarily mean that the statistical uniformity throughout the lateral will be more homogenous.
Keywords: error theory, statistical approach, manufacturing coefficient.

1 Introduction

In microirrigation systems, generally the longer the lateral pipeline implemented in a sub-main unit is, the less the final cost of the designed project and hence the overall profitability of the system is improved. In practice, two approaches may be used to optimise the lateral length while maintaining the desirable uniformity of applied water: hydraulic and statistical. Considering the latter, the coefficient of variation of pressure head (*CVHp*) term appears in the model due to the concept of discharge variance, where the source-point discharge is assumed to be susceptible to oscillations in both the pressure head (*H*) and emitter coefficient (*K*). The latter is inherent to the existing variability of emitters coming from the

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manufacturing processes. Specifically, discharge variation values are assumed to follow a bivariate normal distribution without correlation between independent variables and hence these oscillations need to be contemplated when designing laterals. Small differences between what appear to be identical emitters may result in significant discharge variations.

The manufacturer's coefficient of emitter variation is a measure of the variability of discharge of a random sample of a given make, model and size of emitter, as produced by the manufacturer and before any field operation or ageing has taken place (ASAE [2]). It reflects the degree of precision with which the emitters are made by the manufacturer. With respect to average source-point discharge, one can use Taylor's series for deducting the variance with some difficulty degree or, alternatively, it may be deduced more easily via error theory. In order to support the theoretical basis and contribute to the improvement of educational capability issues relating to irrigation design, this work uses a simple engineering algebraic manipulation and thereby presents an alternative deduction of the variance of discharge as a bidimensional variable and a brief application of Anyoji and Wu's [1] model for statistically design laterals in micro-irrigation.

2 Model description and alternative deduction

To describe the emitter discharge (q, L h^{-1}), Keller and Bliesner [8] suggest the widely used power equation as a function of pressure head (H, m):

$$q = K \cdot H^x \tag{1}$$

The variance of discharge (δ_q^2) throughout lateral is given by:

$$\delta_q^2 = \frac{1}{N} \cdot \sum_{i=1}^{N} (q_i - q_m)^2 \quad ; \quad \delta_q^2 = \frac{1}{N} \cdot \sum_{i=1}^{N} (dq)^2 \tag{2}$$

in which q_i is the discharge of the ith emitter; q_m is the mean discharge; and dq is the deviation of discharge values (continuous variable); and N is the number of emitters along a lateral.

Likewise, the variances of independent variables are:

$$\delta_K^2 = \frac{1}{N} \cdot \sum_{i=1}^{N} (dK)^2 \tag{3}$$

$$\delta_H^2 = \frac{1}{N} \cdot \sum_{i=1}^{N} (dH)^2 \tag{4}$$

in which dq, dK and dH are the deviations of discharge, emitter constant and pressure head, respectively.

The coefficient of correlation (ρ) between the random variables K and H measures the magnitude and direction in which they linearly straight ahead together (Moore [10]), being possible to present it as:

$$\rho = \frac{\sum_{i=1}^{N}(K_i - K_m)\cdot(H_i - H_m)}{\sqrt{\sum_{i=1}^{N}(K_i - K_m)^2}\cdot\sqrt{\sum_{i=1}^{N}(H_i - H_m)^2}} \tag{5}$$

Rearranging eq. (5) in a convenient manner:

$$\sum_{i=1}^{N}(dK)\cdot(dH) = \rho\cdot\sqrt{\sum_{i=1}^{N}(K_i - K_m)^2}\cdot\sqrt{\sum_{i=1}^{N}(H_i - H_m)^2} \tag{6}$$

According to the error theory, the total derivative of eq. (1) corresponds to the total error implied when it is used. Particularly for q, this may be described by:

$$dq = \frac{\partial q}{\partial K}\cdot dK + \frac{\partial q}{\partial H}\cdot dH \tag{7}$$

where:

$$\frac{\partial q}{\partial H} = K_m \cdot x \cdot H_m^{x-1} \tag{8}$$

$$\frac{\partial q}{\partial K} = H_m^{x} \tag{9}$$

Whereas the higher-order partial differentiations are given by:

$$\frac{\partial^2 q}{\partial K^2} = 0 \tag{10}$$

$$\frac{\partial^2 q}{\partial H^2} = K_m \cdot x \cdot (x-1)\cdot H_m^{x-2} \tag{11}$$

Substituting eq. (7) in eq. (2):

$$\delta_q^2 = \frac{1}{N}\cdot\sum_{i=1}^{N}\left(\frac{\partial q}{\partial K}\cdot dK + \frac{\partial q}{\partial H}\cdot dH\right)^2 \tag{12}$$

Expanding eq. (12) by taking the binomial quadratic expansion:

$$\delta_q^2 = \frac{1}{N}\cdot\sum_{i=1}^{N}\left[\left(\frac{\partial q}{\partial K}\cdot dK\right)^2 + 2\cdot\frac{\partial q}{\partial K}\cdot dK\cdot\frac{\partial q}{\partial H}\cdot dH + \left(\frac{\partial q}{\partial H}\cdot dH\right)^2\right] \tag{13}$$

$$\delta_q^2 = \frac{\sum_{i=1}^{N}\left(\frac{\partial q}{\partial K}\cdot dK\right)^2}{N} + \frac{\sum_{i=1}^{N}2\cdot\frac{\partial q}{\partial K}\cdot\frac{\partial q}{\partial H}\cdot dK\cdot dH}{N} + \frac{\sum_{i=1}^{N}\left(\frac{\partial q}{\partial H}\cdot dH\right)^2}{N} \tag{14}$$

All terms involving partial differentiation on the equation above are possible to be taken out of sum operators because they do not contemplate indices i[th]. This might be verified by reviewing eq. (8) and eq. (9). Thus:

$$\delta_q^2 = \left(\frac{\partial q}{\partial K}\right)^2 \cdot \frac{\sum_{i=1}^{N}(dK)^2}{N} + 2 \cdot \frac{\partial q}{\partial K} \cdot \frac{\partial q}{\partial H} \cdot \frac{\sum_{i=1}^{N} dK \cdot dH}{N} + \left(\frac{\partial q}{\partial H}\right)^2 \cdot \frac{\sum_{i=1}^{N}(dH)^2}{N} \tag{15}$$

Substituting eq. (6) on eq. (15):

$$\delta_q^2 = \left(\frac{\partial q}{\partial K}\right)^2 \cdot \frac{\sum_{i=1}^{N}(dK)^2}{N} + 2 \cdot \frac{\partial q}{\partial K} \cdot \frac{\partial q}{\partial H} \cdot \frac{\rho \cdot \sqrt{\sum_{i=1}^{n}(K_i - K_m)^2} \cdot \sqrt{\sum_{i=1}^{n}(H_i - H_m)^2}}{N} + \left(\frac{\partial q}{\partial H}\right)^2 \cdot \frac{\sum_{i=1}^{N}(dH)^2}{N} \tag{16}$$

Assuming that the variables K and H are independent each other, especially when system pressure is low and emitter material is rigid enough to not provoke deformation, ρ is zero and the second term of second member vanishes in the equation above. Hence, eq. (16) reduces to:

$$\delta_q^2 = \left(\frac{\partial q}{\partial K}\right)^2 \cdot \frac{\sum_{i=1}^{N}(dK)^2}{N} + \left(\frac{\partial q}{\partial H}\right)^2 \cdot \frac{\sum_{i=1}^{N}(dH)^2}{N} \tag{17}$$

Substituting equations (3), (4), (8) and (9) on the equation above, we obtain:

$$\delta_q^2 = \left(H_m^x\right)^2 \cdot \delta_K^2 + \left(K_m \cdot x \cdot H_m^{x-1}\right)^2 \cdot \delta_H^2 \tag{18}$$

Providing that the coefficients of variation are considered, it highlights that:

$$CVq = \frac{\sqrt{\delta_q^{\ 2}}}{q_m} \tag{19}$$

$$CV_K = \frac{\sqrt{\delta_K^{\ 2}}}{K_m} \quad ; \quad \text{or} \quad \delta_K^{\ 2} = \left(K_m \cdot CV_K\right)^2 \tag{20}$$

$$CVH = \frac{\sqrt{\delta_H^{\ 2}}}{H_m} \text{ ; or } \delta_H^{\ 2} = \left(H_m \cdot CVH\right)^2 \text{ ; or } CVH^2 = \frac{\delta_H^{\ 2}}{H_m^2} \tag{21}$$

Substituting eq. (20) and eq. (21) on eq. (18):

$$\delta_q^2 = \left(H_m^x\right)^2 \cdot \left(K_m \cdot CV_K\right)^2 + \left(K_m \cdot x \cdot H_m^{x-1}\right)^2 \cdot \left(H_m \cdot CVH\right)^2 \tag{22}$$

Reorganising the equation above in a convenient way:

$$\delta_q^{\ 2} = K_m^2 \cdot H_m^{2 \cdot x} \cdot \left(CV_K^2 + x^2 \cdot CV_H^2\right) \tag{23}$$

The deducted equation above is going to be utilised in the coming steps, and represents the variance of point-source of the discharge as a bidimensional variable, that depends upon both emitter coefficient (K) and pressure head (H) if clogging and temperature effects on emitter discharge are neglected (i.e. design stage).

3 Applicability of variance of the bidimensional discharge

The expectation [$E(q)$] of q utilising a Taylor's series expansion is:

$$E(q) = K_m \cdot H_m^x + \frac{1}{2} \cdot \left[\frac{\partial^2 q}{\partial K^2} \cdot \delta_K^2 + \frac{\partial^2 q}{\partial H^2} \cdot \delta_H^2 \right] \tag{24}$$

When all events have the same probability of occurrence, the expectation (or expected value) equals to arithmetic average of the values, but the second term of second member stand for the variation of discharge due to the effect of both K and H variation. The expectation of q can also be called q_m.

Substituting equations (10), (11) and (21), into (24) becomes:

$$q_m = K_m \cdot H_m^x \cdot \left[1 + 0.5 \cdot x \cdot CVH^2 \cdot (x-1)\right] \tag{25}$$

Substituting both eq. (25) and eq. (23) in eq. (19), the latter comes to:

$$CVq = \frac{\sqrt{K_m^2 \cdot H_m^{2 \cdot x} \cdot \left(CV_K{}^2 + x^2 \cdot CVH^2\right)}}{K_m \cdot H_m^x \cdot \left[1 + 0.5 \cdot x \cdot CVH^2 \cdot (x-1)\right]} \tag{26}$$

As done in this paper for the expectation of q (see eq. (24)), Anyoji and Wu [1] have demonstrated eq. (26) by using the approach of Lindley [9] about Taylor's Series to estimate the variance of a variable that is function of two independent variables, therefore, differently from the herein presented approach, based on error theory and fully deducted for the variance. The approach of Lindley [9] is certainly and fully valid but is more complex for deducting the variance than error theory, whereas the closed equation for deducting expectation through Taylor's Series can be achieved more easily.

The variable *CVK* is often referred as *CVf*. Additionally, *CVH* might be called as *CVHp* (coefficient of variation of pressure on project) at the design stage, most likely to be different from the *CVH* of the working irrigation system. By considering this and conveniently rearranging the equation above, we finally have the expression that, indeed, is the first step to statistically design laterals in irrigation systems:

$$CVq \cdot \left[1 + \frac{1}{2} \cdot x \cdot CVHp^2 \cdot (x-1)\right] - \sqrt{CVf^2 + x^2 \cdot CVHp^2} = 0 \tag{27}$$

Since *CVq* is a user-defined value according to a uniformity criteria, generally not more than 0.07 (or 7%), it means, an emitter flow variation of 20% (Wu [14]). The attributes *CVf* and x are known elements from manufacturers, whereas *CVHp* becomes the variable of interest. As it is not possible to explicit derive *CVHp*, it must be found through any iterative method (i.e. Newton-Raphson, Bisection or Secant).

Even though Frizzone et al [6] have utilised eq. (26) for their calculations, which is basically the same of that one developed by Anyoji and Wu [1] and the rearranged eq. (27), the latter is mathematically more suitable than the former in terms of programming because expresses a typical algebraic equation to be solved in zero [i.e. f(*CVHp*) = 0]. Burden and Faiures [4] provide more details about the advantages and limitations of some of the main iterative methods in terms of facility and speed of convergence. As far as we have found, such value might be easily obtained through root-finding algorithm existing in Microsoft Excel® (Tools > Goal Seek…) with no difficulty of convergence.

If the second term inside the brackets of the denominator of eq. (26) is small to note that when x tends to 0.5 (i.e. working in full turbulence), the expression in the brackets $\left[1+0.5\cdot x\cdot CVH^2\cdot(x-1)\right]$ tends to 1 so that the expectation is provided directly by the original functional form of q with the mean values of K and H [see eq. (1)] and the CVq can be reduced to same equation of that one derived by Bralts [3].

When emitters are ideally compensating (i.e. x = 0), there is no solution for eq. (27) because this becomes meaningful. When x = 0.5 (orifice-type emitters), the value of $[1 + 0.5 \cdot x \cdot CVH^2 \cdot (x - 1)]$ tends to be higher than that one when $0 < x < 0.5$. In other words, compensating emitters are more influenced by the value of $[1 + 0.5 \cdot x \cdot CVH^2 \cdot (x - 1)]$ as a consequence of higher resulting CVH values. However, this is only noted when $[1 + 0.5 \cdot x \cdot CVH^2 \cdot (x - 1)]$ is calculated after eq. (27) is solved. When x = 1, the value of $[1 + 0.5 \cdot x \cdot CVH^2 \cdot (x - 1)]$ is zero.

After finding *CVHp*, it is used as an input value to optimise the length of laterals according to (Anyoji and Wu [1]):

$$\frac{(m+1)^2}{(2\cdot m+3)\cdot(m+2)^2}\cdot Hf^2+\frac{1}{12}\cdot\Delta Z^2+\frac{(m+1)}{(m+2)\cdot(m+3)}\cdot Hf\cdot\Delta Z-(CVHp\cdot Hm)^2=0 \quad (28)$$

in which m is the exponent with respect to discharge in friction equations; H_m is the mean value of pressure (m) throughout lateral – calculated by inverting eq. (25); Hf is the total friction loss (m) throughout lateral; and ΔZ is the level difference (m), assuming a negative value only in case of downhill slopes.

The following equations must be substituted in eq. (28) prior to solve it:

$$\Delta Z=\frac{So}{100}\cdot L \quad (29)$$

$$Hf=\frac{Khf}{3600000^m}\cdot\frac{q^m}{D^n}\cdot\left(\frac{Se+hfe}{Se^{m+1}}\right)\cdot\frac{L^{m+1}}{m+1} \quad (30)$$

in which L is the length (m) of lateral – the goal value to be found iteratively; So is the soil slope (%), assuming it as uniform; hfe the local head losses (m – equivalent length) at emitter insertions; Khf the friction constant; Se the emitter spacing (m); D the internal lateral diameter (m); n the adjusted empirical coefficient.

Analogously to eq. (27), we found no problem to solve eq. (28) by using either of most traditional methods (Newton-Raphson, Secant or bisection). In addition, it is necessary to comment that the current approach disregards higher order terms of Christiansen coefficient (F), in which $F = 1 / (m+1)$, valid for laterals contemplating a large number of emitters (i.e. N > 20). However, the exact length of lateral and consecutively the number of emitters (N) are not known *a priori* (i.e. L may be very short under high slope situations if at the same time H_m is low and CVq is rigorously low), which then would require the full equation for calculating F regardless of its former simplification.

As N must be a non-decimal number, it is worthwhile to remind that once L value is found from eq. (28), N must be truncated (for N_R) preferably towards to the nearest smaller integer value in order to obtain a little gain in terms of pressure throughout the lateral, which has the addition a benefit of preventing clogging of emitters due small particles of sediments. As a consequence, a new lateral length (L_R) is then recalculated:

$$L_R = N_R \cdot Se \tag{31}$$

Thus, the lateral inlet pressure (H_{inl}, m) and the pressure related to each i emitter (H_i, m) may be calculated, respectively, according to:

$$H_m = H_{inl} - \left(\frac{m+1}{m+2}\right) \cdot Hf_R - 0.5 \cdot \Delta Z \tag{32}$$

$$H_i = H_{inl} - \left[1 - (1 - CR_i)^{m+1}\right] \cdot Hf_R - CR_i \cdot \Delta Z \tag{33}$$

where Hf_R and ΔZ_R are the recalculated ΔZ and Hf, in an analogous way provided by equations (29) and (30), respectively, by replacing L_R instead of L. Likewise eq. (28), ΔZ is here is also assumed as negative for declivity situations; CR_i is the relative length, i.e. the length between the first and the i^{th} emitter, is given by:

$$CR_i = \frac{i \cdot Se}{L_R} \tag{34}$$

Consequently, the discharge (q_i, L h^{-1}) in each emitter may be obtained by:

$$q_i = K \cdot H_i^x \tag{35}$$

An equation that accounts for both emitter variation and system pressure variation is one denoting the uniformity of emission, which was modified and redefined for design purposes, as pointed out by Bralts [3]. However, despite the existence of a number of equations to assess the uniformity (i.e. absolute emission uniformity, statistic uniformity, etc.), Favetta and Botrel [5] have compared all these equations and have shown a strong correlations among them, which allows the adoption of that one that most suits the user's needing. At the design stage, the statistical uniformity (US, %) of water depth can be used (Juana et al [7]), according to (Wilcox and Swailess [13]):

$$US = 100 \cdot (1 - CVq) = 100 \cdot \left(1 - \frac{\sum_{i=1}^{n}\left(q_i - \frac{1}{N} \cdot \sum_{i=1}^{N} q_i\right)}{\frac{1}{N} \cdot \sum_{i=1}^{N} q_i}\right) \tag{36}$$

There are more factors affecting the uniformity of micro-irrigation than that of sprinkler irrigation besides manufacturer's variation (Wu [14]). These other factors might be grouping of emitters, plugging and temperature. The effect of temperature on emitter flows can be neglected when a turbulent flow emitter is used (Peng et al [11]), whereas plugging of emitters is developed with respect to time and occurs in the form of partial plugging and complete plugging. As partial pluggings are difficult to evaluate, plugging evaluations are taken by using completely plugged emitters (Wu [14]).

For illustrative purposes values for length were simulated from variations in the input variable *CVf* (2%, 4%, 6%, 8% and 10%), considering a low-pressure irrigation system designed in a 2% downhill uniform slope (*So* = – 2). The emitter spacing was 1 m (i.e. coffee plantation paddock design) mounted throughout a ½" diameter (or 0.0165 m) lateral and was considered grouped in one per tree; the local head pressure losses due to emitter insertions (*hfe*) was 0.1 m; the project mean point-source discharge of 4.0 L h^{-1} was considered in principle, given that the emitter equation $q_i = 1.1134\ H_i^{0.5}$ was provided (orifice-type emitters working under full turbulent flow, according to x value); and the Blasius constants were assumed (*Khf* = 0.00078; m = 1.75; n = 4.75). As a project criterion, it was considered a relaxed *CVq* of 10%. The rest of variables are calculated; such as *CVHp*, lateral, losses by friction, inlet head pressure and uniformity.

The results of the simulation are presented in Table 1. The user can perform the same simulations by running the following procedure: i) calculate *CVHp* from eq. (27); ii) calculate *Hm* from eq. (23); iii) calculate L from the combination of equations (28), (29) and (30); iv) calculate H_{inl} from eq. (32); v) calculate H_i from the combination of equations (31), (34), Hf_R through eq. (30) and eq. (33); vi) calculate q_i (eq. (35)); and finally vii) calculate discharge statistic uniformity of design according to eq. (36).

Table 1: Main results of the simulation performed.

CVf	*L* (m)	H_{inl} (m)	Hf / H_{inl}	*US*	*CVHp*
2.00%	218.00	21.76	43.04%	91.18%	19.52%
4.00%	214.00	21.35	41.69%	91.55%	18.37%
6.00%	205.00	20.45	38.68%	91.55%	15.90%
8.00%	187.00	18.94	32.43%	93.98%	11.88%
10.00%	90.00	13.98	5.88%	99.12%	2.79%

It might be seen from Table 1 that required inlet head pressure tend to decrease as *CVf* increase and the same is verified the relationship between *Hf* and H_{inl}. The statistic uniformity of discharge tends to follow *CVf* increasing, as *CVHp* decreases. Interest to note that, when *CVf* = 10%, all results increase or decrease abruptly. From this value on (i.e. *CVf* > 0.1), there was no convergence for *CVHp*. This *CVf* could be considered a critical value, for example, for the scenario assumed (*So* = – 2, *D* = 0.0165 m, etc.).

The better manufacturing of emitters implies a lower *CVf* value, which allows for lengthening of laterals for a given soil slope, as shown in Figure 1 [the combination of the pairs i vs. q_i Eq. (35)], however this does not necessarily

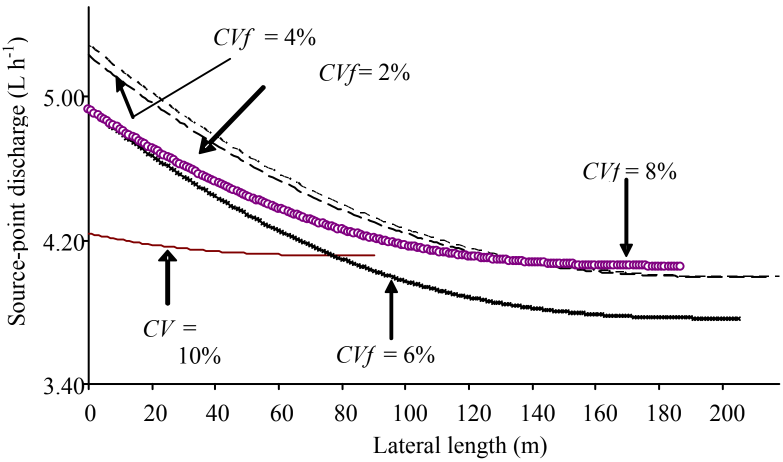

Figure 1: Point-source discharge variation throughout laterals designed according to the statistical approach.

mean that the discharge statistical uniformity throughout lateral will be more homogenous. It is possible to note that the flatter is the shape of the curve the shorter it is the simulated lateral length. Corroborating to this, it is also highlighted a decreasing tendency in statistical uniformity as emitters are more uniformly made and hence resulting in lower *CVf* values (Figure 2).

As outlined by Wu [14], the sum of squares relationship shown in the second term existing into eq. (27) indicates the effect of hydraulic design will be less significant when the emitters have high manufacturer's variations. Hence, the flatter slopes obtained from the higher *CVf*s in Figure 1, corroborated by Figure 2, reinforce such relationship. Another important characteristic to verify in Figure 1 is not only the distribution of source-point discharge values but also the limits of the corresponding head pressure values (H_i), often leading to quite distant values of q_i from the suggested mean discharge (4.0 L h^{-1}). The designed mean source-point discharge decreased from 4.36 to 4.15 L h^{-1}, as well as has happened to the emitter flow relative variation [(generally called $q_{var} = (q_{max} - q_{min})/q_{max}$], going from 24.48 to 2.91%.

From the lowest (i.e. least variable) to the highest value of *CVf*, this simple emitter attribute alteration has reduced the lateral length for more than a half although a 2% *CVf* still led to an acceptable value of micro-irrigation project uniformity (i.e. greater than 90%). The choice of a 10% *CVf* would lead to the lowest pressure head at the beginning of the lateral (13.98 m), a value 64% lower than the necessary pressure head if a 2% *CVf* emitter were selected. Thus, a 10% *CVf* emitter would result in a more economical system with respect to energy expense considered alone, but would be the shorter lateral line on the other hand. Wu [14] report that when the manufacturer's variation of selected emitters is selected to be < 10%, which is very easy to achieve, the system uniformity < 20% in CV can be achieved when the q_{var} is 30%.

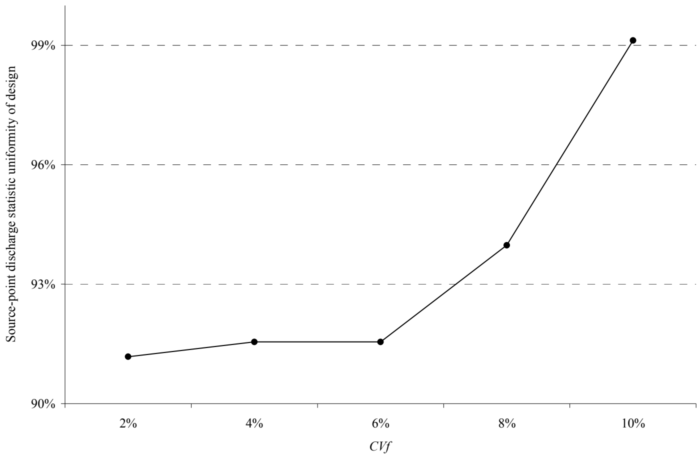

Figure 2: Variation of statistic uniformity as a function of *CVf* for a specific condition of irrigation project design.

Factors such as *CVf*, *CVH*, *CVq*, H_i and *x* should be carefully considered when a micro-irrigation system is designed and managed in order to ensure an application of water that is as uniform as possible. However, regular evaluation of irrigations should take place when systems are working so that they can be correctly maintained and can closely perform according to its original design (Pereira [12]). Moreover, one can also create an optimisation model to find the best trade-off between lateral length and *CVf*, thereby maximise profit, however this is not the primary goal of this work. The costs of different emitters, supposedly varying with the level of manufacturing, need to be compared as well, since it is useful to assess the profitability of an irrigation system by taking into account the fix costs involved. It is often necessary when dealing with manufacturing variation of a particular emitter to reinforce its applicability in practical situations of systems design. Therefore, it is always worthwhile to outline some crucial concepts and methodologies by bringing them in an accessible manner for educational purposes, i.e. by deducting eq. (27) in order to support that not always will be the lowest (or even highest) *CVf* the best choice.

4 Conclusions

From partial differential equations applied to error theory, it is possible to deduct the variance of discharge in order to obtain the coefficient of variation of pressure on project instead of using the deduction from Taylor Series for bidimensional variables, then to simulate lateral length in microirrigation projects. The larger the manufacturing coefficient variation, the longer will be the simulated length, which can be economically interesting until certain limits of discharge uniformity at project stage, as this performance index will be lower for longer laterals.

Appendix

List of symbols:

q	Source-point discharge [L^3 t^{-1}]
K	Coefficient in emitter discharge formula [L^{3-x} t^{-1}]
x	Exponent of emitter discharge equation
H	Head pressure [L]
ρ	Correlation
δ^2	Variance
d	Deviation or differential
CVq	Coefficient of variation in discharge
CVH	Coefficient of variation of pressure heads
$CVHp$	Coefficient of variation of pressure heads at design stage
$CVK = CVf$	Coefficient of variation in discharge due to manufacture
$E(q)$	Expectation of q [L^3 t^{-1}]
L	Lateral length of emitter insertion [L]
Khf	Constant in empirical equation of head loss
m	Exponent of discharge in empirical equation of head loss
n	Exponent of diameter in empirical equation of head loss
N	Number of emitters
Hf	Total head losses along a working lateral [L]
ΔZ	Elevation or level difference [L]
So	Slope [L, if divided by 100; otherwise is dimensionless]
Se	Emitter spacing [L]
hfe	Equivalent length of emitter [L]
D	Lateral internal diameter [L]
F	Christiansen' factor
CR	Relative length
US	Statistic uniformity of discharge

List of subscripts:

H	Head pressure
i	Order of a number of series
inl	inlet
K	Coefficient in emitter discharge formula
m	Mean
R	Recalculated
x	Exponent of emitter discharge equation

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