The vehicle routing problem in urban networks: an approach based on a network fundamental diagram

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Abstract

This paper presents a formulation of the vehicle routing problem (VRP) based on the concept of a network fundamental diagram (NFD). The proposed model for estimating the (weighted) link costs considers variables describing the average traffic conditions related to homogeneous portions of the network. They are synthetized in a NFD. The proposed VRP computes the optimum path between each couple of clients by taking into account link costs and average values of density, which may be considered as proxies of travel time reliability. An application to a real dimension network allowed the testing of the proposed method.
Keywords: vehicle routing problem, network fundamental diagram, time-dependent link costs, genetic algorithm.

1 Introduction

The vehicle routing problem (VRP) has been object of several studies in literature and many solution procedures were proposed in last years (see [1]). Dantzig and Ramser [2] presented in the seminal paper the first VRP formulation: the problem was called a truck dispatching problem and the aim was to mimimize the total distance traveled by trucks. Later, the problem was extended and reformulated to

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take into account different elements related with the freight delivery/pick-up operations. As an example (the papers in this field are several, in this work only few oh them are recalled), some problem formulations consider constraints in the freight delivery (pick-up) time period [3, 4]. In other formulations [5, 6], travel demand can change during the travel time period (may be necessary to change route), or the delivery and pick-up operations are made jointly [7, 8].

In many cases the proposed approaches are based on static estimation of costs (in term of travel time), and only in few cases the estimation of costs is based on traffic flow theory and/or on traffic system simulation in real time [9–11]. Advancements in VRP formulation consider time-dependent evolution of the network [12–14], where link costs (to use as input in VRP) are obtained by means of a (static or dynamic) assignment procedure.

The paper proposes a formulation of the VRP based on the concept of network fundamental diagram (NFD). In traffic flow theory, the NFD [15, 16] correlates global values of vehicular flows and densities. In some cases, the assignment models [17, 18] may support the estimation of link costs, considering also their structural conditions [19].

An enhancement proposed in this paper concerns the definition of NFDs associated to a homogenous portions of the network, obtained after a zoning of the study area. The aim is to estimate an average value of density related to each portion, which may be considered as a proxy of travel time reliability. Moreover, link costs are evaluated for each time slice (e.g. 10 minutes) of the total period of analysis (e.g. hours). The VRP is formulated in order to minimize the total travel time, as the sum of (weighted) link travel times estimated by taking into account the above elements. An application on a real dimension problem is proposed in order to evaluate the method.

The paper is articulated as follows. Section 2 reports the VRP formulation. Section 3 presents an application to an urban network of real dimensions. The last section concerns the research perspectives.

2 Problem formulation

This section presents the proposed problem formulation reporting the assignment procedure, the link costs formulation and the routing problem.

The main inputs of our formulation are (Figure 1): the *transport network*, a study area divided into homogenous *zones*, a *NFD* for each portion of the network. The NDFs are estimated from data obtained from a microscopic traffic simulator (the expected value of link cost is provided). *Link costs* are estimated; considering, for each zone, the NFD and the time slice. Particularly, link costs are a function of the expected costs and of a variance depending on the cost distribution in the time. Then, a *path search* algorithm (Dijkstra algorithm) finds the *best paths* on the network, considering that the path cost depends on time slice and zone. Finally, a *vehicle routing* optimization model identifies the *best routes* for freight vehicles considering the time variability of path costs with a discrete approach. In [20] the problem is solved considering continuous cost functions.

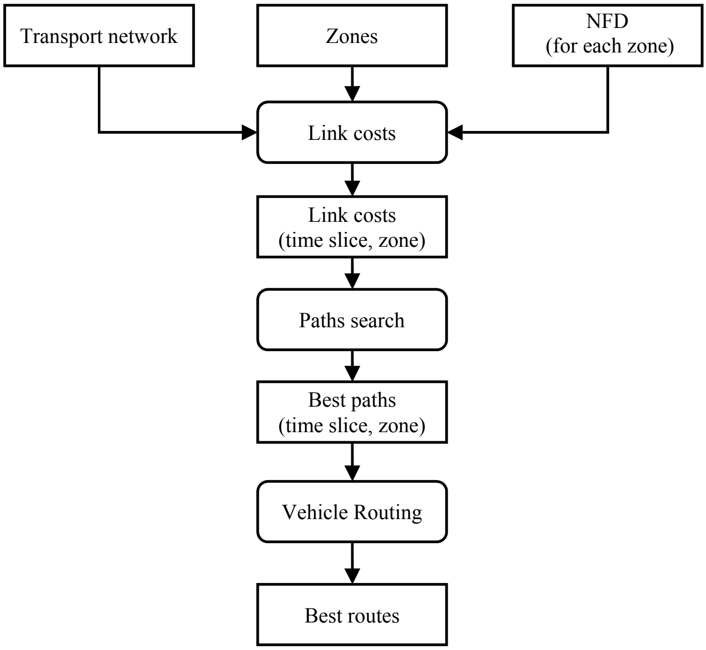

Figure 1: Flowchart of the proposed formulation.

2.1 Network Fundamental Diagram (NFD)

Two classes of vehicle are considered: cars and freight vehicles. Four indicators are specified in order to build the NFD [15, 16, 21]:

- Total travel distance (TTD)

$$TTD = \Sigma_i f_j \cdot l_j \qquad j \in A$$

- Total number of vehicles (TNV)

$$TNV = \Sigma_j k_i \cdot l_j \qquad j \in A$$

- Average flow (AF)

$$AF = \Sigma_j f_j \cdot l_j / L \qquad j \in A$$

- Average density (AD)

$$AD = \Sigma_j k_j \cdot l_j / L \qquad j \in A$$

with (for each link j):
f_j, vehicular flow;
k_j, vehicular density;
l_j, link length;
$L=\Sigma_j l_j$, total network length;
A, link set.

The above indicators can be evaluated for the entire network or for a portion of it; in the second case links j considered in the indicators evaluation belong to a specific zone ($j \in A'$ with $A' \subseteq A$).

It is possible to define a critical value of AD (AD^{cr}_z in Figure 2) as the value of the density corresponding to the maximum value of flow AF^{max}_z. As the NFD may be related to a zone of the urban area, it is possible to define a set $B=\{\ldots AD^{cr}_z, \ldots\}$ containing the values of critical density for each zone $z \in Z$ (being Z the set of the zones).

Moreover, for each time slice, each zone is characterized by a couple of flow-density values in the NFD (an example is reported in the Figure 2).

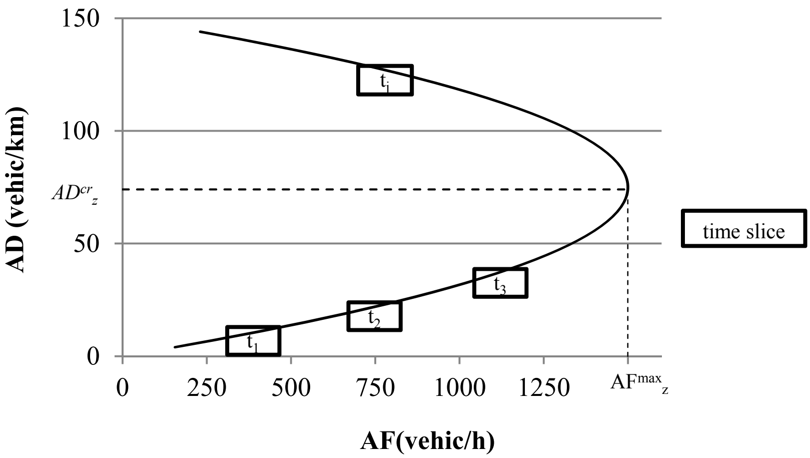

Figure 2: Schematic plot of the NFD.

2.2 Link costs

The dynamic traffic assignment (DTA) model provides, for each link j and for each time slice m, a value of the expected link cost $\overline{c_{mj}}$ ($\overline{c_{mj}}$ is the expected value of the link cost distribution).

The weighted link cost is evaluated as a function (φ) of the expected value of the cost $\overline{c_{mj}}$ and the relative variance σ_{mj}, assumed as a function of the link density k_{mj}:

$$\widehat{c_{mj}} = \varphi(\overline{c_{mj}}, \sigma_{mj}, k_{mj}) \qquad (1)$$

A specification of eq. (1) may be reported in the following:

$$\widehat{c_{mj}} = \overline{c_{mj}} \cdot \left(1 + \beta_1 \cdot k_{mj} / k^{cr}_j + \beta_2 \cdot AD_{mz} / AD^{cr}_z\right) \tag{2}$$

where
k_{mj} is the density of link j in the time slice m;
k^{cr}_j is the critical density according to the macroscopic flow-density diagram of link j;
AD_{mz} is the average density for network portion of zone z in time slice m according to the NFD;
AD^{cr}_z is the critical density for network portion of zone z according to the NFD;
β_1 and β_2 are parameters.

In time slice m, the term $\beta_1 \cdot k_{mj}/ k^{cr}_j$ of Eq. (2) takes into account the reliability of link cost due to the local traffic conditions on the link; the term $\beta_2 \cdot AD_{mz}/AD^{cr}_z$ takes into account the reliability of the link cost due to the average traffic conditions related to the portion of the network belonging to the zone.

According to this approach, it is necessary to estimate a value of link cost for each time slice. This means that when a vehicle travel on the network if changes the time slice it is necessary to re-computing the shortest paths.

2.3 Routing

The vehicle routing problem is formulated with the aim of minimizing the total travel cost for freight vehicles. The assumptions are:

1) weighted link costs are estimated by means of Eq. (2),
2) path cost is weighted during the travel, considering the variation of the link costs varying the time slice.

In this way, the cost $g_{k(r,s)}$ of a path $k_{(r,s)}$ between origin r and destination s is:

$$g_{k(r,s)} = \Sigma_j \delta_{kjm} \cdot \zeta(\widehat{c_{mj}}) \tag{3}$$

where:
δ_{kjm} is equal to 1 if the link j belong to the path k, in the time slice m; 0 otherwise;
$\zeta(.)$ is a function to identify the time slice in which estimate the link cost.

The formulated objective function is an extension of the one presented in [14]:

$$\Phi(X) = \Sigma_v(\Sigma_r \Sigma_s (g_{k(r,s)}) \cdot x_{k(rs)v}) \tag{4}$$

where:
v is a generic vehicle;
(r, s) is the origin destination pair;
$x_{k(rs)v}$ is the design binary variable (is equal to 1 if the vehicle v use the path $k_{(r,s)}$ 0 otherwise).

The constraints of the problem, as reported in [11], are:

- only one vehicle can visit a node;
- all vehicles leave from the depot and come back to it;
- capacity constraint;
- the variable $x_{k(rs)v}$ can assume the value zero or one.

3 Application

This section reports an application of the routing problem (4) to the network of Villa San Giovanni, a town in the South Italy. The network has 359 links and 136 nodes (Figure 3). The study area is subdivided into three zones (A, B and C): a NFD (Figure 4) and a critical value of critical density AD^{cr}_z are available [21] for each zone z.

Figure 4 shows the scatterplots between the estimated values of AF and AD for each of the three zones of the study area. Moreover, the values of AD^{cr}_z are reported. It emerges that zone B presents values of AF and AD^{cr}_z greater than the ones of zones A and C. This is due to the fact that zone B covers the central part of the town with higher levels of congestion.

The origin-destination demand matrix is obtained by means of a survey [22]; average trips per hour in the study area are 3631 (vehicles/hour), with 1627 trips with internal origins and destinations and 469 trips having external origins and/or destinations.

The clients to be served are 28 (named from 2 to 29 in Figure 4) and they are located in the three zones of the study area. The depot is in the node 1. The optimization procedure is based on a genetic algorithm, assuming a fleet of two homogeneous freight vehicles that travel from the depot to the clients and three time slices. A Dijkstra algorithm is used to find the optimum path between each couple of clients [23].

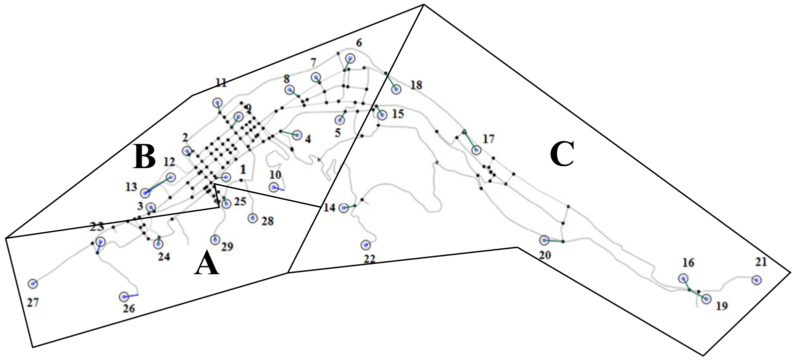

Figure 3: Urban network of Villa San Giovanni and identification of three zones.

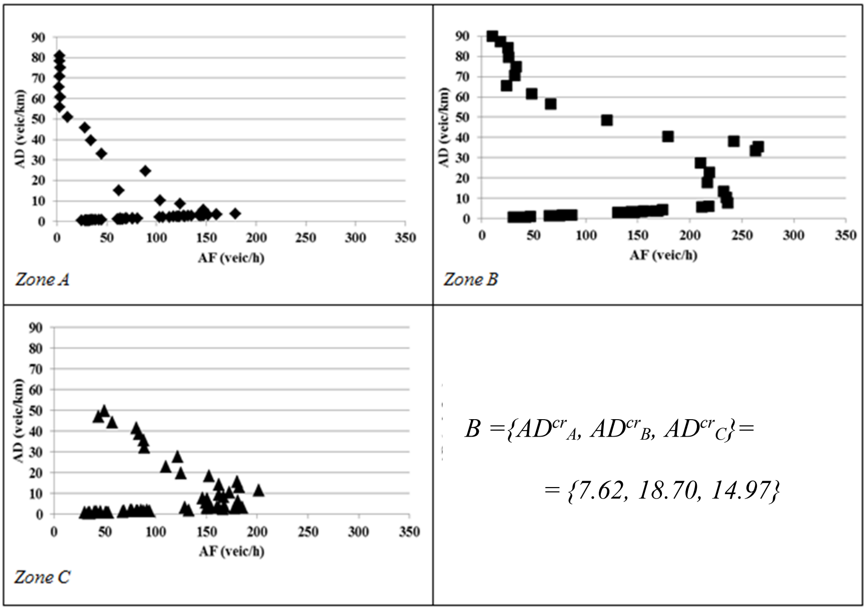

Figure 4: NFD for each zone and values of set *B* [21].

The values of AD_{mz} are estimated for each zone *z* and time slice *m*. The value of AD_{mz} is major or minor than the critical value AD^{cr}_{z}, according to the NFD defined for each zone. The preliminary results obtained are reported in Table 1.

The optimal solution consists of two routes with sequence of clients, expected and weighted costs presented in Table 2.

The optimization procedure tends to find a solution in which clients belonging to a zone *z* are visited in a time slice where $AD_z < AD^{cr}_z$.

Table 1: Average density vs. critical density for each time slice and zone.

Zone *z* \ Time slice *m*	1	2	3
A	$AD_{1A} < AD^{cr}_A$	$AD_{1A} \geq AD^{cr}_A$	$AD_{1A} < AD^{cr}_A$
B	$AD_{1B} \geq AD^{cr}_B$	$AD_{1B} \geq AD^{cr}_B$	$AD_{1B} < AD^{cr}_B$
C	$AD_{1C} < AD^{cr}_C$	$AD_{1C} \geq AD^{cr}_C$	$AD_{1C} < AD^{cr}_C$

The path generated with the expected time (e-path) is different from the path generated with the weighted time (w-path). Considering the expected travel time, it is lower in the e-path than the w-path (about 3%); considering the weighted travel time, it is higher in the e-path than the w-path. The use of w-path guarantees a bit higher expected travel time but it is more reliable in relation to the flow instability. Paths generated confirm it. Table 3 presents an analysis that correlate clients, zones and time slices.

Table 2: Optimal routes (sequence of clients and costs).

Optimization with	*Route*	*Client sequence*	*Travel time (s)*	*Travel and delivery (s)*
Expected costs (s)	1	27-26-23-24-3-13-12-2-11-9-4-10-29-28-25	804.51	6078.51
	2	15-18-17-21-19-16-20-22-14-5-6-7-8	1030.09	5428.09
Total (s)			**1834.60**	**11506.60**
Weighted costs (s)	1	28-25-29-24-26-27-23-3-13-12-2-11-9-8-7-6	1015.65	6655.65
	2	22-14-17-20-19-21-16-15-18-5-4-10	1256.34	5288.34
Total (s)			**2271.99**	**11943.99**

Focusing on zone A, there is no change in relation to the number of nodes visited in each time slice. Focusing on zone B, the e-path connects eight nodes (3-13-12-2-11-9-4-10) in time slice 3 with stable flow conditions and four nodes (5-6-7-8) in time slice 1 with instable flow conditions. In the same zone, the w-path connects all nodes in time slice 3 with stable flow conditions (9 nodes with route 1 and 3 nodes with route 2).

Focusing on zone C, the e-path connects three nodes (17-21-19) in time slice 2 with instable flow conditions and six nodes (15-18-16-20-22-14) in time slices 1 and 3 with stable flow conditions. In the same zones, the w-path connects only two nodes in the time slice 2 with instable flow conditions.

Table 3: Scenario 2: visited clients per zone and time slice.

	Zone	A			B			C		
	Time slice	1	2	3	1	2	3	1	2	3
Expected costs	Clients of route 1	27-26	23-24	29-28-25			3-13-12-2-11-9-4-10			
	Clients of route 2				5-6-7-8			15-18	17-21-19	16-20-22-14
Weighted costs	Clients of route 1	28-25	29-24	26-27-23			3-13-12-2-11-9-8-7-6			
	Clients of route 2						5-4-10	22-14	17-20	19-21-16-15-18

4 Conclusions

This paper proposes a formulation of the vehicle routing problem based on the network fundamental diagram. A set of indicators describes the average traffic conditions on portions of the network according to a NFD. The link cost is function of an expected cost (provided by a micro-simulation model) and a weighted cost (related with the cost variance). A cost function to evaluate the link cost is specified, considering in the formulation the density indicators related to the links and the zones. The aim of this formulation is to take into account the reliability of the link cost in terms of average density of link and zone.

Finally, an application in a real dimension network allowed to evaluate the effectiveness of the proposed method. A NFD for each of the three zones is estimated in order to determine the traffic regime (stable or unstable) of the zone in each time slice. A set of clients to be served is considered and a vehicle routing problem is solved by means of a genetic algorithm.

For each time slice, it is evaluated if a zone is in the stable or unstable regime comparing the density of the zone with the critical density according to the estimated NFD. Moreover, for each scenario, the solution obtained by considering the expected link costs is compared with the one obtained by considering the weighted link costs. The preliminary results demonstrate that the optimized routes, in general, tend to avoid heavily congested links and portions of the network.

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