Contact problems in geomechanics focused on bumps occurrence

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Abstract

The phenomenon known as bumps or rock bursts are one of the greatest dangers to human health during mining operations. They are caused by the accumulation of energy of forces (stresses) in close vicinity to a face of excavations of seams. This causes a sudden release of energy, which is strongly dependent on a development of material properties in both rock overburden and coal. In this case the coal material behavior can be neglected as for the time development. Hereditary problems have to be taken into account, as velocity of excavation is one of the biggest problems in this field of research. The faster the velocity is, the shorter time for redistribution of forces can be reached. This causes a more dangerous situation, as for the bumps initiation. These ideas are valid for pure openings, but not for wall reinforcement during mining, support of the ceiling of mines, etc. These additional elements also have to be taken into account, but not in this paper. We are concentrating only on time dependency of redistribution of stresses in the vicinity of seam face.

Keywords: rock bumps, coupled modeling, certain time-dependent problems, bumps on dislocations, physical scale modeling, numerical solution.

1 Introduction

In the paper, a time-dependent contact problem is formulated and solved. The main application is focused on the field of geomechanics, namely on bumps occurrence in deep mines. The time factor, which plays decisive role in the solution of the problem of bumps, depends on wide scale of circumstances. One of the most important items describing the sudden failure of the rock, i.e. influencing the real behavior of the rock, and consequently leads to the bumps

occurrence is the velocity of excavation of mines in extreme depths. In the deep mines the way of depositing of packs and its mechanical properties are also decisive. Their mutual coupling of these items can principally influence the safety against bumps. For correct understanding of behavior of the rock aggregate (coal seam vs. overburden) rheology has to be reliably described, i.e. creep and relaxation is necessary to formulate in a proper way. Before formulate the above stated problem, large extent of laboratory experiments and “in situ” measurements have to be conducted. Since the on site tests are too expensive, and virtually impossible, laboratory models are prepared and the results from these tests are transmitted to mathematical models.

Contact conditions between particles of the rock are very important in the formulation of distinct element methods, which is a powerful tool for solving cracking in materials and its deterioration, softening, damage and other hereditary problems, i.e. time-dependent behavior. The discrete methods have been solved in many papers of the second author for static case. Either lagrangian multipliers or penalty formulation were used. New formulation has to be submitted in terms of extended lagrangian principle. Creep is involved into formulation instead of eigenparameters, which can be substituted for quantities describing both hereditary development of the change of material and plastic behavior of the compacted rock. As we concentrate our attention on bumps occurring in coal deep mines, brittle behavior of coal is considered and the plasticity and time-dependent development of stiffness in rock material is involve only in surrounding rock. From couple of examples it was shown in static case that the behavior at the face of longwall mining is close to that near the crack tip, and the differences in material properties of coal and overburden are also not negligible. These factors are also expected to be important in the case of dynamical problems. Some examples will show applications of the procedures proposed.

2 Physical modeling

Principles of a new projecting method of underground construction in soft rocks can be formulated on the basis of results from extensive tests on physical models which were carried out in experimental department of Pardubice University.

Properties of these rocks differ from site to site. Rocks are frequently separated by discontinuity surfaces. These surfaces and the weakening zones cause disintegration or susceptibility to disintegration of the rock mass into structural units of various forms, size and properties. Their properties change also with the stress mode and depend on the stressing force, to which the rock was exposed in the past.

It results from the above-mentioned facts that the properties of the rock environment cannot be measured either on small rock samples or by isolated sporadic tests in situ. In the first case, we cannot evaluate the effect of weakening planes, and in the other one an unpleasant dispersion variance of material properties is caused by inhomogeneities of the rock mass. In both cases, the necessary conditions of the physical similitude are usually not observed. These

conditions would require measuring the rock properties under stress conditions equal to stressing force, to which they are exposed during construction and after completion of construction works. These requirements can very easily be observed on physical models from equivalent materials.

Figure 1: Side of the physical model.

Results from laboratory tests can be used directly as impute data for mathematical solution provided the constitutional relation between the stress tensor and strain (deformation) tensor is linear and provided the relative homogeneity and isotropy (e.g. of solid rocks) are supposed. This is not our case and large simulations in laboratory have to be carried out.

On the other hand the stresses are difficult to obtain from the physical modeling. This is the moment when the mathematical model can help and coupling of both models can approximate the real state of the rock continuum and the structure.

A very important conclusion results from some selected rock mechanics problems using the method of physical and mathematical modeling, and from their mutual comparison. It is possible, by means of a test on a physical model, to establish conditions, which are of primary interest to us (e.g. when the resistance of internal forces against failure is optimal). For conditions thus established, the deformation and stress of the rock environment in the neighborhood of the underground opening are determined by the finite element method, boundary element method, or with the aid of the combination of both. A part of problem can therefore be resolved by tests on a physical model and the results obtained can be used as input values for final solution by some numerical methods. In this way, both procedures can be adequately combined and completed, advantages and drawbacks of both of them acting complementarily.

With respect to the extension of the modeling equipment, material properties of physical material, measurement tools, and time factor it has been selected geometric ratio 1:200 and time scale 1:120. The mutual relation between real parameters and model properties of the material is given by adequate rules of similarity. The modeling stand (basin with glassed front side reinforced by steel frame was 6 m high, 2 m long and 1 m wide). The rear side has been constructed from plastic plates with longitudinal openings in the shape of distinguished seams. Modeling equipment enables researchers to load the terrain by pneumatic pillows. Front view of the model is depicted in Fig. 1. The progress of the failure is visible in Fig. 2.

The results of movements of the equivalent material in the front plane of the stand are determined for vertical and horizontal displacements and have been measured by photogrammetric method. This method enables one to get stereo couples of photographic pictures; one of them is in the starting (virgin) position. From the stereo couples of the model states the movement vectors at selected points for given stages of physical model are derived.

3 Solution of underground continuum by the BEM

In this part we briefly describe an implementation of boundary element method to the solution of specific problems of underground continuum, for which the numerical method appears to be extraordinary advantageous.

The method, among others, reduces the problem by one. Further good application of the boundary element method is the optimization and/or contact problems which concern the boundary only. Then, in spite of the finite element method it suffices to study a change of location of boundary elements only.

The problem is solved as two-dimensional, i.e. a possible dislocation is long enough, and narrow seam is considered. Moreover, the nonlinear behavior is considered in the region, which is sufficiently close to the dislocation, according to Mises theory. Suspicious dislocation is given from the experimental model from physically equivalent materials.

In our following consideration we will concentrate to the physically nonlinear problems (nonlinear evolution is also included in boundary conditions). Let us solve the problem in domain Ω . We originate from the Cauchy equations:

Figure 2: View of the experimental stands with the physical model failure.

$$(\alpha + \mu) \frac{\partial}{\partial x_i} \operatorname{div} u + \mu \Delta u_i + X_i + \sum_j \frac{\partial \sigma_{ij}^0}{\partial x_j} = 0, \quad i=1, \dots, 3 \quad (1)$$

where

$$\operatorname{div} u = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2}, \quad \Delta = \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2}$$

and $u = (u_1, u_2)$ is the displacement field, (X_1, X_2) are components of the volume weight and σ_{ij}^0 are components of the tensor of initial stress.

These equations will be solved in coordinate system Ox_1, x_2 . In the sense of BEM, (1) may be reformulated in an equivalent form:

$$c_{kl}(\xi) u_l(\xi) = (\varepsilon_{ijk}^*, \sigma_{ij}) + [p_{ik}, u_i] - [u_{ik}^*, p_i] - (u_{ik}^*, x_i) \quad (2)$$

where $[.]$ are boundary integrals, $(.)$ are plane integrals, c_{kl} is the matrix of coefficients depending on a position of ξ , p is the vector of external forces, ε is the tensor of deformations and quantities with asterisk denote the relevant quantities of fundamental solution. Such the function was derived by Melan and can be found in publication [1], for instance.

Now the trick starting with the polarization tensor used in [2] is applied for describing the nonlinear behavior of the whole massif. In the polarization tensor

also initial stress can be involved. From the Cauchy equations (respecting shearing stresses to be zero) we have well known relations for virgin state to get initial stresses:

$$u_1^0 = -\frac{1-2\nu}{4\mu(1-\nu)} X_1 x_1^2 + \text{const.}, \quad u_2^0 = 0$$

$$\sigma_{1,1}^0 = -X_1 x_1, \sigma_{1,2}^0 = 0, \sigma_{2,2}^0 = -\frac{\nu}{1-\nu} X_1 x_1 \quad (3)$$

4 Contact problem

Before we start the analysis preliminary considerations will be introduced. In order to explain the process of computation two-dimensional problem will be treated. The three-dimensional problems are solved similarly.

Let the problem be described from experimental study. The field of vertical displacements is depicted in Fig. 3, vertical stress in Fig. 4. In numerical version the problem is illustrated in Fig. 5 by domain Ω , Γ_c is a part of boundary splitting the plane into left Ω^l and right half plane Ω^r , along Γ_p the distribution of given surface forces is done and Γ_c is fictitious slip surface (dislocation) either prescribed (this is our case, the location is estimated from the physical model), or the location is to be searched by an enlarged numerical process. Ω^{seam} is the domain of the seam, for which the dislocation and the bearing capacity is to be assessed.

After discretization of (2) in the sense of boundary element method the problem leads to the system of algebraic equations:

$$\begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & A_{23} \\ H_{31} & H_{32} & A_{33} \end{bmatrix} \begin{Bmatrix} u \\ u_c^- \\ u_c^+ \end{Bmatrix} - \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} + G_{11}^g \begin{Bmatrix} g \\ g_c^- \\ g_c^+ \end{Bmatrix} = \begin{Bmatrix} F \\ F_c^- \\ F_c^+ \end{Bmatrix} \quad (4)$$

where the upper index - denotes “from the left” and + denotes “from the right”, g is the vector of prescribed surface forces along the boundaries Γ and Γ_p , p_c is the vector of surface forces on fictitious contact Γ_c and F includes the effect of volume weight.

As the vector g contains known quantities we can rearrange the previous equations to obtain:

$$\begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & A_{23} \\ H_{31} & H_{32} & A_{33} \end{bmatrix} \begin{Bmatrix} u \\ u_c^- \\ u_c^+ \end{Bmatrix} - \begin{bmatrix} G_{12}p_c^- + G_{13}p_c^+ \\ G_{21}p_c^- + G_{23}p_c^+ \\ G_{31}p_c^- + G_{33}p_c^+ \end{bmatrix} = \begin{Bmatrix} F + G_{11}^g \\ F_c^- + G_{21}^g \\ F_c^+ + G_{31}^g \end{Bmatrix} \quad (5)$$

Figure 3: Cross section of the massif with vertical displacements (physical model).

Suppose now that for example u_c^- and u_c^+ is known. Then the problem is uniquely solvable, so that the matrix H_{11} is regular. For the similar reason the matrices H_{kk} are regular, too. Also, the same assertion holds for the matrices G_{1i} , $i = 1, 2$. This is the general result of solvability of linear problems of elasticity by boundary element method. We can conclude that the matrix H is singular, but the last submatrices are regular matrices. This is why it is possible to rearrange the system in the sense of matrix canonical transformations (in algorithm we use Gaussian elimination) to obtain:

Figure 4: Cross section of the massif with vertical stress (physical model).

Figure 5: Domain and denotation of the example under study.

$$\begin{bmatrix} H_{11} & H_{12} & H_{13} \\ 0 & A_{22} & A_{23} \\ 0 & A_{32} & A_{33} \end{bmatrix} \begin{Bmatrix} u \\ u_c^- \\ u_c^+ \end{Bmatrix} - \begin{Bmatrix} B_{11} \\ B_{21} \\ B_{31} \end{Bmatrix} \{p_c\} = \begin{Bmatrix} C_{11} \\ C_{21} \\ C_{31} \end{Bmatrix} \tag{6}$$

where the balance condition

$$p_c = p_c^- = -p_c^+ \tag{7}$$

was employ. The matrices are known while the vectors u and p remain unknown. From the last form the reducibility follows and we can employ the following system of equations:

$$\begin{aligned} A_{32}u_c^- + A_{33}u_c^+ - B_{31}p_c &= c_{21} \\ A_{22}u_c^- + A_{23}u_c^+ - B_{21}p_c &= c_{22} \end{aligned} \quad (8)$$

Generally, along the contact line only balance condition holds and the compatibility is prescribed with the aid of more complicated relations. For example, suppose that at each nodal point along the contact line holds:

$$\begin{aligned} [u]_n &= u_n^1 - u_n^2 \geq 0 \\ |p_t| &\leq T p_n + c \\ |p_t| \leq T p_n + c &\Rightarrow E\lambda > 0, [u]_n = -\lambda p_t \end{aligned} \quad (9)$$

where T and c are prescribed coefficients (they may vary along the contact), p_n and p_t are projections of tractions to the normal and tangential direction with respect to the contact line, respectively. Then Uzawa's algorithm can be applied to the contact problem – see, e.g., [3].

5 Conclusions

In this paper a complex study based on coupled numerical and experimental studies has been proposed and applied to a real structure of underground work. It is aimed to assess the bearing capacity of dislocation, its position has been detected by experimental studies on scale models, and the results involving positioning of the dislocation have then been taken as input data of a great importance to observing stability (particularly to bumps) in the coal seams.

Many others studies have been carried out and in the future we intend to test with the aid of this procedure another approaches to improve the information about the three-dimensional behavior of the system of dislocations in the rock mass. This will lead to much more realistic information and helps to designers of underground works to prepare better plans.

The approach is very close to a back analysis, which is widely used in other parts of civil and underground structures.

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References

- [1] Brebbia, C.A., Teles, J.C.F. & Wrobel, L.C., Boundary element techniques. Springer Verlag, Berlin, Heidelberg, NYC, 1984.
- [2] Prochazka, & P. Šejnoha, J., Behavior of composites on bounded domain. BE Communications, Vol. 7, No. 1, pp. 6-8, 1996.
- [3] Prochazka, P. & Sejnoha, M., Development of debond region of lag model. Computers and Structures, Vol. 55, No. 2, pp. 249-260, 1995,
- [4] Dolezel, V., Experimental measurement on physical models of the deep mine in Ostrava. Research report, CSAV, Prague, 1991.