Uncertainty in resistance models for historic cast-iron columns

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Abstract

Numerous processing and manufacturing mills, workshops, warehouses, bridges and other industrial buildings belong to industrial heritage. Their origin dates back to the 19th and 20th century when cast iron became a widely used construction material. It has been recognised that existing structures including cast-iron structures do not fulfil requirements of present codes of practice. A key step of reliability assessment is modelling of resistance of load-bearing members made of cast iron. The present paper investigates several empirical or physical models for resistance of historic cast-iron columns. Outcomes of the models are critically compared with experimental results obtained for solid and hollow cylindrical, and square columns from English grey cast iron. Imprecision of the models is expressed by means of model uncertainty for which appropriate probabilistic models are proposed. As tensile strength of cast iron is considerably lower than compressive strength, it dominates resistances of columns centrically loaded in compression with slenderness ratio over 60. In such cases model uncertainty can be described by a two-parameter lognormal distribution with the mean of 1.25 and coefficient of variation of 0.15. For columns with lower slenderness ratios compressive strength is decisive and the mean of model uncertainty decreases to 1.2.
Keywords: industrial heritage, cast-iron columns, reliability assessment, probabilistic methods.

1 Introduction

Numerous processing and manufacturing mills, workshops, warehouses, bridges and other industrial buildings belong to modern heritage, termed also as industrial

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heritage. Such structures are mostly of significant architectural, historic, technological, social, or scientific value [1]. Their origin dates back to the 19th and 20th century when cast iron became a widely used construction material [2].

It has been recognised that existing structures including cast-iron structures do not fulfil requirements of present codes of practice. Decisions about adequate construction interventions should be based on the complex assessment of a structure considering actual material properties, environmental influences and satisfactory past performance [3]. A key step of this assessment is modelling of resistance of load-bearing members made of cast iron [4].

That is why the present contribution investigates empirical or physical models for resistance of historic cast-iron columns. Outcomes of the models are critically compared with experimental results obtained for solid and hollow cylindrical, and square columns from English grey cast iron. Imprecision of the models is expressed by means of model uncertainty for which appropriate probabilistic models are proposed.

2 Model uncertainty

The concept of the model uncertainty proposed in [5–7] is adopted here. The uncertainties in resistance models are obtained from comparisons of physical tests and model results; real structure-specific conditions need then to be taken into account when they significantly deviate from test conditions. General framework of the uncertainty assessment for models of cast-iron columns with examples of influences affecting test and model results is given in Figure 1. Computational options seem to be irrelevant in this study since simple analytical models are considered.

Treatment of the test uncertainty was proposed in [6]. It was shown that unbiased test results with coefficient of variation around 0.05 can be assumed for tests of common reinforced concrete members. In the absence of statistical data these indications are accepted for cast-iron columns. The test uncertainty was proved to be of low significance and negligible when higher coefficient of variation of model uncertainty (say, greater than 0.1) is observed [6]. As this is the case in the present study, the test uncertainty is hereafter neglected.

If needed appropriate modifications of the model uncertainty such as increasing variability and/or adjustments of the mean value should be accepted to reflect real conditions of a structure (Figure 1). In most cases expert judgements are inevitable and general quantification of the effect of structure-specific conditions is hardly possible. Detailed discussion on structure-specific conditions is beyond the scope of this study.

The model uncertainty θ is here treated as a random variable. The multiplicative relationship for θ can be assumed [8]:

$$R(\boldsymbol{X},\boldsymbol{Y}) = \theta(\boldsymbol{X},\boldsymbol{Y})\, \mathrm{R}_{\mathrm{model}}(\boldsymbol{X}) \tag{1}$$

where R = response of a structure – real resistance estimated from test results; $\mathrm{R}_{\mathrm{model}}$ = model resistance – estimate of the resistance based on a model; $\boldsymbol{X}$ = vector

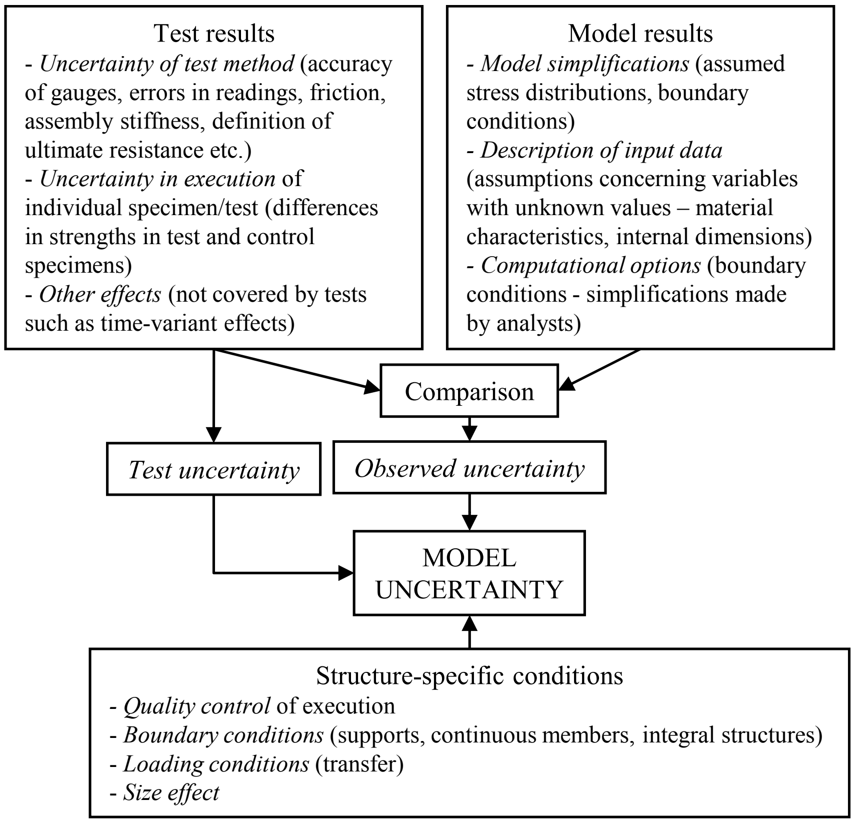

Figure 1: General framework of the model uncertainty assessment and examples of influencing factors for models considered in this study.

of basic (random) variables X_i included in the model; and $\boldsymbol{Y}$ = vector of variables neglected in the model, but possibly affecting the resistance. Modulus of elasticity is the example of a variable $\boldsymbol{Y}$ for some models for resistances of cast-iron columns.

In this study the model uncertainty is assessed using the following procedure [5, 7]:

(1) Compilation of a database of model uncertainty observations: – any design bias is excluded from the calculation of R_{model}, for instance real cast-iron strengths instead of characteristic values are applied – ranges of test parameters such as a slenderness ratio are made available to represent the sample space of experimental observations for which model uncertainty is investigated.

(2) Statistical assessment of the dataset including tests of unbiased sampling, outliers and goodness of fit of the probability distribution; in this study Grubb's test of outliers is performed to identify test results possibly affected

by errors, incorrect records, etc. considering a significance level of 0.05 [9, 10].

(3) Suitable probabilistic description of the model uncertainty; lognormal distribution with the origin at zero is commonly an appropriate probabilistic model [5, 7, 8] and is accepted in this study.

When generalising the model uncertainty beyond the scope of the database, trends in its mean and dispersion should be carefully considered. Extrapolation with respect to basic variables for which significant trends are observed may be dubious.

3 Resistance models for cast-iron columns

The resistance models provided in *EN 1993-1-1:2006* for design of steel structures can hardly be directly applied for cast-iron columns due to:

- Different stress-strain relationship of cast iron and mild steel,
- Missing yield stress of cast iron, and
- Lower tensile strength of cast iron as compared to its compressive strength.

Stress-strain diagram of cast iron is similar to that of aluminium or stainless steel.

Resistance of centrically loaded, cast-iron columns is primarily affected by fragility and susceptibility to brittle fracture without development of plastic deformation at higher slenderness ratios. Stability of columns with geometrical and material imperfections in connection with compressive and tensile strengths of cast iron should be adequately reflected in assessment of load-bearing capacity of cast-iron columns.

The imperfections are mostly caused by unknown technology of casting such as hand casting or forging. Due to casting in a horizontal position cross sections have inner eccentricities and different wall thicknesses. Together with lack of straightness these imperfections govern the stability of slender columns.

Model proposed in [11] determines strength σ_{model} of cast-iron columns exposed to buckling as a minimal value of its compressive σ_c and tensile strength σ_t:

$$\sigma_{\text{model}} = \min(\sigma_c; \sigma_t) \tag{2}$$

Tensile strength becomes decisive for columns with a high slenderness ratio.

Two models, denoted hereafter as *Approach 1* and *Approach 2*, can be used to estimate compressive strength. Using *Approach 1* [11], σ_c is obtained as:

$$\Sigma_c = \chi_c \times \sigma_{0.2} \tag{3}$$

where $\sigma_{0.2}$ = nominal strength based on the stress-strain curve proposed in [12]; and χ_c = slenderness reduction factor obtained similarly as recommended in *EN 1993-1-1:2006* with considerations for specific properties of cast iron. The nominal strength of 375 MPa is recommended for cast iron [11]. For low slenderness ratios, $\lambda < 25$, *Approach 1* numerically fails as the reduction factor exceeds unity. In such cases $\sigma_c = \sigma_{0.2}$ is here taken into account. However, these cases are of low practical significance.

Approach 2 [13] is valid for any slenderness ratio:

$$\sigma_c = 552 \text{ MPa} / (1 + \lambda^2 / 1600) \tag{4}$$

The tensile strength is assessed as follows [11]:

$$\sigma_t = \chi_t \times f \times \sigma_{0.2} \tag{5}$$

where χ_t = reduction factor accounting for slenderness ratio; and f = ratio between tensile and compressive strength. Equation (5) apparently takes basis in *Approach 1*. The representative value f = 0.2 is accepted in [11] as a conservative value for English grey iron. In practical cases it is recommended to derive a value of the parameter f from tensile tests.

Assuming $\sigma_{0,2}$ = 375 MPa, f = 0.2 and the reduction factors χ_c and χ_t according to [11], it can be shown that:

- For $\lambda \leq 37$, σ_c obtained by *Approach 1* is negligibly lower (by about 2%) than that based on *Approach 2*,
- For $37 < \lambda < 66.5$ *Approach 2* leads to σ_c-values lower than *Approach 1*; the maximum difference of 10% is observed for $\lambda \approx 50$; the difference vanishes with increasing slenderness ratio,
- A limiting value of slenderness ratio above which tensile strength becomes decisive for σ_{model} in Equation (2) is λ_{lim} = 55.7 for *Approach 1* and λ_{min} = 66.5 for *Approach 2*.

4 Database of experimental results

Uncertainty assessment for the considered models is based on comparison of test and model outcomes. Database of experimental results includes 72 tests of cast-iron columns with different slenderness ratios. The outcome of a test σ_{test} represents compressive stress corresponding to a force causing the failure of a specimen. All columns have been made of English grey iron with the expected content of carbon between 3.5–5% and small amount of additives. The content of carbon is dependent on a manufacturing process. The database is divided into three samples according to cross sections of the columns (Table 1).

Table 1: Database of experimental results.

Cross section	Sample size n	Slenderness ratio λ	Column strength (MPa)
Solid cylindrical	50	26–242	14.8–537
Hollow cylindrical	18	50.8–242	31.9–186
Solid square	4	154–204	24.2–43.6

The database includes solid and hollow cylindrical columns with slenderness ratios uniformly covering the range from 25 to 240 (Figures 2 and 3). The sample for solid square columns is small (n = 4); only specimens with high slenderness ratio are included. The database contains no information about cross-section characteristics, eccentricities and imperfections.

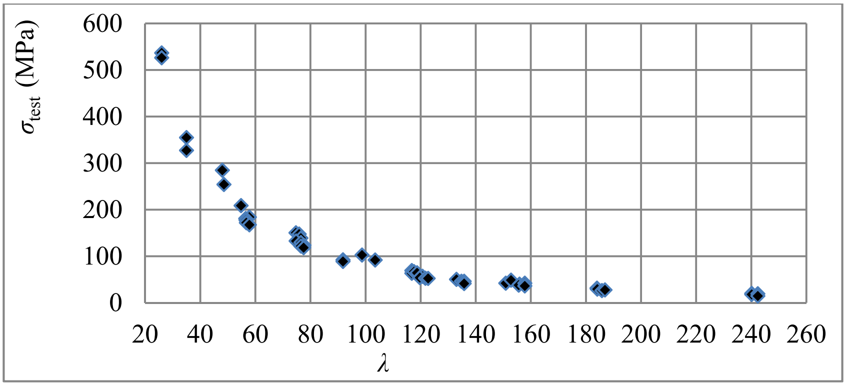

Figure 2: Variation of σ_{test} with λ for solid cylindrical columns.

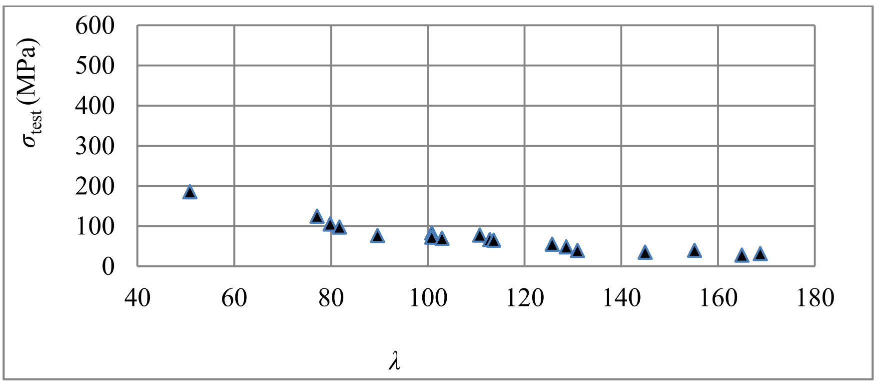

Figure 3: Variation of σ_{test} with λ for hollow cylindrical columns.

5 Statistical evaluation of model uncertainty

Model uncertainty values are obtained using Equation (1), $\theta_i = \sigma_{test,i}/\sigma_{c(t),i}$. Statistical parameters of model uncertainty based on the method of moments [10] are given in Table 2. Note that tensile strength is dominating strength according to Equation (2) for all hollow cylindrical and solid square columns as $\lambda \geq 50.8$.

It follows from Table 2 that the model for tensile strength given in Equation (5) is more conservative ($\mu_\theta \approx 1.12$–1.43) than *Approaches 1 and 2* for compressive strength ($\mu_\theta \approx 1.11$–1.18). This indicates that the considered value $f = 0.2$ be inappropriate for the investigated database and should be revised. Dispersion of model uncertainty as expressed by its coefficient of variation ranging mostly between 0.1 and 0.15 corresponds well to buckling resistance of steel columns [14, 15]. However, the sample sizes for *Approaches 1 and 2* and solid cylindrical columns and for tensile strength and solid square columns are small and obtained characteristics of model uncertainty should be considered as indicative only.

Table 2: Statistical characteristics of model uncertainty θ.

Cross section	Model	Sample size	λ	Mean μ_θ	Coefficient of variation V_θ
Solid cylindrical	*Approach 1* (σ_c)	7	26.5–55.7	1.18	0.13
	σ_t*	43	55.7–242.4	1.2	0.14
	Approach 2 (σ_c)	12	26–66.5	1.11	0.13
	σ_t**	38	66.5–242.4	1.24	0.11
Hollow cylindrical	σ_t	18	50.8–242.4	1.12	0.11
Solid square	σ_t	4	153.8–204.1	1.43	0.08

*Combined with *Approach 1*. **Combined with *Approach 2*.

Taking into account the limited amount of data, the following recommendations are provided on the basis of the results given in Table 2:

- Model uncertainty characteristics $\mu_\theta \approx 1.2$ and $V_\theta \approx 0.15$ should be considered when compressive strength is decisive in Equation (2),
- $\mu_\theta \approx 1.25$ and $V_\theta \approx 0.15$ should be considered when tensile strength is governing resistance of a cast-iron column.

These characteristics can be directly applied when deriving model uncertainty factor for assessments using the partial factor method as provided in *EN 1990:2002* for basis of structural design [16, 17].

Figure 4 shows variation of model uncertainty values with slenderness ratio for solid cylindrical columns. *Approaches 1 and 2* seem to be conservative particularly for low slenderness ratios, $\lambda < 30$. However, these cases are rare in practical situations. In most cases $\lambda > 70$ applies and the model for tensile strength is decisive for resistance of columns. Figure 4 indicates that this model may be also conservative with considerable dispersion of outcomes. The conservative bias may be reduced by specifying an appropriate value of the ratio *f*. The dispersion is attributed to varying effects of eccentricities and imperfections that seem to be inadequately taken into account by the model for tensile strength. A more advanced model is proposed in [18]. The considered models may overestimate real resistances for $55.7 < \lambda < 66.5$ when compressive and tensile strengths become comparable.

6 Defects of cast-iron columns

Real conditions of cast-iron columns may be different from those included in the test database. Ultrasonic methods are commonly applied to detect imperfections, cracks or cavities in cast-iron columns. Particularly the phased-array method is an efficient tool. Common defects of cast-iron columns include [19]:

- Shrinkage defects,
- Gas porosity comprising nitrogen blowholes or hydrogen pinholes,
- Pouring metal and metallurgical defects,
- Slag and sand inclusions.

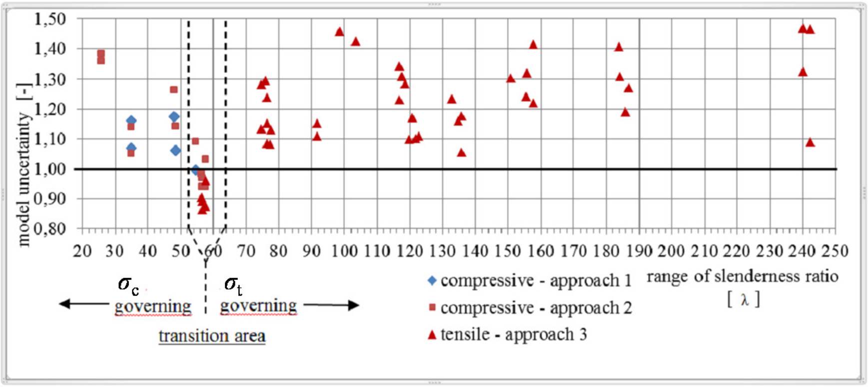

Figure 4: Variation of model uncertainty with slenderness ratio for solid cylindrical columns.

These defects should be then included in the assessment of load-bearing capacity of a column. Typically, measures mitigating some of these defects have caused the others insufficiencies. However, detailed discussion of defects and their consideration in structural analysis is beyond the scope of this contribution.

7 Concluding remarks

Reliability assessments of historic cast-iron columns should be supported by inspection including collection of appropriate data. Imperfections, cracks or cavities of columns should be investigated and identified defects should be adequately considered in reliability analysis. Uncertainties in resistance models can become a crucial aspect of reliability verifications.

As the tensile strength of cast iron is considerably lower than compressive strength, it is a variable dominating resistances of centrically loaded columns with slenderness ratio over 60. In such cases model uncertainty can be described by a two-parameter lognormal distribution with the mean of 1.25 and coefficient of variation of 0.15. For columns with lower slenderness ratios compressive strength is decisive and the mean of model uncertainty decreases to 1.2.

Further research should be focused on uncertainties in resistance of columns exposed to eccentric forces and investigation of uncertainties related to advanced numerical models (such as the Finite Element Methods).

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