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Regret model for resilience in road design

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Abstract

The goal of this study is to identify if regret theory may be used to decide between different road design alternatives to ensure the resilience of the design given future uncertainties. It formulates a matrix for the regret factor in the decision-making process. There are different, and often conflicting, parameters and variables that are taken into account whenever different road designs are considered. This paper uses an example of five variables and constraints such as planning (i.e. optimal usage of land use), traffic flow, traffic safety (i.e. to ensure low crash rates), along with constraints such as construction economics and time to completion. It is assumed that each variable may have its own performance indicator based on its own modelling techniques and each is weighed based on the objectives of the decision-makers. However, each model may have inherent uncertainties that may propagate in a multi-attribute decision system. The proposed theoretical framework allows the system to analyze different roads construction alternatives to aid in decision-making. This study does not induce a specific strategy to be utilized by decision-makers, but provides a systematic methodology to help making decisions once an objective-driven strategy has been established. Using a multi-attribute matrix, the framework considers how to practically use regret theory in deciding between different road designs given the uncertainties inherent in each model. It is found that the usage of regret theory can be beneficial. However, furthermore research needs to be conducted to illustrate how this framework would be utilized in a real-world scenario and how it may impact decision in road designs.
Keywords: decision theory, multi-attribute utility, regret theory, road design.

1 Introduction

Decisions pertaining to road construction projects are complex. There are associated risks of going over budget due to unstudied decisions or *uncertainties* whether in the short- or long- terms.

There are limited systematic methods to formulate decisions of road constructions. Mostly, decisions are restricted by current budgetary requirements and making the best traffic flow that suits the budget. Due to restrictions, which may or may not be intended, such as uncertainties or those due to budget constraints, any decision may be regretted. However, most current models do not include the regret factor in the decision process. In this study, a regret factor is modeled to aid in taking decision for roads construction. The model presented in this paper can be extrapolated for the use of any decision process with performance indices. However, due to the limitations of space, a numerical example is not presented in this paper.

Due to uncertainty, there is room for regret. Therefore, probabilistic modeling is used to further understand the regret factor. Variables of uncertainties are defined in this study. Those variables form the basis of any roads construction study and models that are used today. They include traffic data, transportation data, since major urban cities move towards multi-modal transportation networks, geotechnical data, especially important when the roads project involves the construction of tunnels or bridges, buildings in the surrounding areas, environmental data that would possibly be impacted by different road construction alternatives, utilities data, as it forms an integral part of the road corridor, land use (planning) data to ensure that the planning of surrounding area can integrate the new roads construction seamlessly or whether certain conditions would be important to meet some requirements, such as building noise walls separating the highway from residential areas, as well as the possible effects of the zoning of the area, and of course, financial data, which, to many authorities, is the real and final decision-maker, no matter what other variables might conclude.

Different decisions carry various rewards and risks associated with it. Within regret theory, reward and risk are factors of the regret function. Decision theory identifies the best decision to take, assuming informed ideal conditions are available. However, still taking into account uncertainties of the risks associated. Therefore, it is necessary to first determine the risks and rewards for each road design alternative, before the regret factor that is associated to such a decision is evaluated.

Different alternatives might have different rewards, since transportation models utilize multi-criteria evaluations, which include traffic flow and safety, planning and environmental impact, and financial costs. This, in decision-theory, determines the dominance of the action taken. Somehow, decisions are objective-driven with different weights given to each. By such, balancing between risks and rewards, regret is born.

The model introduced in this study is intended to quantify the level of regret of decisions pertaining to the choice of different alternative road designs. It is best utilized for design of an intersection, segment of road, or highway within a limited geographical area, in order to decrease the level of bias between the data available for each alternative. This means that if there were data missing, it would be missing on all alternatives. As with any model, the accuracy of the results is dependent on the accuracy of the data provided. However, the model proposed in this study has the capacity to consider the regret due to lack of information or misinformation by quantifying the confidence level within the regret matrix. In other words, the results would quantify the level of accuracy expected from the model.

The model is not intended to outline a strategy for decision-making. It assumes that a strategy has already been assessed by decision-makers through strategic planning and goals. Thus, once a decision for a new road or the reconstruction of an existing one has been established and there are different alternative designs for the road, then this model has the capability to translate those designs into quantifiable figures and compare the best alternative, based on the strategy applied by the decision-makers by providing importance levels to different performance indicators. However, the model does provide a basis to start developing on optimization of the strategy used, which can be used as a starting point for further optimization research. It is possible to regret the strategy used and the model proposed in this research looks into the sensitivity of the model due to regretting the strategy applied to reach a winning alternative.

For example, if a specific intersection has a high rate of crashes and engineers in the city plan to re-design the intersection, then this model can be used to choose between different designs, after the goal of enhancing safety has been established. The decision-maker would be able to provide a higher weight for traffic safety and a little lower on traffic flow, since their goal is to reduce the number of crashes, and not necessarily enhance the traffic flow, which may not even be a problem with the current design.

2 Decision criteria

There are different criteria for decisions, such as the maximax, minimax, Hurwicz, Savage minimax regret, modal, and Laplace insufficient reason [1–5]. The maximax decision rule means that a maximum reward is sought no matter how high the risks are. When evaluating alternative road designs, and especially road safety, this criterion would not be ideal. The minimax criterion attempts to minimize the maximum loss. The connotation of this rule is to have the lowest risk no matter how attractive are the rewards of other alternatives. When dealing with roads alternatives and road safety issues, taking a conservative approach might be crucial, but not perfect, as it ignores many rewards.

Hurwicz criterion attempts to compromise between the maximax and minimax rules, by giving each a percentage weight [2]. However, neither the maximax nor the minimax rules apply the regret function developed in the same year by Savage [3]. To make sure that the regret function is included in decision-making, a criterion known as Savage Minimax Regret criterion has been developed. This criterion focuses on avoiding regrets that may result in composing a non-optimal decision.

Maximum Likelihood (Modal) criterion considers only the state of nature most likely to occur as the basis for the decision, excluding all other outcomes [4]. Since this criterion almost ignores most of the data, the decisions are being made from a position of ignorance.

Laplace Insufficient Reason criterion is an alternative approach in decision theory. This criterion makes use of explicit probability assessments regarding the likelihood of the sates of nature [5]. Therefore, it makes use of almost all available data. To address uncertainty, probability theory is used for each state of nature. This study considers this criterion to identify the decision process of different roads alternatives.

As decision theory is based on an expected utility function, regret theory is based on non-expected utility function, first proposed by Bell [6] and Loomes and Sugden [7]. Compared to different non-expected utility theories and under uncertainty, regret theory is consistent with violations of transitivity and have been observed experimentally [8]. Other theories, such as the prospect theory, are not consistent with violations of transitivity [9, 10]. Many studies have tested the predictions of regret theory in a qualitatively.

The relationship between regret and decision process are intertwined. Larrick [11], Zeelenberg *et al.* [12], and Zeelenberg [13] have provided psychological evidence on the impact of regret on decision making under uncertainty. Although regret has not been analyzed in decision making for road construction in particular, as well as expected traffic flow and safety, it has been analyzed for its role in medical decision-making [14, 15]. In terms of construction cost, regret theory has been incorporated into models of asset pricing and portfolio choice by Gollier and Salanié [16] and Muermann and Volkman [17].

Regret theory has been used to explain stock market investments [18]. Hedging behaviour has also been shown to be due to regret [19]. Regret has also been found to explain disposition effect, where investors may sell winning stocks and hold losing ones [17].

Some researchers have argued in favour of the perspective validity of regret theory and intransitive choice [7, 20–22], while others oppose this argument due to the possibility of intransitive preferences. Nevertheless, in this paper, it is crucial to define a method that allows quantifying regret theory for practicality. Quantification of regret has been introduced in several ways. Some research suggests that a trade-off method similar to that used in prospect theory to quantify utility may also be used within regret theory [23].

Error propagation proposed by Fechner [24] has been modeled in decision theory by Hey and Orme [25] and successfully applied in behavioural game theory [26, 27].

2.1 Comparison of road design alternatives

Many models exist that can evaluate performance indicators for planning, traffic flow and safety, construction economics, and time. Those performance indicators can be, but are not necessarily objective driven. However, to reach an optimal decision, regret theory needs to be considered for each of the performance indicators.

Consider PI_{ij} is a performance indicator, such that the following may apply:
PI_{1j}: Performance Indicator for planning (e.g., land use);
PI_{2j}: Performance Indicator for traffic flow;
PI_{3j}: Performance Indicator for traffic safety;
PI_{4j}: Performance Indicator for construction economics;
PI_{5j}: Performance Indicator for time of completion.

Conceptually, this methodology can be applied to as many *m* performance indices as required. Since these performance indicators may be evaluated using different model techniques, they would need to be normalized, such that it would be comparable with one another. Nevertheless, each individual performance indicator must be applied to all the alternatives. This is essential to remove any bias in the evaluation of a performance indicator that may arise to the usage of different quantification methods between different models.

In this study, it is assumed that the larger the performance indicator, the better, and also assumes that performance indicators are positive values. However, since this may not be true to some existing models, which reverses the method. Then, for performance indicators where the smaller value is better shall be inversed, prior to normalizing each performance indicator for consistency purposes. The worst performance indicator is presumed to be zero.

Assuming there are *n*-number of alternatives to evaluate, normalizing between them would require the following step for each performance indicator converting it to a normalized performance indicator, NI_{ij}:

$$NI_{ij} = \frac{PI_{ij}}{MAX(PI_{ij})} \tag{1}$$

This would mean that each normalized performance indicator is bound by the following:

$$0 \leq NI_{ij} \leq 1.$$

Essentially, normalized performance indicators are only evaluated to express the relationship between different alternatives in the same study. Important to note, it cannot be used to compare between one alternative in one study group and another alternative from a different study group. Normalized performance indicators can be expressed in a matrix, where each column is an alternative showing the result of evaluating all performance indicators to it.

$$\begin{bmatrix} N_{11} & \cdots & N_{1n} \\ \vdots & \ddots & \vdots \\ N_{m1} & \cdots & N_{mn} \end{bmatrix}. \tag{2}$$

Models, whether planning, traffic, or others, are not perfect. They may simulate what the outcome may be but there is always an amount of uncertainty. Different models have different levels of uncertainty. The likelihood the model is correct may be determined through different criteria determined by the levels of uncertainty or statistics. This means there exists a probability, P_{ij}, that a positive outcome is reached for each of the performance indicators of every alternative. However, those probabilities would not necessarily be the same even if the same model is used to evaluate each alternative.

This is further expounded by the following example. If the parameters using a specific model give a high predictability for a four-legged intersection, its predictability of a roundabout may not be as accurate. Therefore, the probability that a positive outcome is reached, if the design for a four-legged intersection, is higher than when evaluating for a roundabout, using this specific model. Therefore, we can identify that different models may be biased. For example, a model can be liberal when analyzing a four-legged intersection and more conservative when analyzing a roundabout. A probability matrix for a positive outcome for each performance indicator can be expressed as follows:

$$\begin{bmatrix} P_{11} & \cdots & P_{1n} \\ \vdots & \ddots & \vdots \\ P_{m1} & \cdots & P_{mn} \end{bmatrix}. \tag{3}$$

Due to uncertainty, as there is a probability for a negative outcome, there is, therefore, a probability to regret the decision made defined as $(1 - P_{ij})$. In some models, the probability of not having a positive outcome may be known. Therefore, in such cases, decisions are under risk. However, most decisions are made under uncertainty.

Since different models have been tested and possibly calibrated, it is not fair to say that evaluation of different road design alternatives are under strict uncertainty. Thence, this study will mainly consider decisions under uncertainty, but not to the extent of ignoring known risks. Therefore, partial probabilistic knowledge of the outcomes is recognized. Within decision theory, a utility function is generated. This study focuses on the utility function, and more accurately on the expected utility function, which is based on a probability-weighted utility theory. The weight for each performance indicator is given expressed as follows:

$$w \in \begin{bmatrix} w_i & \cdots & w_n \end{bmatrix}. \tag{4}$$

Bell explains that regret is measured as the value difference between the assets actually received and the highest level of assets produced by other alternatives. However, regret can also be understood as the desire to avoid uncertainty [6].

This study approaches regret from a multivariate process, since there are different objectives, which may or may not conflict with each other. Although this study does not dwell into optimization, it is appropriate to identify that optimizing the performance indicators might be useful. Regret, however, is identified as the amount of gain that might have been expected for each performance indicator, if a different alternative was used.

The quality of a decision depends on the quality of alternatives and the ability to develop a detailed analysis for selection. However, as in any analysis, an analysis is only as good as the data inputs. If the data is not accurate or insufficient, the analysis will therefore be similarly inaccurate. When working with data, it is important to consider the macro-level and micro-level data for the selection analysis.

The process for evaluating alternatives starts by proposing an action. Then, it is necessary to define the objective of such an action, as in what is the purpose. Then to develop different alternatives, which are considered reasonable and meet the objective. The alternatives are then evaluated from a macro-level point of view, which include the planning and environmental models. This analysis will eliminate some of the alternatives. The remaining possible candidates are then analyzed in the micro-level, which include a full traffic study. Finally, the alternatives are given a score each according to the criteria and the weight given for each of the major factors.

If the expected utility function within the context of decision theory does not agree with the actual state of nature, θ, then regret is defined. Though models can be used to give us an expectation of the future state of nature, accuracy is always a concern.

There is a probability that the/ future state of nature (θ) would be worse than expected. This is not only due to unexpected planning and traffic flow, but also due to unexpected overrunning the budget of the cost of construction, and other factors.

Classically, regret is defined as the value lost, if another alternative has a higher yield for a specific performance indicator. Therefore, loss is evaluated as follows:

$$L_{ij} = N_{ij} - MAX\left\lfloor N_{ij} \right\rfloor. \tag{5}$$

This evaluation of loss assumes that the relationship between reward and risk are compared in linear form. Hence, the loss matrix is expressed as follows:

$$\begin{bmatrix} L_{11} & \cdots & L_{1n} \\ \vdots & \ddots & \vdots \\ L_{m1} & \cdots & L_{mn} \end{bmatrix}. \tag{6}$$

If the action of a specific performance indicator in an alternative is already the highest among other alternatives, then according to the equation above, it receives a value of *zero*, since no known opportunity in terms of regret is expected. However, this definition of regret is used loosely. In reality, this definition is restricted to the value that would have been gained for a specific performance indicator, if another alternative yielded a better result. However, when looking at regret, though this valuation is partially correct, it does not factor the probability that the model used to get the performance indicator will actually yield the expected values.

As discussed earlier, there is a probability that the results of road design may not yield the expected utility. For example, if an alternative is the best in each performance indicator, then according to the equation of regret above, it will have a value of *zero* regret. However, after completion of the construction, if the performance was lower than expected, then it would still be regretted. Also, the worse the actual state of nature (θ) is, the higher the regret. This means that regret in itself is a function in which the value would normally increase the lower the expected utility is.

For example, the performance indicator of cost is evaluated between two alternatives, A and B, with all other performance indicators equal. Alternative A costs $10,000,000 and alternative B costs $15,000,000. If alternative A was chosen, then according to the definition of regret above, the regret value for the cost performance indicator is *zero*. However, if during construction it is found that due to unexpected projections, the cost would overrun its budget by another $10,000,000, then the cost of alternative A is actually $20,000,000, which is $5,000,000 above the cost projected for alternative B. Assuming the cost of alternative B is not affected by the unexpected projections, then the choice of alternative A is regretted. Therefore, the definition of regret, as defined earlier, falls short from being accurate.

On another note, the higher the cost is overrun, the higher the value of regret. Thence, a probability function of expected values needs to be used to evaluate the value of the regret function. It is apparent that for all the parameters discussed in this research the worse an unexpected event occurs, the higher the value the regret. Therefore, regret is assumed as a continuous function and not necessarily discrete.

The definition of regret as stated earlier can be loosely expressed as the maximum amount of opportunity lost due to the existence of an alternative that might have been chosen. However, this denotes as comparing between different alternatives and no other. For this decision flow to make sense, it is strictly used as a comparison tool between different alternatives, and it assumes that if there is an unexpected event that might occur in the future, then it is as likely to occur in any of the alternatives equally. For example, if the traffic volume increases to unexpected values in the future, then it is assumed the same number would increase for any alternative.

Nevertheless, this assumption is flawed. If the road capacity is restricted and there are other road alternatives, then the volume of traffic may increase less compared to what would happen if the road has a higher capacity. The reality of this event depends on the driver's behaviour. If an alternative exists and can accommodate a higher capacity, more drivers would like to travel through it than on an alternative route; whereas, if the capacity of the road is restricted, more drivers would opt to choose an alternative route with a higher capacity. On another note, if the capacity of a road is restrictive, then development along the road will be restricted as well, and therefore, even if an increase of traffic volume occurs to unexpected values, it may not be as high as it would if the road design would have allowed more development.

Furthermore to this analysis, there could be an error in the computation of the expected performance indicators for each alternative. There is a possibility that none of the alternatives compared would have had a scenario for an unexpected event. Otherwise, it would not have been unexpected. Thence, there is room for regret due to this uncertainty.

As a solid definition of regret, it is the loss of the maximum opportunity. From a deterministic point of view, the actual future state of nature (θ) is compared with the assumed values that may still be objective-driven for each performance indicator. The value of each performance indicator can be re-evaluated at any future time and compared with the original assumed values. Once the values become known, regret is easily identified, given the initial priori conditional objectives.

The weighted value of the opportunity lost can be evaluated as regret and expressed as follows:

$$R_{ij} = w_{ij} \cdot \left(1 - P_{ij}\right) \cdot \left(L_{ij}\right) \tag{7}$$

This expression includes the value of regret due to an expected opportunity lost, when comparing different road design alternatives. It does not include the opportunity lost compared to the future state of nature (θ).

The true state of nature (θ) will only affect the regret function if the true state of nature (θ) exceeds the maximum expected value of the performance indicator in any of the alternatives, which would mean there is a higher value in the opportunity lost. Also, the true state of nature (θ) will also affect the regret function if the models used to evaluate the performance indicators are not accurate and therefore the values are skewed or were in error (ξ). Error continues to be assumed with linear relationship and as a continuous function for regret. The final regret matrix, based on a weighted probability of the opportunity lost based on eqn. (7) is represented as follows:

$$\begin{bmatrix} R_{11} & \cdots & R_{1n} \\ \vdots & \ddots & \vdots \\ R_{m1} & \cdots & R_{mn} \end{bmatrix} \tag{8}$$

3 Conclusion

Due to space limitation, there needs to be further study on the behavior of error propagation within the matrix and a model sensitivity along with a working example to be discussed. Any road project may have a direct influence on the planning of the area, which would also include the environmental aspects, such as the effects on the flora and fauna, as well as air quality issues, which is one of the most important aspects to consider in construction of roads. Different data and models have been introduced in this study and to illustrate the extensive horizon in the complexity of decision-making. This is to demonstrate how values obtained from these performance indicators using whichever model the decision-maker chooses are implemented in the regret model.

The main factors that have been a focus of this study are i) planning, ii) traffic flow, iii) traffic safety, iv) construction economics, and v) time. However, the regret model introduced does not necessarily only envelope those factors, neither does it require all the above factors to exist for a better evaluation. However, the decision-maker has the liberty to choose between the above factors or even add more as it would be deemed necessary. Each performance indicator is given a weight that provides its relative importance with other factors evaluated. It can be imagined that political leaders may add a performance indicator to the above that would include the attitude of the people towards such a project.

All decisions made can be regretted, including the decision of not making any. When evaluating alternatives, people regularly look at what would give them the highest gain. Those people are the optimistic and opportunistic type. However, this type does not necessarily be practical, because they would be interested in the highest gain no matter at what cost. A more practical method that investors do is by forming indicators that make them understand the relationship between cost versus benefit. Most investors calculate ratios, such as Return-on-Investment (ROI) index, to understand how much profit is expected from the investment they make. Other individuals seek the least loss. They are known as the pessimistic type, and that as well may not be the best method to evaluate investments on infrastructure using public funds.

In this study, a regret function has been formed to have a broader understanding of how the regret factor influences decisions in a more practical way for infrastructure investments involving public funds. The main concept behind this is that when dealing with infrastructure investments, flexibility of the design is very important for future expansions, in case they would require a higher capacity in the future. In general, the rule is if we do not want to regret a decision, or if we want to reduce the degree of regret, then the best decision would be the decision that we can easily change. This is very important, since traffic forecasting may not always be accurate. However, this statement is not to be said liberally, because it would still depend on the cost of making a flexible decision compared to the cost of being inflexible. Similarly in the business field, discounted tickets purchased are usually unchangeable.

The mathematical concepts of decision theory have been found to be the best quantitative method to be used in the study and include the regret factor in the decision-making process. A weighted regret matrix has been formulated that provides a systematic ranking system in the evaluation of various road construction designs. The synthesis of probability of potential outcomes into the formulation of the performance indicator provides a powerful tool in the evaluation of the performance indicators.

Due to uncertainties, it is very difficult to reach perfection in any decision. However, evaluating those uncertainties are the basis of formulating a regret matrix that allows decision-makers understand the complexity of the confidence level given to each performance indicator used in the assessment of alternatives.

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