Fuzzy utility models: possible applications in evacuation conditions in smart cities

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Abstract

In this paper the fuzzy utility models approach is proposed in relation to route choice in a network in evacuation conditions. Fuzzy logic is introduced and the main advantages are presented: the key concept of possibility allows a better representation of human minds. As a consequence, better results may be obtained in relation to users' behaviour representation. A possible application in evacuation conditions in a smart city is described.

Keywords: evacuation, smart city, fuzzy utility models, possibility, random utility models, route choice.

1 Introduction

Flows, travel times and congestion which occur on a transportation network depend on choices made by users. Generally users make their choices in an uncertain environment. In order to obtain a fair simulation of users' choices, several mathematical models where developed. Two main classes of models can be identified: Random Utility Models – RUM [1, 17] and Fuzzy Utility Models – FUM [3, 4].

Many researchers, investigating the issue of choices on a transport system, used fuzzy logic and mathematics [3, 5–7]. The main difference which characterizes RUM and FUM is related to the way of representing uncertainty: in RUM uncertainty is expressed in terms of probability whereas in FUM uncertainty is expressed in terms of possibility [3, 4].

The main aim of this paper is to specify a mathematical model for route choice in evacuation conditions, in a smart city. Two mathematical formulations



will be proposed: one using RUM and another using FUM. If the analyst is able to forecast users' behaviour, using a suitable model, he may design the network and introduce route guidance policy, in order to improve performances on the network. In this paper we do not want to state that FUM is better than RUM: we only aim at showing differences between them, advantages and disadvantages.

Four sections follow this introduction. Section 2 provides a general view about the state of the art on route choice. Section 3 summarizes RUM. Section 4 summarizes possible applications of FUM in a smart city in emergency conditions. Section 5 summarizes FUM and a model specification is proposed. Conclusions and further developments are put in section 6. In this paper all matrices, vectors and scalars related to RUM are characterized by an R as superscript else, if they are related to FUM, they are characterized by an F as superscript.

2 State of the art

In this section we are going to describe some of the works available in literature: the first part of this section is related to fuzzy logic applications and the second part deals with route choice simulation.

Non-RUM models where born after Zadeh proposed his fuzzy logic theory [8, 9]. Dubois and Prade proposed the possibility theory [10].

Fuzzy logic is used in many works about transportation system simulation:

Atanu and Kumar [11] proposed an algorithm to solve the shortest path problem; Brito *et al.* [12] dealt with the distribution of frozen food using a fuzzy approach in order to obtain an optimal solution; Chang and Chen [13] deal with route choice using a user-optimal approach; Wang and Liao [14] solve a user equilibrium assignment problem.

Several applications of fuzzy logic in relation to choice dimensions are available in literature: for modal choice [7], route choice [3, 4, 6, 15]. Several researchers consider attributes represented by fuzzy numbers and users' behaviour based on the paradigm of rational user, as in RUM.

In relation to route choice simulation, it is worth underlying that models related to choice dimensions should be specified on three levels [16]: generation level, perception level, choice level.

In literature it is also possible finding approaches which simulate two levels instead of three levels: generation and perception levels are aggregated. In other works perception and choice levels are aggregated [17]. In the end some researchers consider a single level, aggregating generation, perception and choice [18].

Coming back to the three levels approach, we are going to describe simulation of each level. At the first level, alternatives are generated. Generation of alternatives can be carried out using different approaches: mono-set or multisets, exhaustive or selective, mono criteria or multi criteria. Alternatives generated compose the choice set or the choice sets. Each alternative which belongs to a choice set is characterized by a specific perception. Hence, at the second level, a perception probability is calculated [17, 19–21]. Perception



probability is calculated using RUM or FUM. In the end, each alternative is characterized by a choice probability, calculated at the third level [11, 17, 23–25].

3 RUM

Random utility theory is the most consolidated theory available in literature for transportation systems simulation. According to RUM each decision maker, assigns to each perceived alternative, belonging to the choice set, a perceived utility U_k^R which is a random variable, in order to take into account aspects not certainly known ([17]). U_k^R is characterized by an expected value $E(U_k^R) = V_k^R$, which is specified through some attributes \mathbf{y}_k^R and some parameters $\boldsymbol{\beta}^R$ and θ^R :

$$\mathbf{V}_{k}^{R} = \nu \left(\mathbf{y}_{k}^{R}; \boldsymbol{\beta}^{R} / \boldsymbol{\theta}^{R} \right) = \sum_{i=1..N} \left(\beta_{i}^{R} / \boldsymbol{\theta}^{R} \right) \cdot \mathbf{y}_{ik}^{R}$$
(1)

In this paper we are focusing our attention on route choice and therefore we are interesting in the possibility to calculate route choice probability using RUM. In the following we are going to present some specification available in literature for route choice calculation like multinomial logit [11], nested logit [26], cross nested logit and link-nested logit [27], probit [28] and modified multinomial logit such as C-logit [21], DC-logit [21], path size logit [23] and mixed logit [17].

Being I the choice set, if U_k^R is independently and identically distributed (iid) according to a Gumbel random variable characterized by mean equal to V_k^R and variance parameter θ^R , the multinomial logit model is obtained [2]:

$$p^{R}(k/I) = (\exp(V_{k}^{R}))/(\sum_{i \in I} \exp(V_{i}^{R}))$$
(2)

The C - logit model specification (Russo et al. 2003):

$$p^{R}(k/I) = (\exp(V_{k}^{R} + CF_{k}))/(\sum_{i \in I} (\exp(V_{i}^{R} + CF_{i})))$$
(3)

In the C-logit model, the introduction of the commonality factor CF_k allows overcoming the independence of irrelevant alternatives (IIA), maintaining a simple mathematical formulation.

The Path Size model [23, 29]:

$$p^{R}(k/I) = (\exp(V_{k}^{R} + PS_{k}))/(\sum_{i \in I} (\exp(V_{i}^{R} + PS_{i})))$$
(4)

In the Path Size model, as in the C-logit, the correction term Path Size (PS) is introduced in the specification, in order to take into account overlapping of alternatives.

The link nested logit model [27]:

$$p^{R}(k/I) = \sum_{l} [\alpha_{lk}^{1/d} \exp(-x_{k}/\phi) \cdot (\sum_{h \in I(l)} \alpha_{lh}^{1/d}] \\ \exp(-x_{h}/\phi))^{d-1}_{l}] / \sum_{l'} (\sum_{h \in I(l')} \alpha_{lh}^{1/d}_{l'} \exp(-x_{h}/\phi))^{d-1}_{l'}$$
(5)

In the link nested logit model, the choice set I is divided in subsets.



Application of the link nested logit model reveals several limits. For example route choice probability depends on the number of links belonging to the considered route.

If random residuals are distributed with a multivariate normal random variable, characterized by zero mean and general variances and covariances, the probit model is obtained [30].

For RUM calibration, the method of Maximum Likelihood is used in many works.

4 Possible application in evacuation conditions in a smart city

In ordinary conditions both applications of RUM and FUM are available. In RUM users' uncertainty is represented by random residuals, whereas in FUM uncertainties are represented by possibility calculation. FUM applications are quite rare; on the contrary RUM use are consolidated.

In emergency conditions, a wide literature is available: travel demand during evacuation was treated in [31]; concerning transport supply, calibration of cost function was carried out in [32]; traffic assignment models in evacuation conditions and DSS for supply-demand interaction were dealt in [33] and [34]; in particular the issues of within-day traffic assignment and signal setting were addressed in [35]; specific attention was paid to path design form emergency vehicles and emergency vehicles routing [36–39]; the ITS may play a key role during an evacuation [40]; moreover, the planning process is dealt in [41, 42] and [43]; the issue of humanitarian logistics, particularly concerning relief vehicles is treated in [44]; moreover, carrying out training activities may improve evacuation planning and implementation [45].

A possible classification of path choice models in emergency conditions is proposed in [46]. Models applied in emergency conditions may be classified in relation to users' classes monoclass or multiclass) and instant of choice (pre-trip, en-route, hybrid). FUM applications in emergency conditions are quite rare. Sun *et al.* [47] dealt with the selection of the best path in emergency conditions considering four fuzzy factors in order to simulate users' choices. In relation to the same theme Wu *et al.* [48], proposed a Multi-target fuzzy decision-making mathematic model.

FUM keep some positive aspects of well-known RUM like simple mathematical formulation and easiness of calculation. One of the main differences between RUM and FUM is related to the main output: probability is the main output of RUM, possibility is the main output of FUM. It is worth noting that in situation characterized by lack of knowledge and high level of uncertainty, users will have a confused idea of alternatives and will associate to each of them a low value of possibility/credibility. This means that likely the sum of possibilities/credibilities will not be equal to one. In relation to this aspect, it is worth underlying that in RUM the sum of probabilities of alternatives belonging to the choice set has to be necessarily equal to one, whereas in FUM the sum of possibilities is not equal to one. In emergency

situations, characterized by a very high level of uncertainty, confusion and quite completely absence of trustful information, possibilities may represent better than probabilities the way human mind works.

Efficacy of FUM application in evacuation conditions is a key issue to be investigated.

Recently, the definition of smart city spread up all over the world. According to the report "Smart cities – Ranking of European medium-sized cities" [49], smart cities definition is based on the definition of six dimensions:

- 1. Smart economy (Innovative spirit, Entrepreneurship, Economic image and trademarks, Productivity, Flexibility of labour market, International embeddedness, Ability to transform);
- Smart mobility (Local accessibility, (Inter-)national accessibility, Availability of ICT-infrastructure, Sustainable, innovative and safe transport systems);
- 3. Smart environment (Attractivity of natural conditions, Pollution, Environmental protection, Sustainable resource management)
- 4. Smart people (Level of qualification, Affinity to lifelong learning, Social and ethnic plurality, Flexibility, Creativity, Cosmopolitanism /Openmindedness, Participation in public life);
- 5. Smart living (Cultural facilities, Health conditions, Individual safety, Housing quality, Education facilities, Touristic attractivity, Social cohesion);
- 6. Smart governance (Participation in decision-making, Public and social services, Transparent governance, Political strategies and perspectives).

In order to make smarter a city several actions should be introduced. Implementation of Information and Communication Technologies (ICTs) may help at reaching the general goal. For example this may help at providing information to users about the actual condition of the network and also may give the possibility to manage traffic providing specific route guidance to users. Many issues related to the kind of information to be provided and users' compliance to guidance should be investigated. It is worth underlying that only if intelligent systems are based on mathematical models which properly represent users' behaviour efficacy of actions implemented would be realized. The possibility to manage traffic in a smart city may lead to a smart mobility and this could allow rescuing many human lives in evacuation conditions.

5 FUM

Fuzzy model specification is based on the assumption that the generic user behave as a quasi-rational decision maker. Each user associates a utility U^F to each alternative: U^F is a fuzzy number characterized by a membership function μ_U may have values in the [0, 1] range. U^F is characterized by a crispy part called V^F , specified through some attributes \mathbf{y}^F and some parameters $\pmb{\beta}^F$ and θ^F .

$$\mathbf{V}^{\mathrm{F}} = \boldsymbol{\omega} \left(\mathbf{y}^{\mathrm{F}}; \, \boldsymbol{\beta}^{\mathrm{F}} \, / \boldsymbol{\theta}^{\mathrm{F}} \right) = \sum_{i=1..N} \left(\boldsymbol{\beta}_{i}^{\mathrm{F}} / \boldsymbol{\theta}^{\mathrm{F}} \right) \cdot \mathbf{y}_{i}^{\mathrm{F}}$$
(6)



Another specification of the crispy part V^F is proposed in [50], where a commonality factor CF was introduced in order to take into account similarity between alternatives: the C-fuzzy model is obtained.

The membership function μ_U numerically coincides with the possibility function [8]. For each alternative it is necessary calculating the possibility that the considered alternative has maximum utility: the possibility is obtained as the possibility of intersection between the perceived utility of the considered alternative and the maximum perceived utility, as expressed by (7).

$$Poss(k) = Poss(U^k \ge U^{max})$$
(7)

From possibility it is possible obtaining probability of alternative k. A method to obtain a choice percentage distribution from possibility is proposed by Klir [51]:

$$p^{F}(k/I) = (q^{F})^{\gamma} / (\sum_{i,N} (q_{i}^{F})^{\gamma})$$
(8)

5.1 Comparison between RUM and FUM

In this section a comparison between RUM and FUM is proposed. At this aim a logit model (superscript L), belonging to the RUM family, and a fuzzy model are considered. First of all, RUM and FUM are characterized by a simple mathematical structure. For this reason, three different steps may be identified. It is worth noting that not all the three steps are available both in RUM and FUM. Anyway the logic process is the same for both FUM and RUM. First of all, the systematic utility has to be specified: at this aim attributes ($y^L - RUM$; $y^F - FUM$) and coefficients ($\beta^L - RUM$; $\beta^F - FUM$), to be calibrated, have to be used. The second step for FUM is the calculation of possibility: when a triangular distribution is chosen for the membership function, the calibrated parameters α_1 and α_2 and the scale parameter θ^F are necessary for possibility calculation. This step is not present in RUM. On the contrary it is necessary in FUM in order to get the last step: the calculation of probability.

Probability is calculated both in RUM and FUM but different inputs are necessary: in RUM probability calculation appears V^L , the variance parameter θ^L ; in FUM probability calculation appears the possibility $q(U^F)$ (obtained in the second step) and the scale parameter γ .

5.2 5.2 Proposed FUM Specification

In order to complete the specification of the fuzzy model the membership function distribution has to be chosen. In this paper we are going to choose a triangular distribution: U^F is a fuzzy interval, characterized by a core value (V^F) and two bounds, U_s on the left and U_d on the right (Figure 1). U_s and U_d are assumed to depend on two shape parameters α_1 and α_2 . U_s and U_d are expressed as a function of V^F , through the shape parameters.

$$U_{s} = \alpha_{1} V^{F} \qquad U_{d} = \alpha_{2} V^{F} \text{ if } U_{c} \ge 0$$
(9)

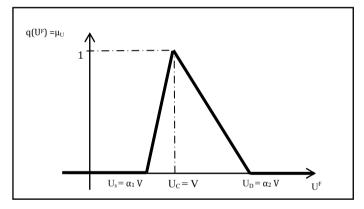
$$0 < \alpha_1 < 1; \alpha_2 > 1$$
 (10)

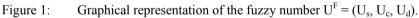


The user perceives several alternatives. Each alternative is represented by a fuzzy interval as the one specified above. Amongst all the perceived alternatives, there is one alternative characterized by the largest perceived utility U^{max} (core value V^{max}). Moreover also the alternative chosen by the user is characterized by a fuzzy interval U^{SC} (core value V^{SC}). In order to calculate the possibility, as explained in section 4, it is necessary to identify an intersection between the triangular fuzzy distribution of the alternative characterized by the maximum utility (U^{max}) and the triangular fuzzy distribution of the chosen alternative (U^{SC}). If no intersections can be found the possibility is equal to zero. On the contrary, if there is an intersection, it is possible calculating it as follows:

$$y = q(U^{max}) = q^{F} = ((U_{C}^{max} \cdot \alpha_{1}) - (U_{C}^{SC} \cdot \alpha_{2})) / ((U_{C}^{SC} \cdot (1 - \alpha_{2})) - (U_{C}^{max} \cdot (1 - \alpha_{1})))$$
(12)

For more detail about possibility calculation please see [50].





5.3 Application on a test network

The model proposed in section 6 is applied on a test network. Moreover also RUM described in section 3 are applied. In figure 5 the test network used for the application is presented: it is characterized by a single o-d pair, four links and three routes, indicated in Figure 2 as a, b and c.

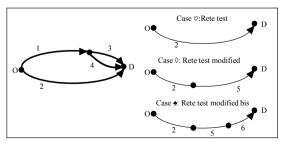


Figure 2: Test network.



Please find the basic test network in Figure 2 indicated as the test network case \heartsuit . Moreover a modified network is considered: it is obtained splitting link 2 in two parts by a new node. In the modified network all paths consists of two links. Please find the modified network in Figure 2 as case \Diamond . In addition to this, a modified network bis is represented in Figure 2 and indicated as case \bigstar : it is obtained, splitting link 2 again by introducing another node. For links costs please see Table 1.

Link			1	2	3	4	5	6
	а	Case ♡	0	1	0	0		
		Case ◊	0	1	0	0	1	
Route		Case 🛧	0	1	0	0	1	1
	b		1	0	1	0		
	c		1	0	0	1		
Link costs			k	10	10-k	10-k+h		

^b1= link belongs to the path; 0= link does not belong to the path.

In the following we are going to discuss results obtained applying models on the test network, represented in Figure 3.

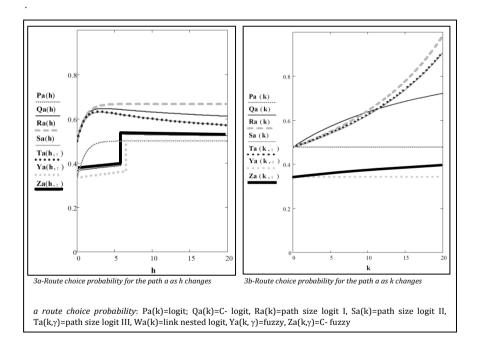


Figure 3: Route choice probability for path a.

Figure 3 is composed of two figures. Figure 3a make us clearer the effect of changes in the value of h on route choice probability of path a. On the contrary, figure 3b make us clearer the effect of changes in the value of k on route choice probability of path a.

In figure 3a, we can observe that if h is great, all models applied provide a probability equal to 0.5. The main difference between figure 3a and figure 3b is the jump which characterizes probability of C – Fuzzy in figure 3a. This is because when possibility overcomes a limit (after which it would be negative that is impossible by definition) it is assumed equal to zero. On the contrary in figure 3b there is no jump because the value of k does not influence route cost.

6 Conclusions

In this paper we proposed a specification for a fuzzy model and compared FUM and RUM. An application of the proposed model on a test network is presented. Moreover possible applications in evacuation conditions in a smart city are described in section 4. Considering the application on a test network, FUM are characterized by a simple mathematical formulation and calculation is not difficult. This makes FUM suitable for practical application.

Features of FUM described in this paper and results obtained from the application on a test network, suggest that FUM application may provide good results also in special context, for instance in evacuation conditions. In addition to this, intelligent systems available in a smart city, jointed with specific abilities of FUM to represent fuzziness in users' behaviour, may allow to realize good practices and lead to obtain good performances on the network, also in evacuation conditions. Uncertainty and lack of knowledge grow up in emergency conditions: therefore introducing possibility instead of probability, using FUM instead of RUM in emergency environments applications may lead to better results.

Delving into advantages and limits of the described methodology is necessary and for this reason the proposed model has to be applied in a real network. Calibration of parameters which characterize FUM and RUM has to be carried out.

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